Aula 2 - Limites laterais e limite de função composta

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$f(x) = x$ e $g(x) = 1/x$

$$\lim_{x \to 0} \left(f(x) \cdot g(x)\right) = 1$$

$$\lim_{x \to 0} f(x) = 0$$

$$\lim_{x \to 0} g(x) \nexists .$$

- Considere a função $f : \mathbb{R} \to \mathbb{R}$ dada por $f(x) = 3x + 2$. Neste caso,

$$\lim_{x \to 1} f(x) = f(1) = 5$$

$$\lim_{x \to 1} f(x) = f(1) = 3 \cdot 1 + 2 = 5$$

Considere a função $f : (0, \infty) \to \mathbb{R}$ dada por $f(x) = x^2 + e^x - 3 \ln x$. Neste caso

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} [x^2 + e^x - 3 \ln x]$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \left(x^2 + e^x - \boxed{3 \ln(x)} \right)$$

$g(x) = x^2$, $h(x) = e^x$, $J(x) = 3 \cdot \ln(x)$

$m(x) = \ln(x)$

$$\lim_{x \to 2} m(x) = \ln(2)$$

$$\lim_{x \to 2} g(x) = g(2) = 4$$

$$\lim_{x \to 2} h(x) = h(2) = e^2$$

$$\lim_{x \to 2} J(x) = J(2) = 3 \cdot \ln(2)$$

$$\Rightarrow \lim_{x\to 2}\left(x^2+e^x-3\ln(x)\right)=\lim_{x\to 2}x^2+\lim_{x\to 2}e^x+\left(\lim_{x\to 2}-3\ln x\right)$$

$$=4+e^2-3\ln 2$$

$$h(x)=\frac{sen(x)}{x^2+1}$$

$$\lim_{x\to \pi/2}\left(\frac{\operatorname{Sen}(x)}{x^2+1}\right)=\frac{1}{\frac{\pi^2}{4}+1}$$

$$f(x)=\operatorname{Sen}(x) \rightsquigarrow \lim_{x\to\pi/2}f(x)=f(\pi/2)=\operatorname{Sen}(\pi/2)=1$$

$$g(x)=x^2+1 \rightsquigarrow \lim_{x\to\pi/2}g(x)=g(\pi/2)=\frac{\pi^2}{4}+1\neq 0$$

Note que não podemos aplicar a **regra** do quociente em

$$\lim_{x\to -1}\frac{x^3+1}{x^2+4x+3}$$

$$\lim_{x\to -1}(x^3+1)=(-1)^3+1=0$$

$$\lim_{x\to -1}(x^2+4x+3)=(-1)^2+4(-1)+3=0$$

$$\rightsquigarrow p(x)=x^3+1 \quad \rightsquigarrow p(-1)=0$$

$$q(x)=x^2+4x+3 \rightsquigarrow q(-1)=0$$

$$\rightsquigarrow \quad p(x) = x^3 + 1 = (x+1)(x^2 - x + 1)$$

$$q(x) = x^2 + 4x + 3 = (x+1)(x+3)$$

$$\boxed{f(x)} = \frac{x^3+1}{x^2+4x+3} = \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}(x+3)} = \frac{x^2-x+1}{x+3} = \boxed{g(x)}$$

$f(x) = g(x)$,

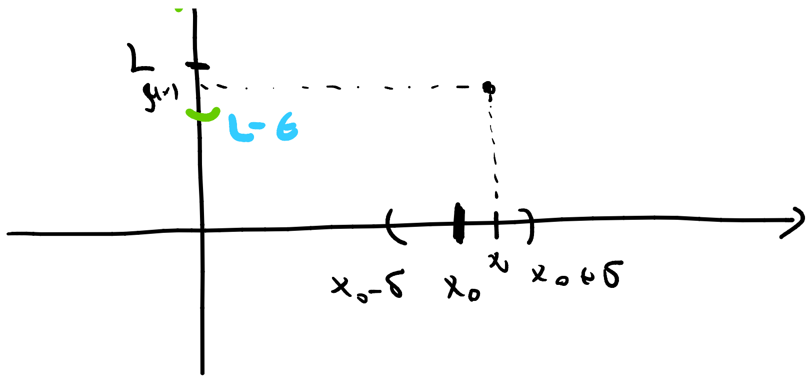
$\forall x \in (-2, 0)$, com $x \neq -1$

