Processo Seletivo Estendido 2016

LISTA FUNÇÕES - 6

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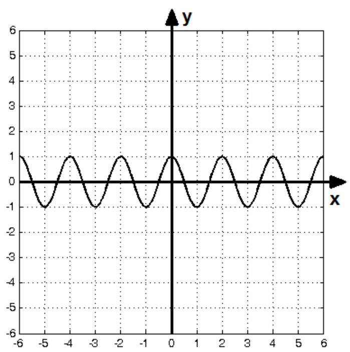
Esta lista foi elaborada pelo professor Lucas Pedroso (UFPR).

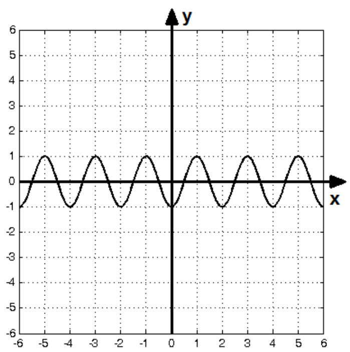
- Nesta lista de exercícios há problemas algébricos e também de modelagem matemática. Em ambas situações o objetivo é recordar e aprofundar o que foi visto no ensino médio a respeito de funções. Alguns tópicos mais diretamente relacionados ao assunto serão também trabalhados
- Quando julgar necessário, utilize uma calculadora, um computador, ou mesmo uma planilha, para fazer estimativas que deem a você uma ideia numérica.
- Matemática é algo que também se aprende junto com outras pessoas. Por isso, discuta em grupo, pesquise e debata suas ideias com os colegas.
- Mais importante que conseguir resolver uma questão é pensar e refletir sobre ela.

Suponhamos que conhecemos o gráfico de uma função f. Dados $A, B, C, D \in \mathbb{R}$, $B, C \neq 0$, queremos saber como é o gráfico de $Cf(Bx + A) + D$. As etapas de 1 a 4 devem ser seguidas na ordem que são mostradas abaixo. Usaremos como exemplo a função $f(x) = \cos(\pi x)$. Vamos, ao final, representar os gráficos de $-\frac{1}{2}f(2x + 1) - 2$ e $2f(-\frac{1}{3}x - \frac{1}{2}) + 1$.

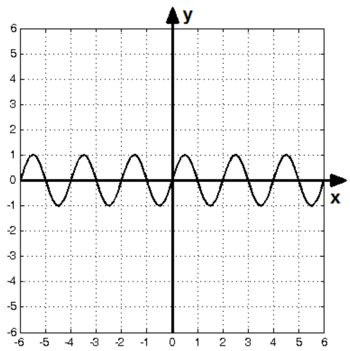


$f(x)$

Passo 1: Deslocar o horizontalmente o gráfico de f em $|A|$ unidades para a esquerda se $A > 0$ ou para a direita se $A < 0$.

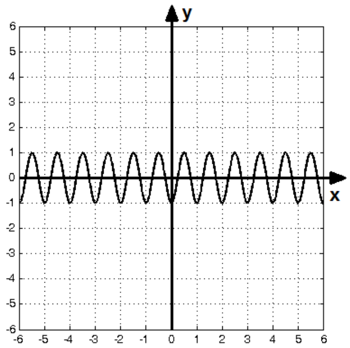


$f(x + 1)$

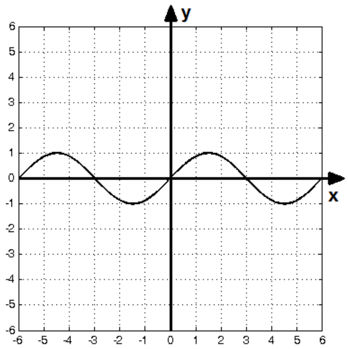


$f(x - \frac{1}{2})$

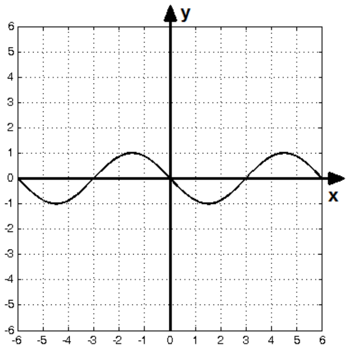
Passo 2: Partindo do gráfico do passo 1, comprimir o gráfico no sentido do eixo x por um fator $|B|$ se $|B| > 1$ ou expandi-lo por um fator $\frac{1}{|B|}$ se $|B| < 1$. Depois disso, se $B < 0$, refletir o gráfico em relação ao eixo y.



$f(2x + 1)$

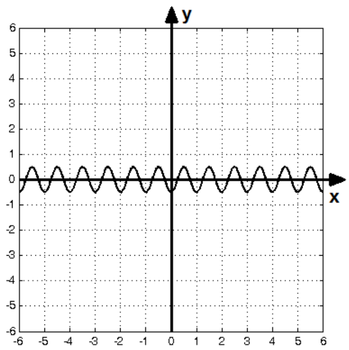


$f(\frac{1}{3}x - \frac{1}{2})$

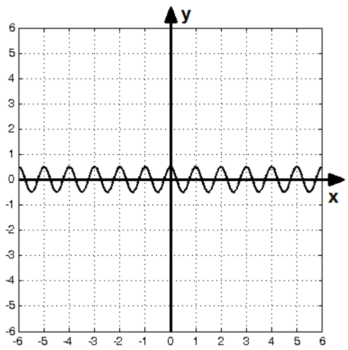


$f(-\frac{1}{3}x - \frac{1}{2})$

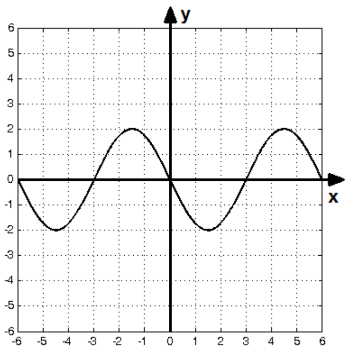
Passo 3: Partindo do gráfico do passo 2, expandir o gráfico no sentido do eixo y por um fator $|C|$ se $|C| > 1$ ou comprimi-lo por um fator $\frac{1}{|C|}$ se $|C| < 1$. Depois disso, se $C < 0$, refletir o gráfico em relação ao eixo x.



