

Derivadas - O Método de Newton

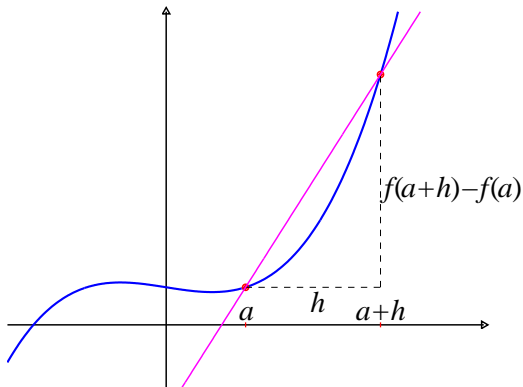
Ademir Alves Ribeiro

Maio / 2007



- 1 Derivada e reta tangente
- 2 Aplicação na otimização
- 3 O Método de Newton

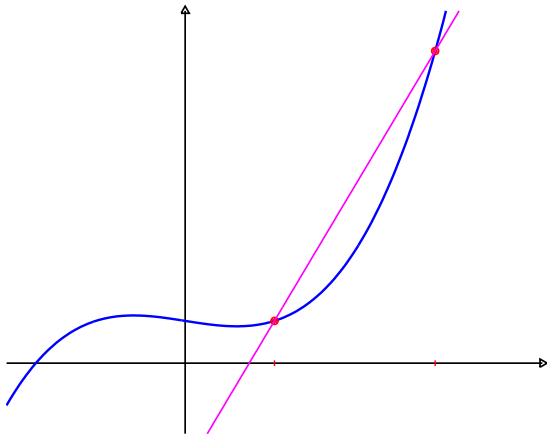
A reta secante



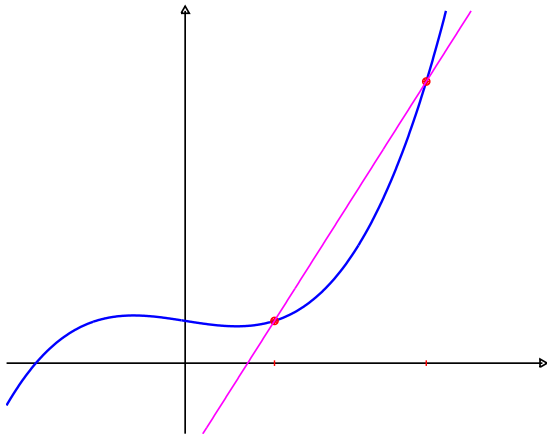
Coefficiente angular

$$m_h = \frac{f(a+h) - f(a)}{h}$$

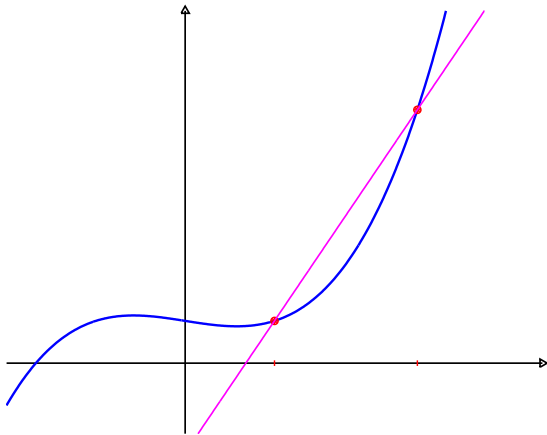
O “limite” das retas secantes



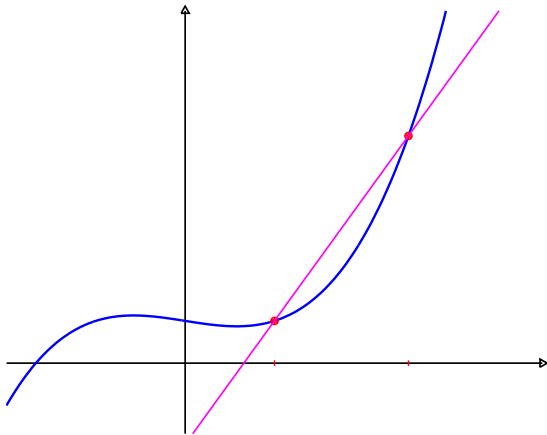
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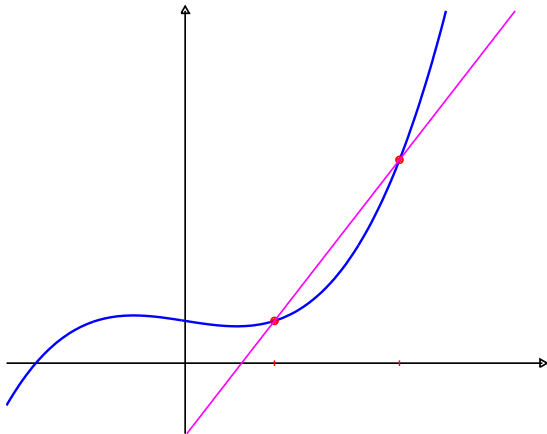