$$\lim_{x \to -1} \frac{x^3+1}{x^2+4x+3} \overset{!!!}{=} \lim_{x \to -1} \left(\frac{x^2 - x + 1}{x+3} \right) = \frac{3}{2}$$

- $\lim_{x \to -1} x^2 - x + 1 = (-1)^2 - (-1) + 1 = 1 + 1 + 1 = 3$

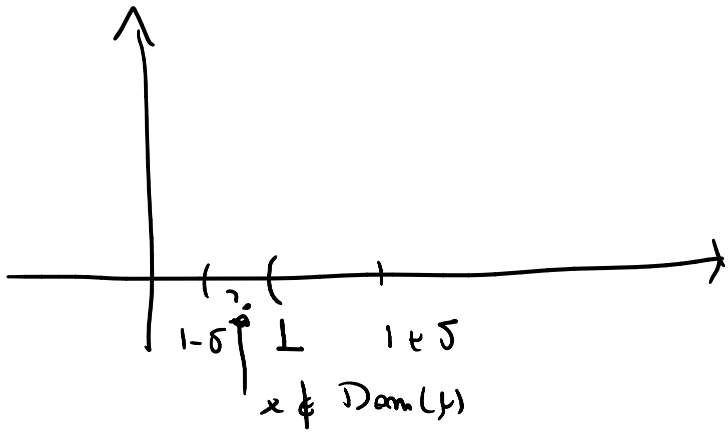
- $\lim_{x \to -1} x + 3 = (-1) + 3 = 2 \neq 0$