$\frac{1}{2}f(2x + 1)$

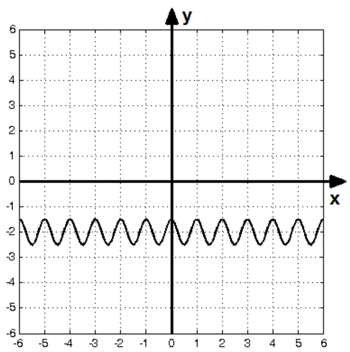


$-\frac{1}{2}f(2x + 1)$

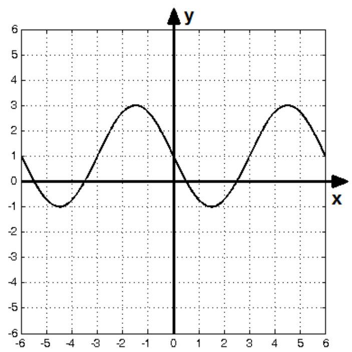


$2f(-\frac{1}{3}x - \frac{1}{2})$

Passo 4: Partindo do gráfico do passo 3, deslocar o gráfico verticalmente em $|D|$ unidades para cima se $D > 0$ ou para baixo se $D < 0$.

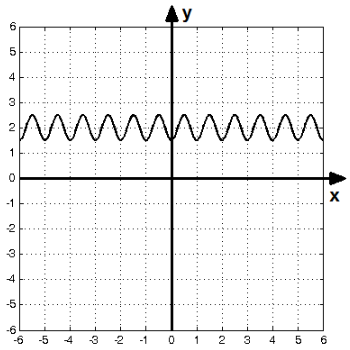


$-\frac{1}{2}f(2x + 1) - 2$

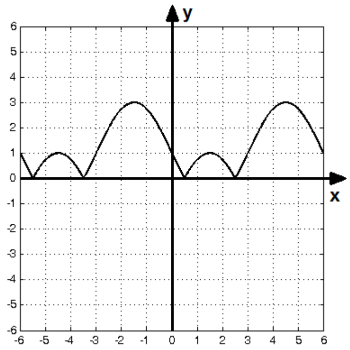


$2f(-\frac{1}{3}x - \frac{1}{2}) + 1$

Gráfico de $|f|$: O gráfico de $|f|$ é igual ao gráfico de f na região onde $f(x) \geq 0$ e é igual à reflexão em relação ao eixo x do gráfico de f na região em que $f(x) < 0$ (pois se $f(x) < 0$ então $|f(x)| = -f(x)$).



$|-\frac{1}{2}f(2x+1)-2|$

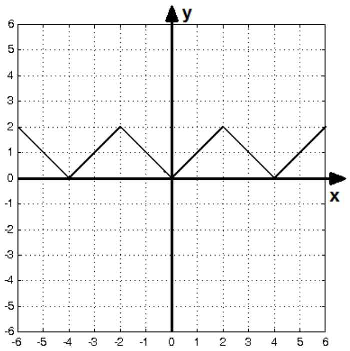


$|2f(-\frac{1}{3}x-\frac{1}{2})+1|$

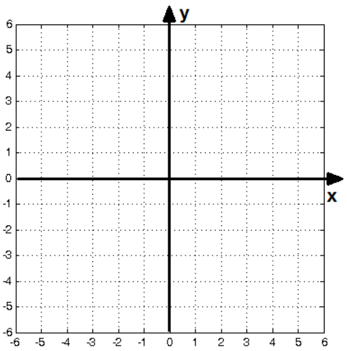
Observações:

- Comparando os gráficos de $f(x)$ e $f(x+A)$, vemos que as raízes (caso existam) estão deslocadas em $|A|$ unidades (para a direita ou para a esquerda, dependendo do sinal de A).
- Comparando os gráficos de $f(x)$ e $f(Bx)$, vemos que o cruzamento do gráfico com o eixo y (caso exista) é mantido.
- Comparando os gráficos de $f(x)$ e $Cf(x)$, vemos que as raízes (caso existam) são as mesmas.
- Comparando os gráficos de $f(x)$ e $f(x)+D$, vemos que o cruzamento do gráfico com o eixo y (caso exista) é deslocado em $|D|$ unidades (para cima ou para baixo, dependendo do sinal de D).

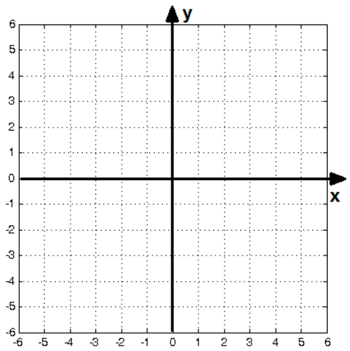
Exercícios: Dadas as funções abaixo, esboce os gráficos das funções pedidas.