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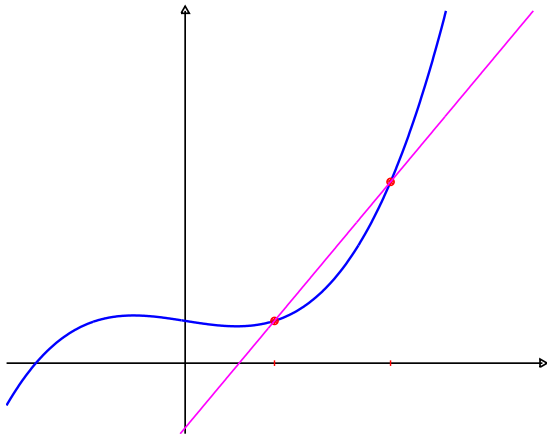
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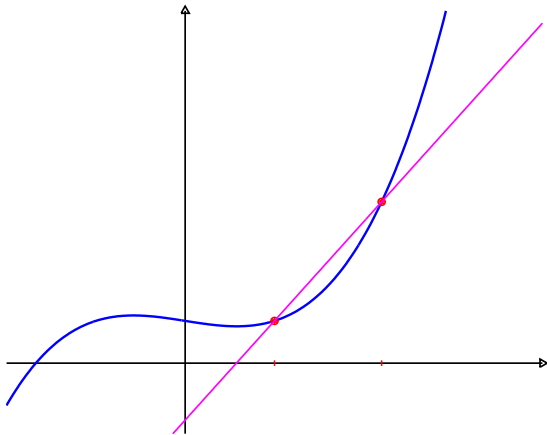
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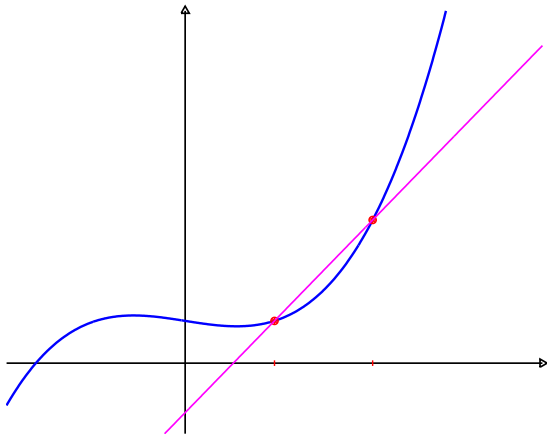
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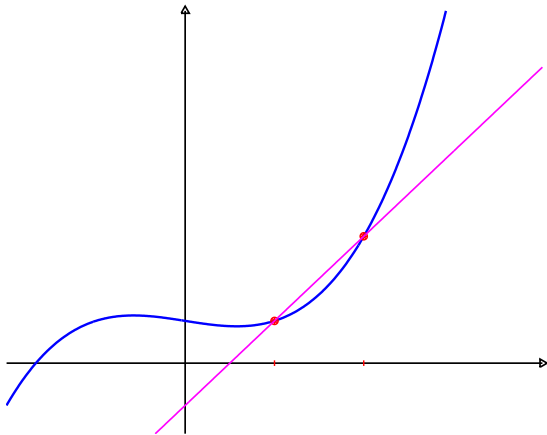
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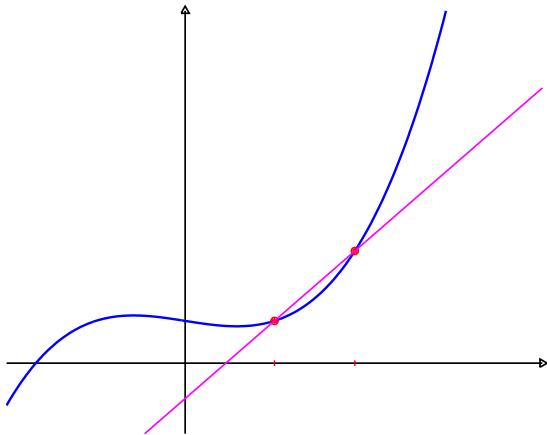
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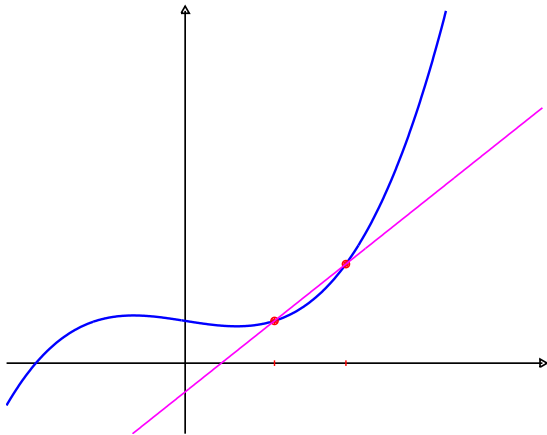