$$\lim_{x \to x_0} f(x) = L$$



$$f : (1, +\infty) \to \mathbb{R}, \qquad f(x) = \frac{1}{x-1}$$

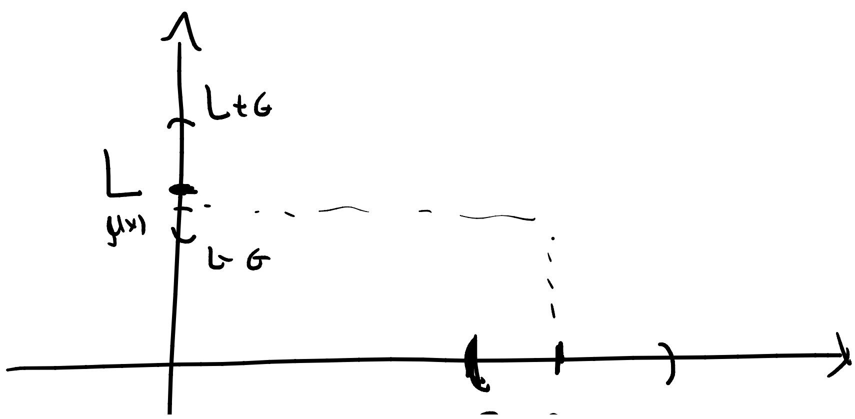
$$\lim_{x \to 1} f(x)$$

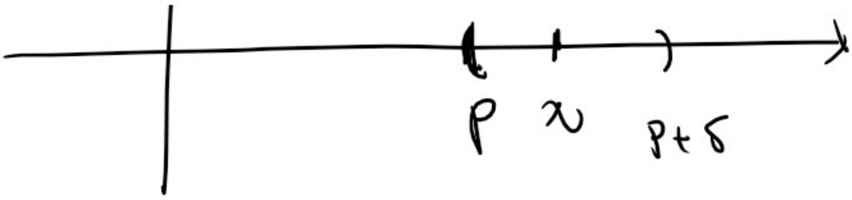


Ou, de modo equivalente,

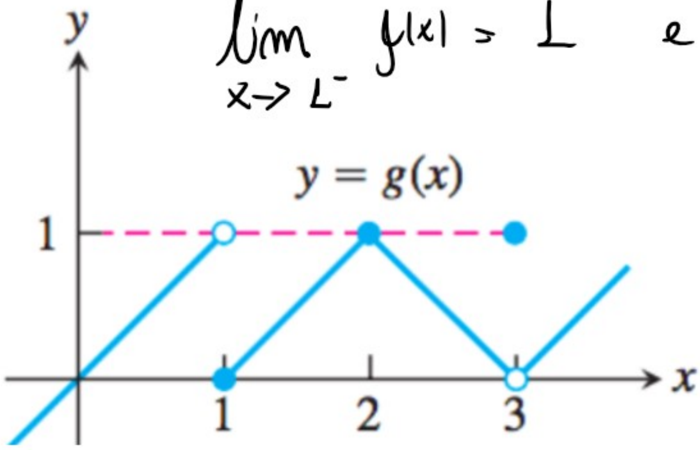
(⋆⋆) dado (qualquer) $\epsilon > 0$, existe $\delta > 0$ tal que

$$x \in (p, p+\delta) \implies f(x) \in (M-\epsilon, M+\epsilon).$$





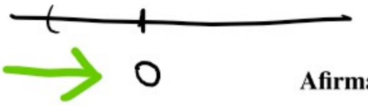
$$\lim_{x \to 1^-} f(x) = 1 \quad e \quad \lim_{x \to 1^+} f(x) = 0$$



Considere a função $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ dada por

$$f(x) = \frac{|x|}{x}$$

$$|t| = \begin{cases} t, & t \geq 0 \\ -t, & t < 0 \end{cases}$$



Afirmação: Não existe $\lim_{x \to 0} f(x)$.

$$\lim_{x \to 0^+} \frac{\boxed{|x|}}{x} = \lim_{\boxed{x \to 0^+}} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$$

$x > 0$

$$\lim_{x \to 0^-} \frac{\boxed{|x|}}{x} = \lim_{x \to 0^-} \frac{\boxed{-x}}{x} = \lim_{x \to 0^-} -1 = -1$$

$$\lim_{x \to 0} sen\left(x^2 + \pi/2\right)$$

$$f \circ g(x) = f(g(x)) = \text{Sen}\left(x^2 + \pi/2\right)$$

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = \text{Sen}(x)$

$g : \mathbb{R} \to \mathbb{R}$, $g(x) = x^2 + \pi/2 \Rightarrow \lim_{x \to 0} \left(x^2 + \pi/2\right) = \pi/2$

$$\Rightarrow \lim_{x\to 0} \operatorname{Sen}(x^2+\pi/2) = \operatorname{Sen}\left(\lim_{x\to 0} x^2+\pi/2\right)$$
$$= \operatorname{Sen}(\pi/2)$$
$$= 1$$

$\lim_{x\to 0} \ln(x+1)$

$$\ln(x+1) = f(g(x))$$

$$f : (0,+\infty) \to \mathbb{R}, \quad f(x) = \ln x$$

$$g : (-1,+\infty) \to \mathbb{R}, \quad g(x) = x+1 \qquad \lim_{x\to 0} (x+1) = 1 \in \operatorname{Dom}(f)$$

$$\Rightarrow \lim_{x\to 0} \ln(x+1) = \ln\left(\lim_{x\to 0} x+1\right) = \ln(1)$$
$$= 0$$

$\lim_{x\to 0} \ln(x+1)$ $\rightsquigarrow$ $\lim_{x\to -1} \ln(x+1)$

$$\rightsquigarrow \lim_{x\to -1} (x+1) = 0$$

$$\ln(0)\ ?$$