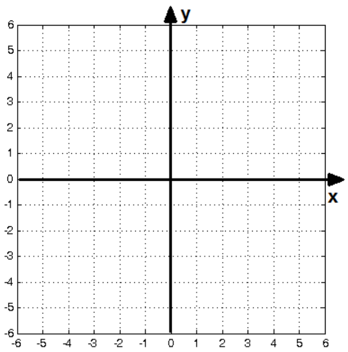
$f_1(x)$



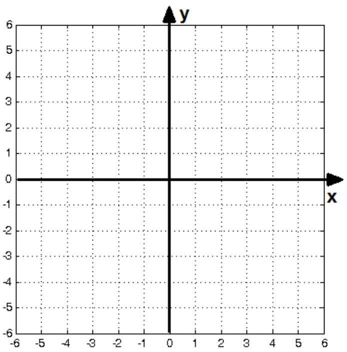
$f_1(x+2)$



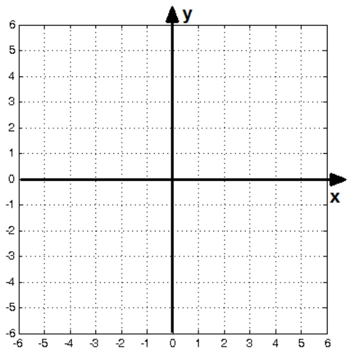
$f_1(2x+2)$



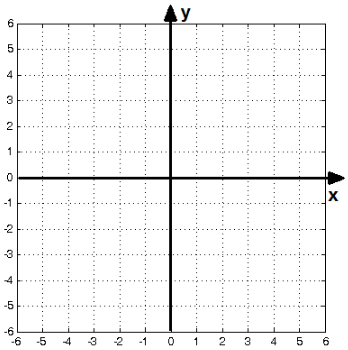
$f_1(x-1)$



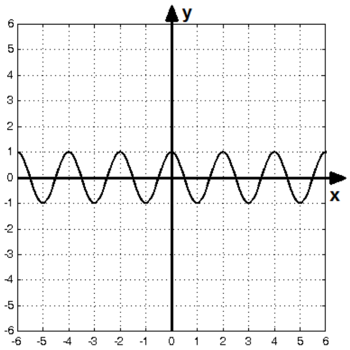
$2f_1(x-1)$



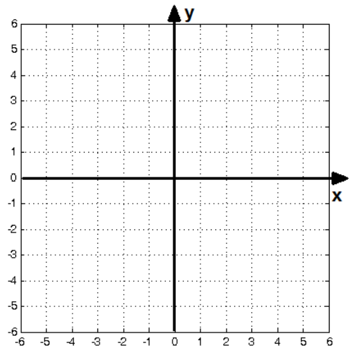
$2f_1(x)-1$



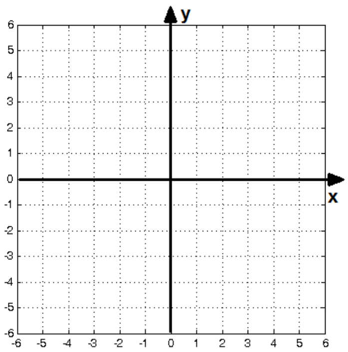
$|2f_1(x) - 1|$



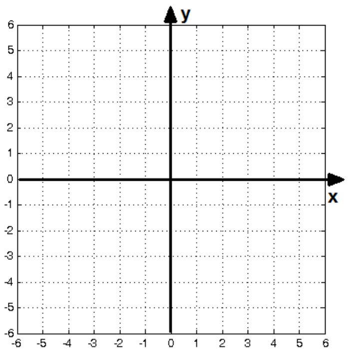
$f_2(x) = \cos(\pi x)$



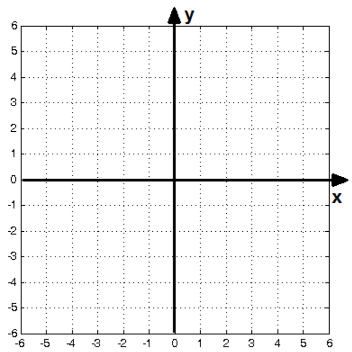
$f_2(\frac{1}{2}x) = \cos(\frac{\pi}{2}x)$



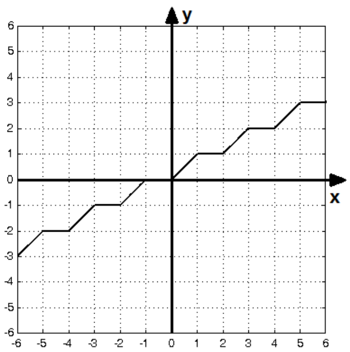
$-2f_2(\frac{1}{2}x) = -2\cos(\frac{\pi}{2}x)$



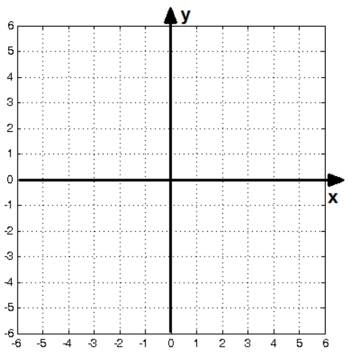
$f_2(x+\frac{1}{2})-2 = \cos(\pi(x+\frac{1}{2}))-2$