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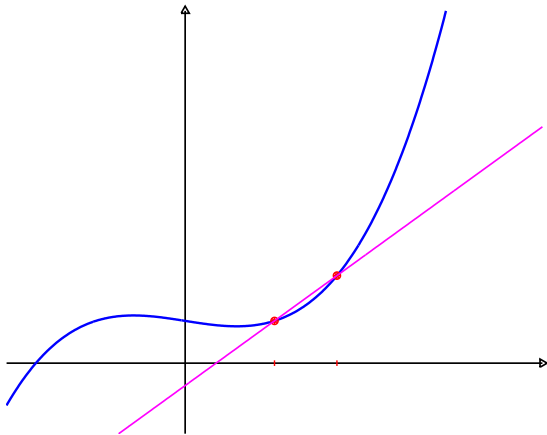
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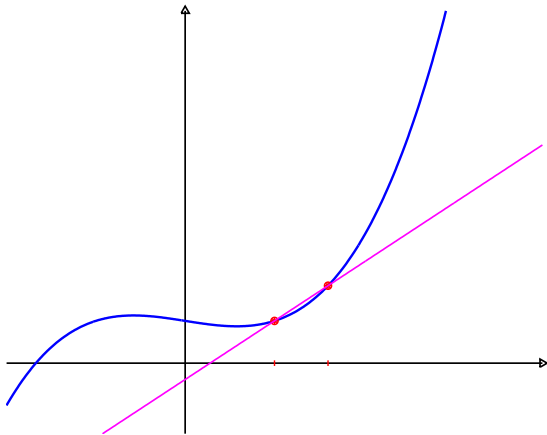
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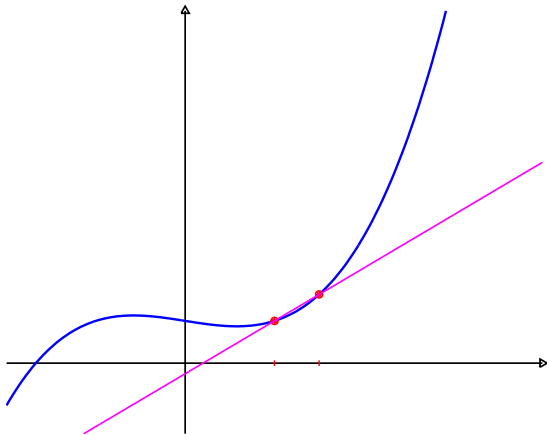
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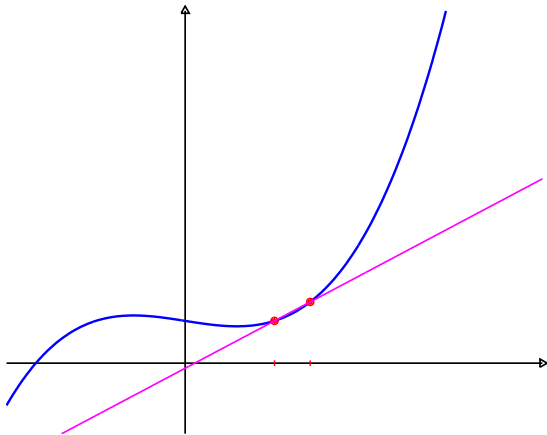
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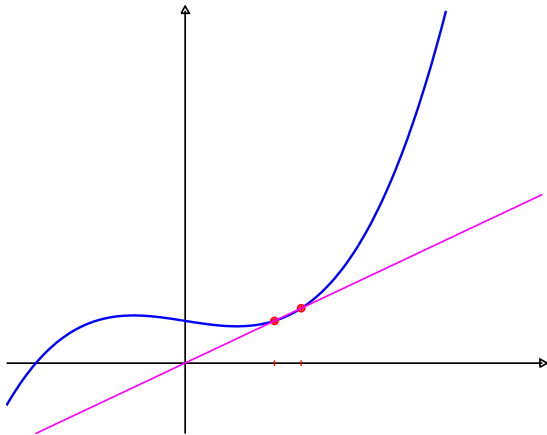
O “limite” das retas secantes