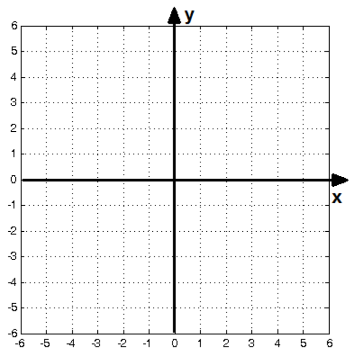
$3f_2(x) + 1 = 3\cos(\pi x) + 1$



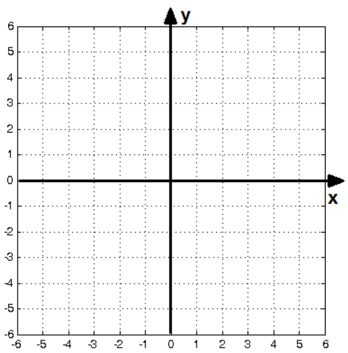
$f_3(x)$



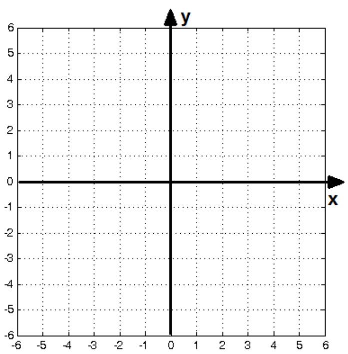
$|f_3(x)|$



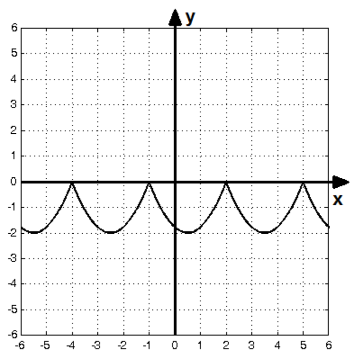
$2f_3(-x)$



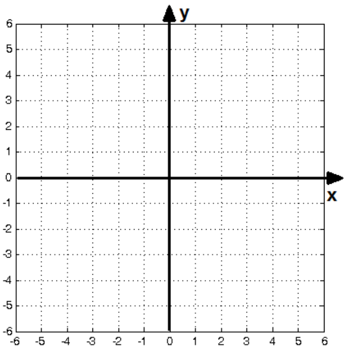
$f_3(\frac{1}{2}x)$



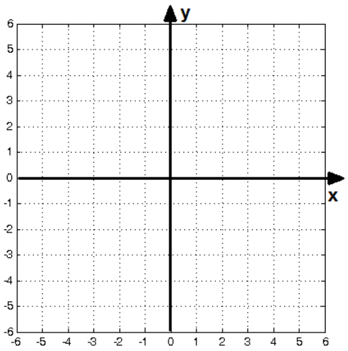
$f_3(x-1) - 1$



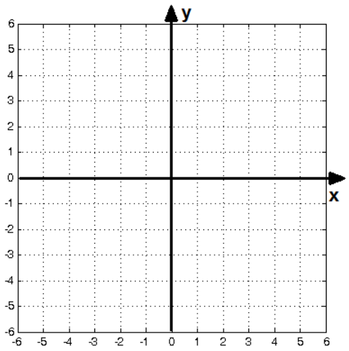
$f_4(x)$



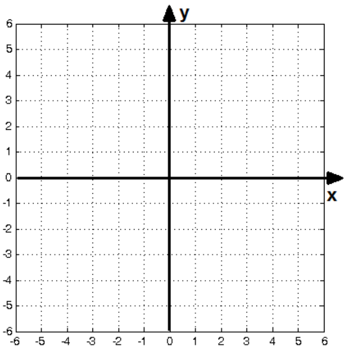
$f_4(2x-1)$



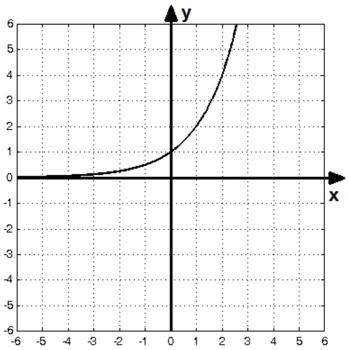
$f_4(\frac{1}{2}x)$



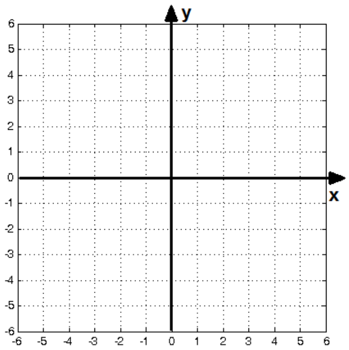
$-2f_4(\frac{1}{2}x)$



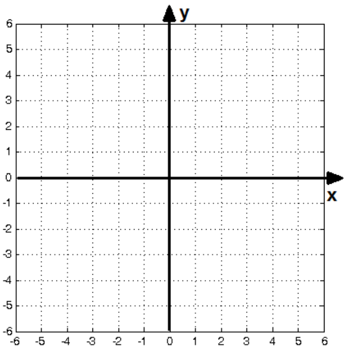
$-f_4(x-1)+1$



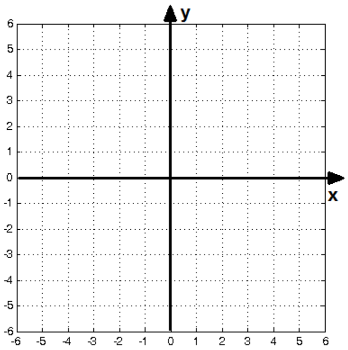
$f_5(x) = 2^x$



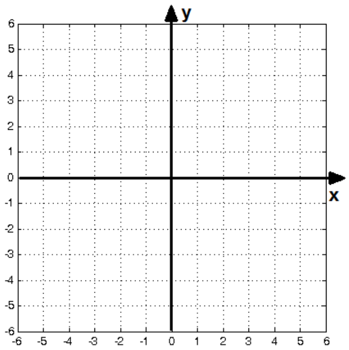
$f_5(-x) = 2^{-x}$



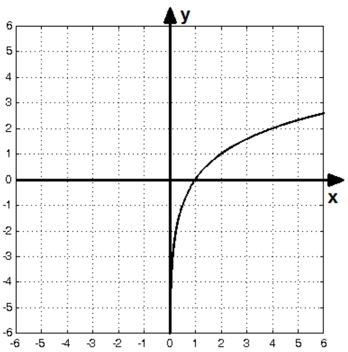
$-f_5(-x) = -2^{-x}$



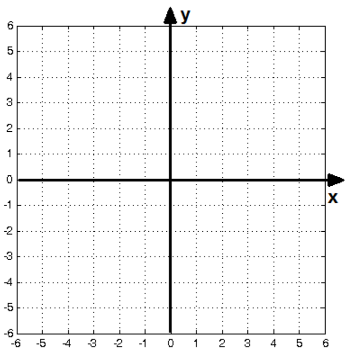
$f_5(-x) - 2 = 2^{-x} - 2$



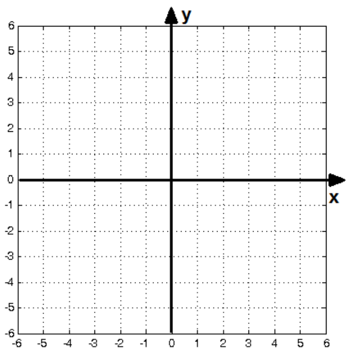
$f_5(x-2) - 3 = 2^{x-2} - 3$



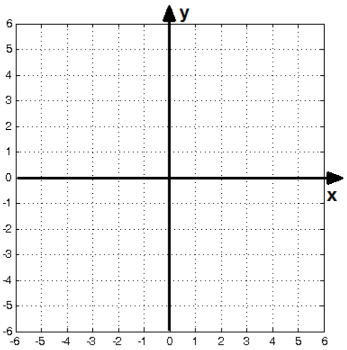
$f_6(x) = \log_2(x)$



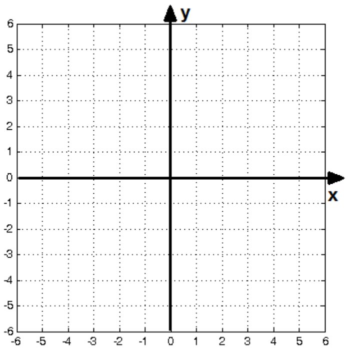
$f_6(x+2)+1 = \log_2(x+2)+1$



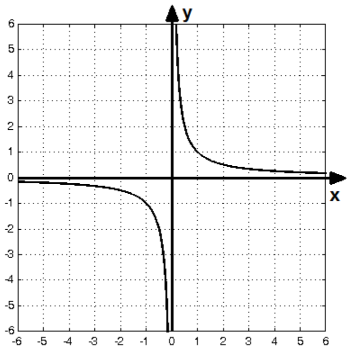
$f_6(4x) = \log_2(4x)$



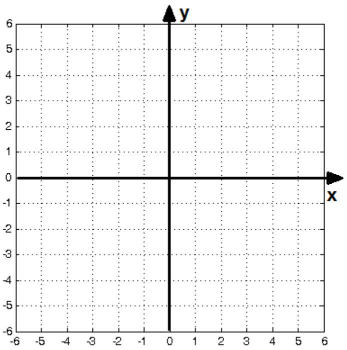
$-f_6(x+1) = -\log_2(x+1)$



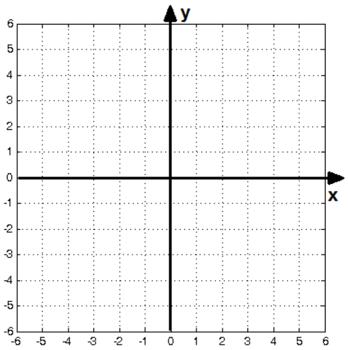
$|f_6(x+1)| = |\log_2(x+1)|$



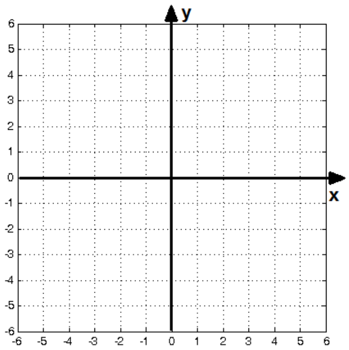
$f_7(x) = \frac{1}{x}$



$f_7(x-1) + 2 = \frac{1}{x-1} + 2$

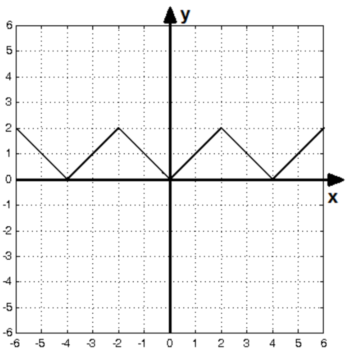


$|f_7(x-1) + 2| = |\frac{1}{x-1} + 2|$

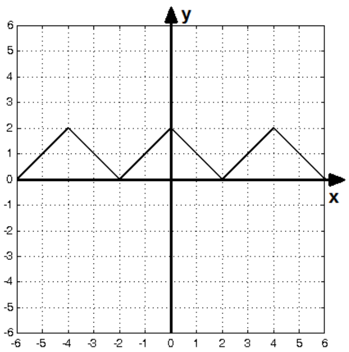


$f_7(-x+2) = \frac{1}{2-x}$

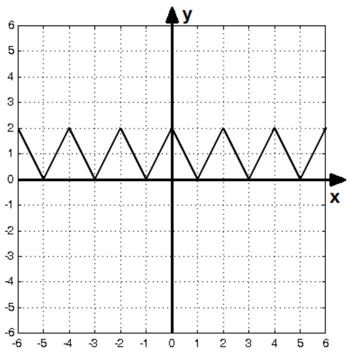
Respostas:



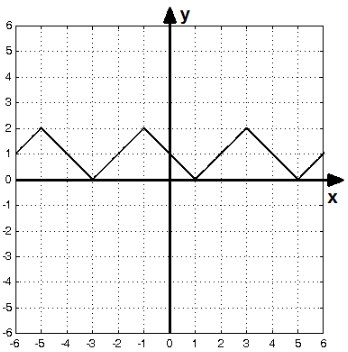
$f_1(x)$



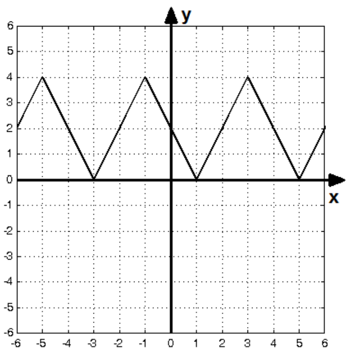
$f_1(x+2)$



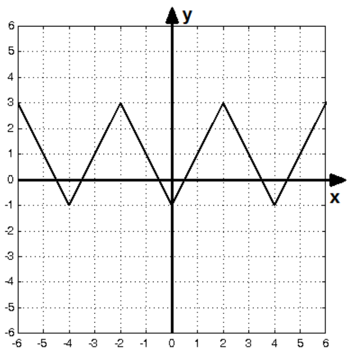
$f_1(2x+2)$



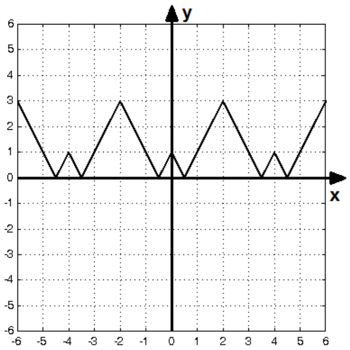
$f_1(x-1)$



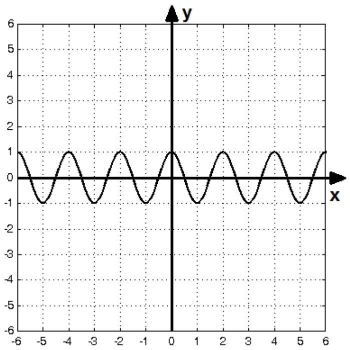
$2f_1(x-1)$



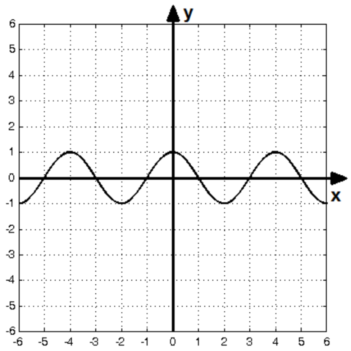
$2f_1(x) - 1$



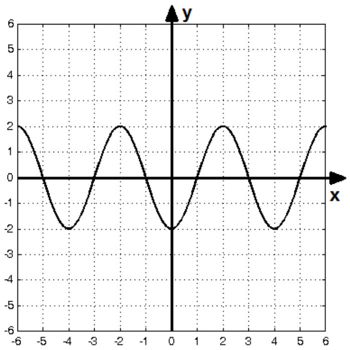
$|2f_1(x) - 1|$



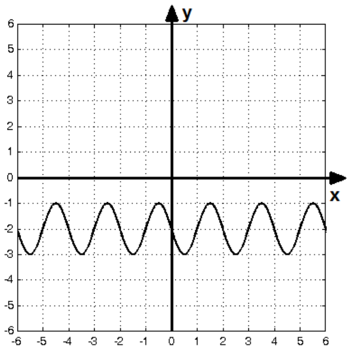
$f_2(x) = \cos(\pi x)$



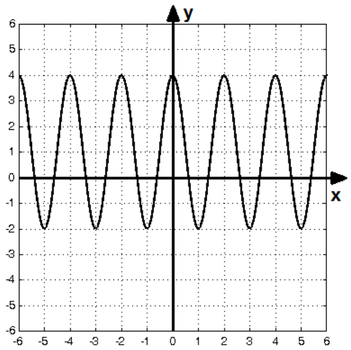
$f_2(\frac{1}{2}x) = \cos(\frac{\pi}{2}x)$



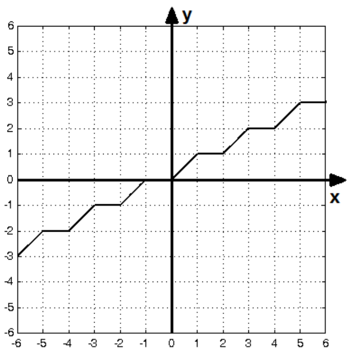
$-2f_2(\frac{1}{2}x) = -2\cos(\frac{\pi}{2}x)$