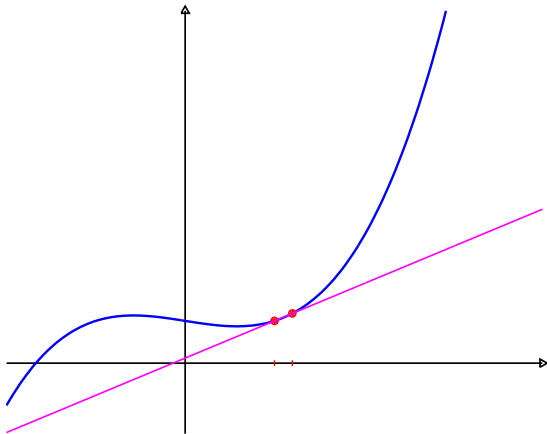
O “limite” das retas secantes



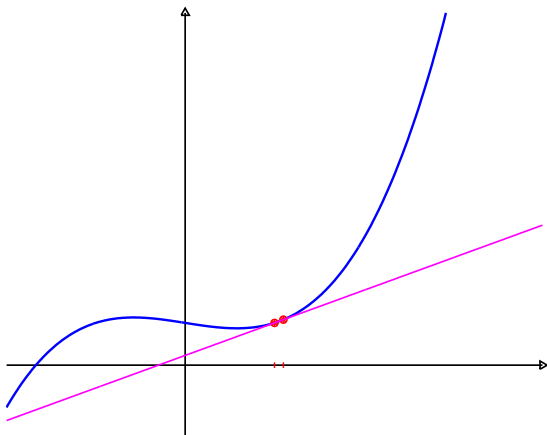
O “limite” das retas secantes



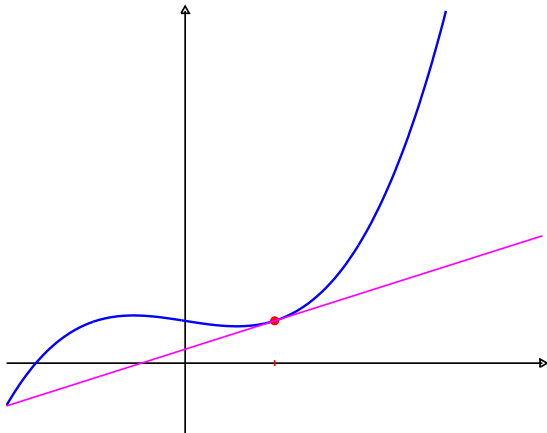
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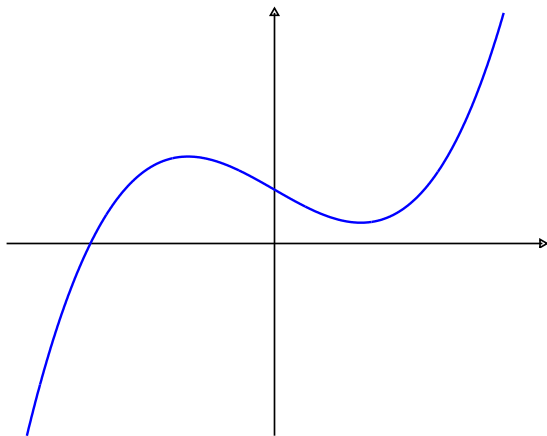
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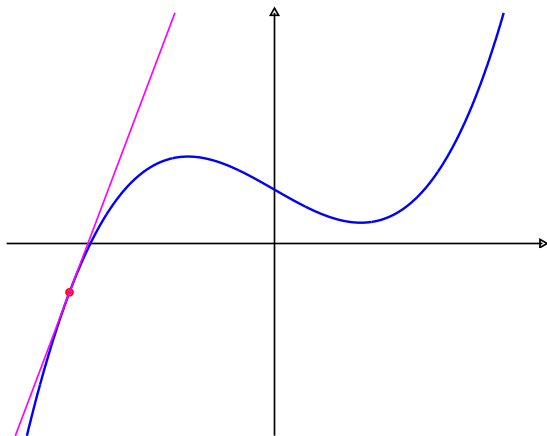
A reta tangente



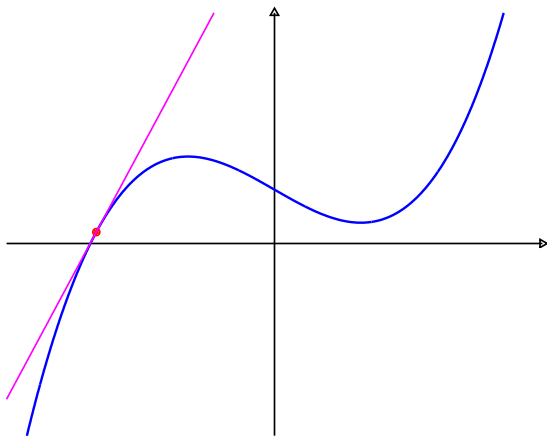
O sinal da derivada



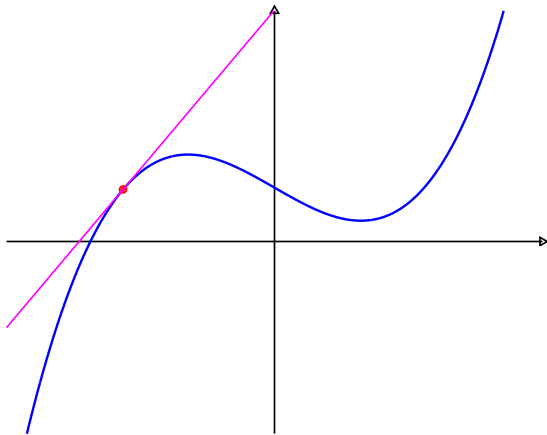
$f' > 0 \Rightarrow$ função crescente



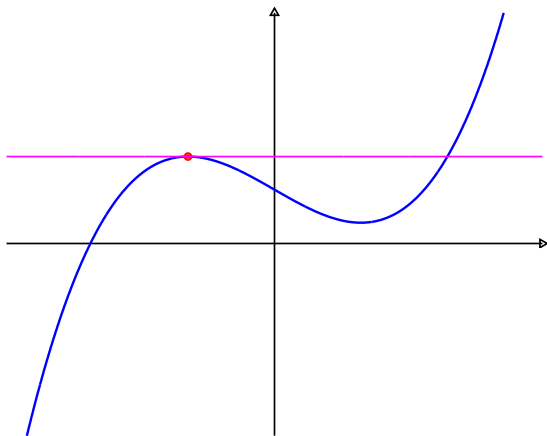
$f' > 0 \Rightarrow$ função crescente



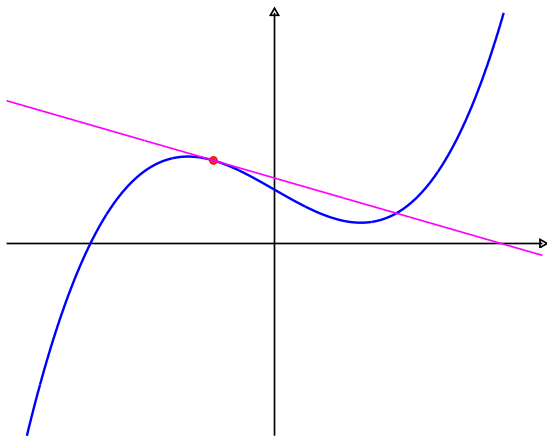
$f' > 0 \Rightarrow$ função crescente



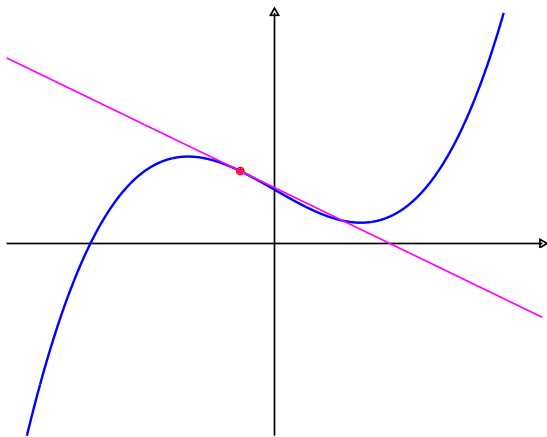
$f' = 0$ (máximo local)