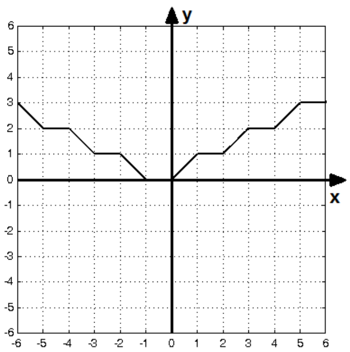
$f_2(x+\frac{1}{2})-2 = \cos(\pi(x+\frac{1}{2}))-2$



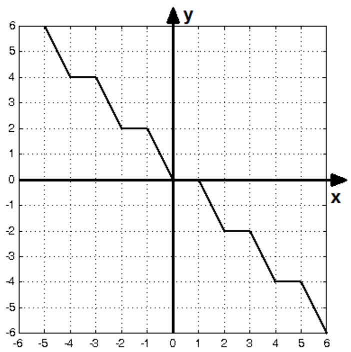
$3f_2(x) + 1 = 3\cos(\pi x) + 1$



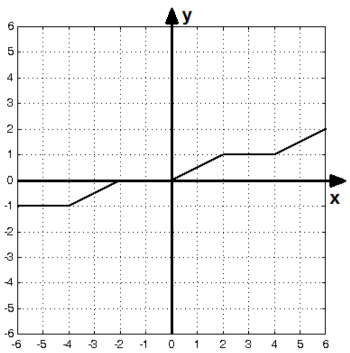
$f_3(x)$



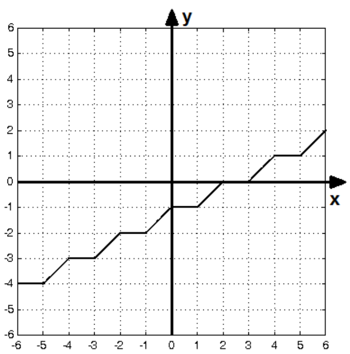
$|f_3(x)|$



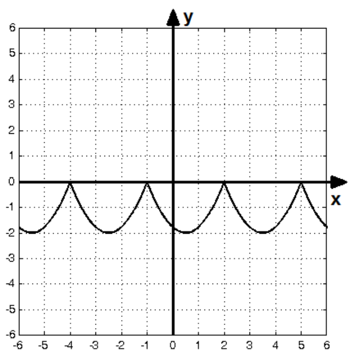
$2f_3(-x)$



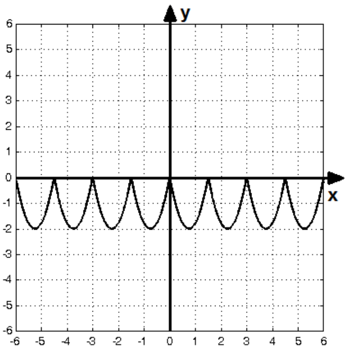
$f_3(\frac{1}{2}x)$



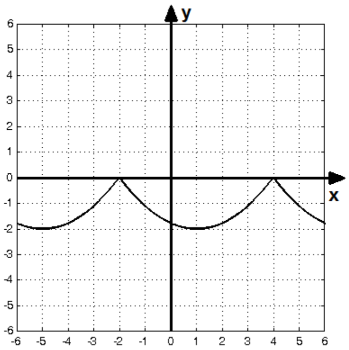
$f_3(x-1) - 1$



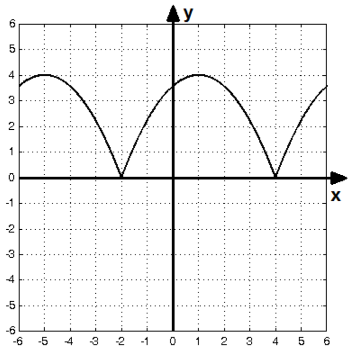
$f_4(x)$



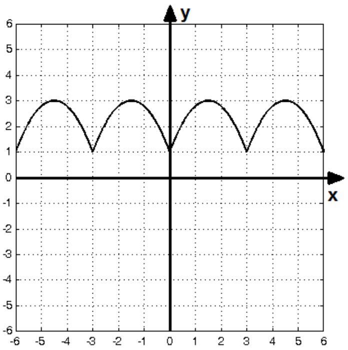
$f_4(2x-1)$



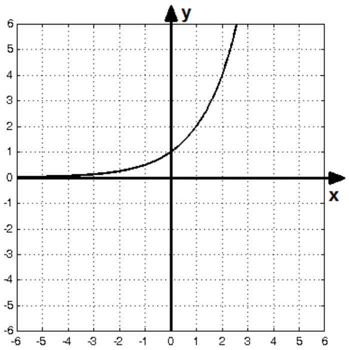
$f_4(\frac{1}{2}x)$



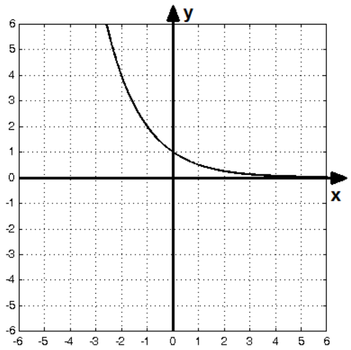
$-2f_4(\frac{1}{2}x)$



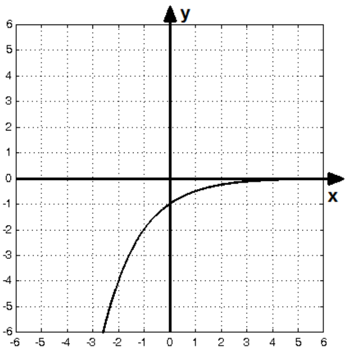
$-f_4(x-1)+1$



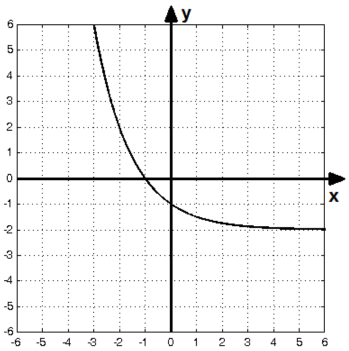