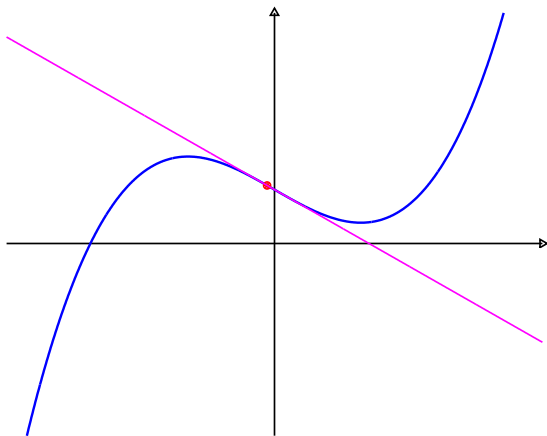
$f' < 0 \Rightarrow$ função decrescente



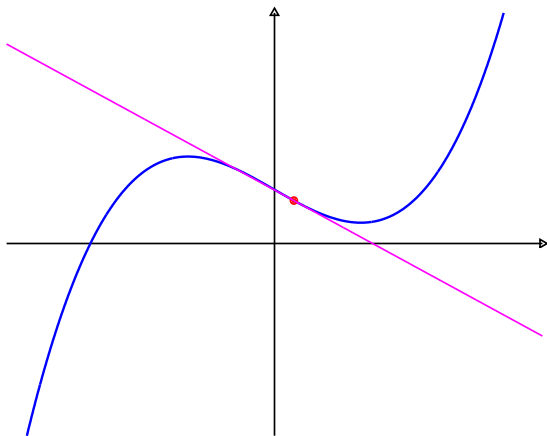
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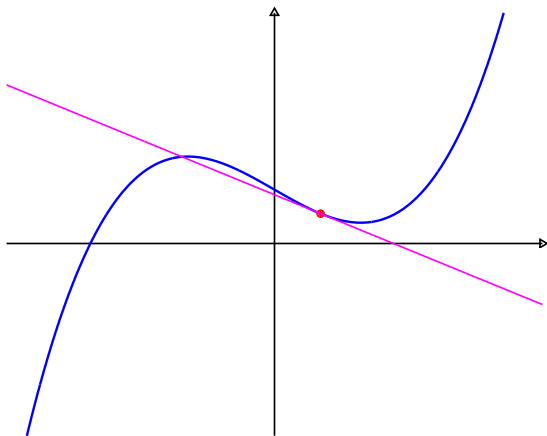
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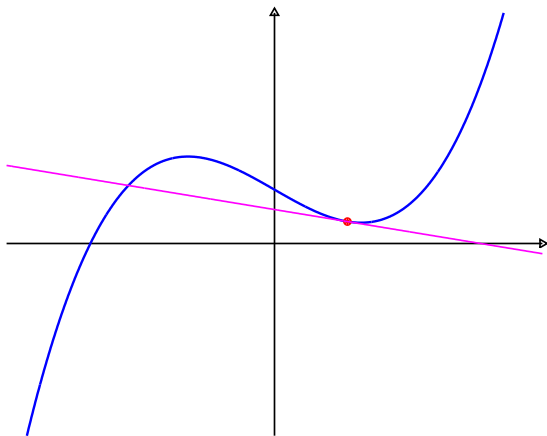
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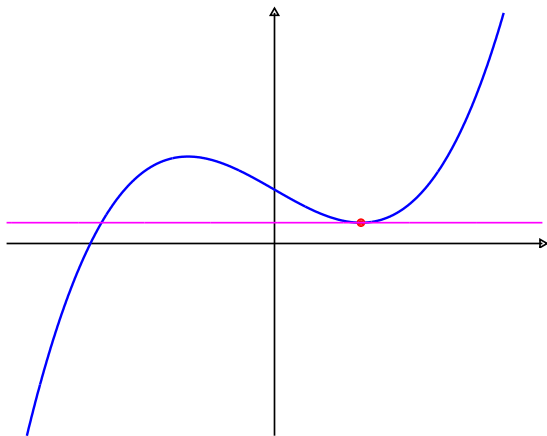
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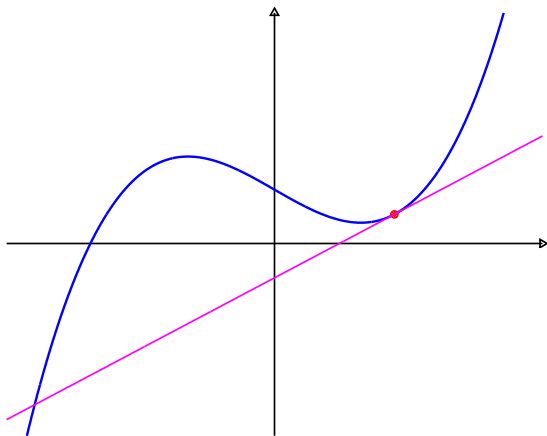
$f' < 0 \Rightarrow$ função decrescente



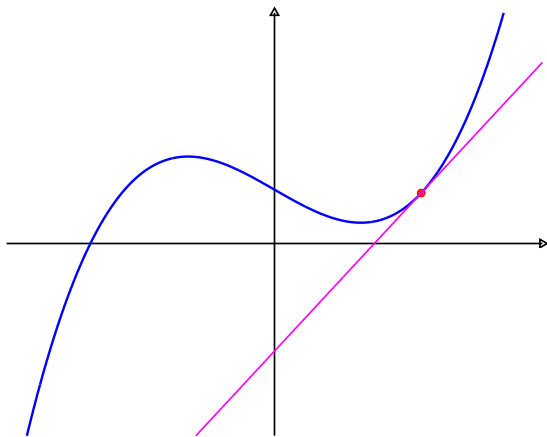
$f' = 0$ (mínimo local)