$f_5(x) = 2^x$



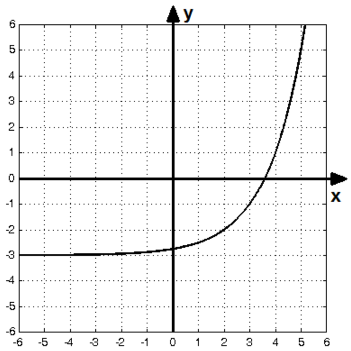
$f_5(-x) = 2^{-x}$



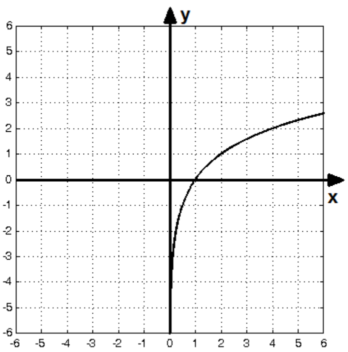
$-f_5(-x) = -2^{-x}$



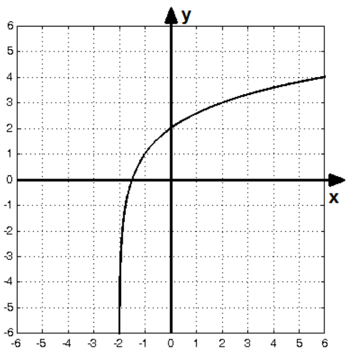
$f_5(-x) - 2 = 2^{-x} - 2$



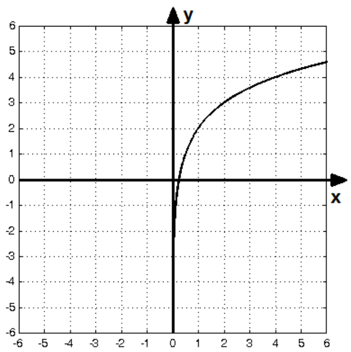
$f_5(x-2) - 3 = 2^{x-2} - 3$



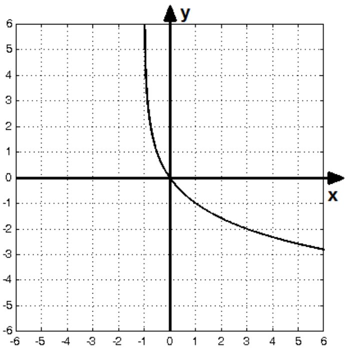
$f_6(x) = \log_2(x)$



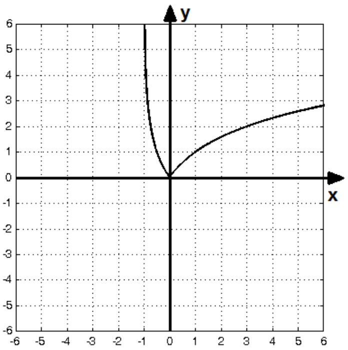
$f_6(x+2)+1 = \log_2(x+2)+1$



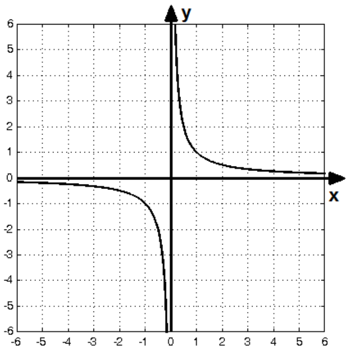
$f_6(4x) = \log_2(4x)$



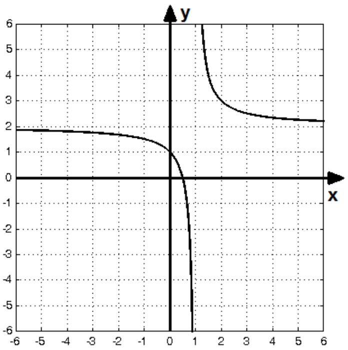
$-f_6(x+1) = -\log_2(x+1)$



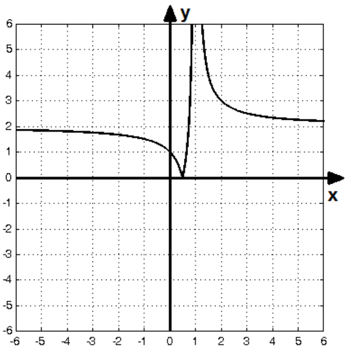
$|f_6(x+1)| = |\log_2(x+1)|$



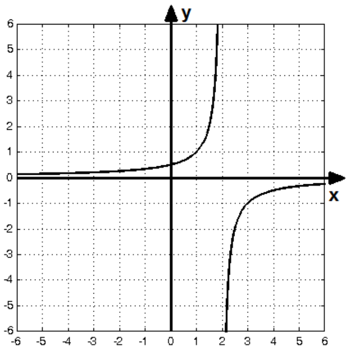
$f_7(x) = \frac{1}{x}$



$f_7(x-1)+2 = \frac{1}{x-1}+2$



$|f_7(x-1)+2| = |\frac{1}{x-1}+2|$



$f_7(-x+2) = \frac{1}{2-x}$