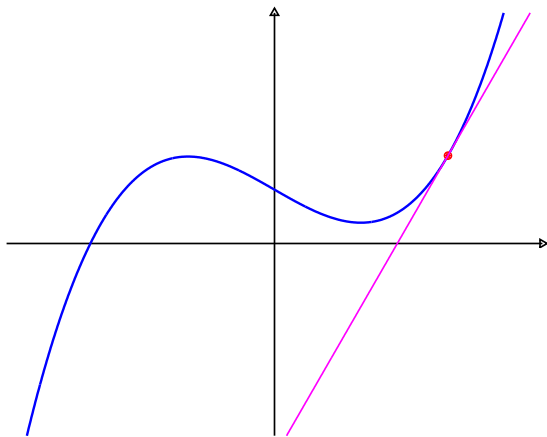
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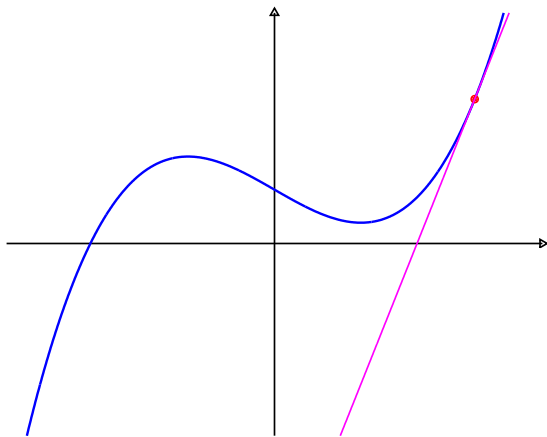
$f' > 0 \Rightarrow$ função crescente



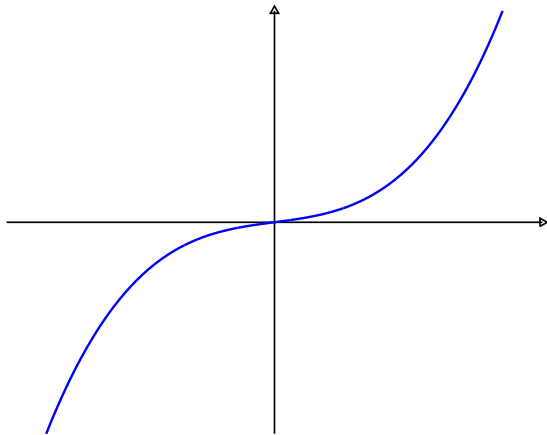
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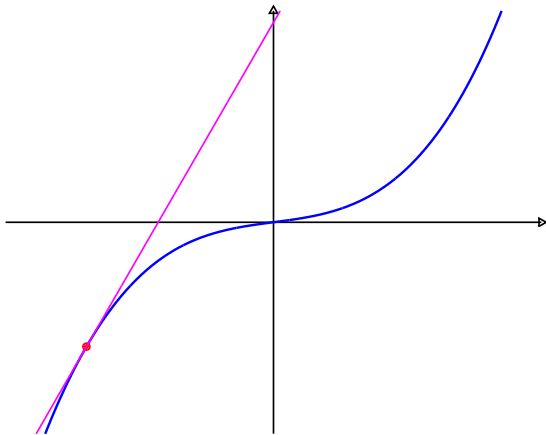
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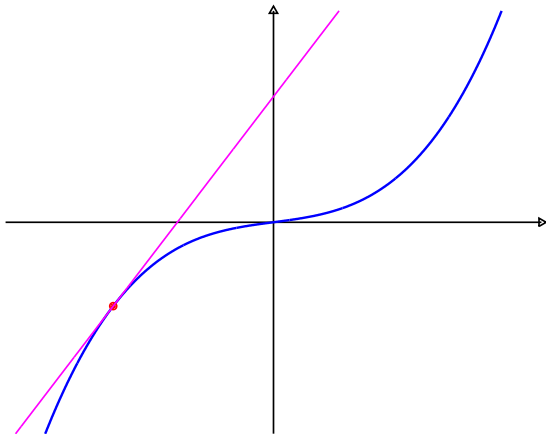
O sinal da derivada segunda



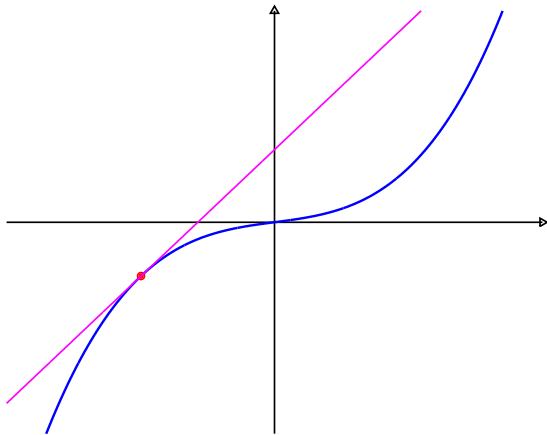
$f'' < 0 \Rightarrow f'$ decrescente $\Rightarrow f$ côncava



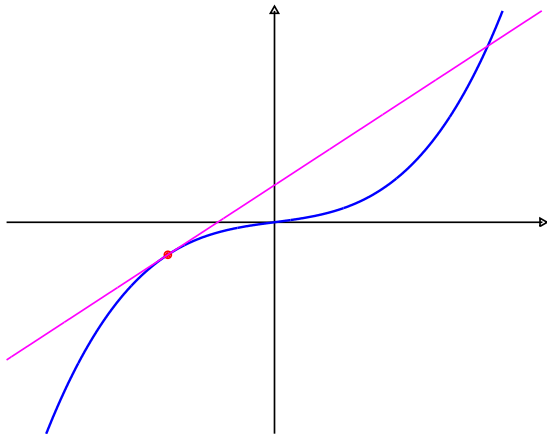
$f'' < 0 \Rightarrow f'$ decrescente $\Rightarrow f$ côncava



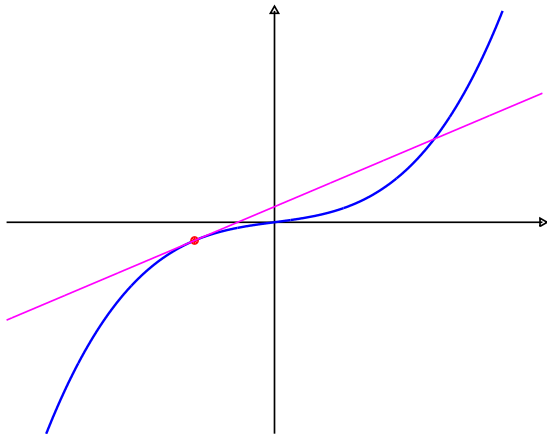
$f'' < 0 \Rightarrow f'$ decrescente $\Rightarrow f$ côncava



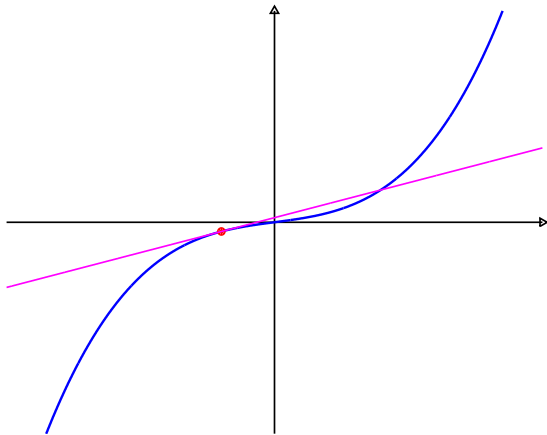
$f'' < 0 \Rightarrow f'$ decrescente $\Rightarrow f$ côncava



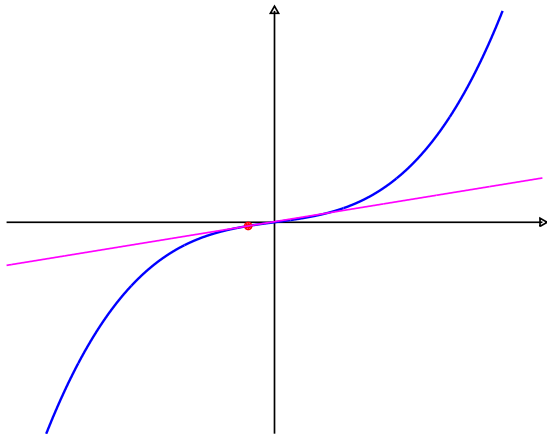
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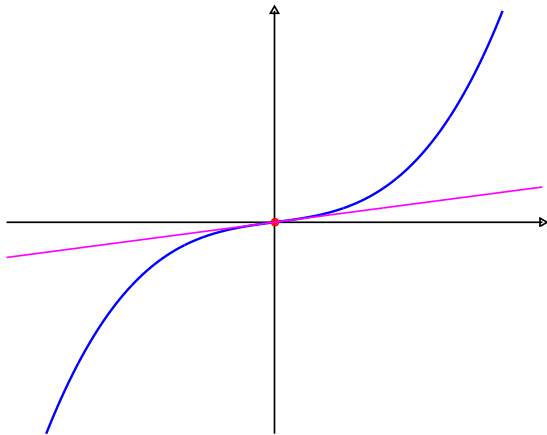
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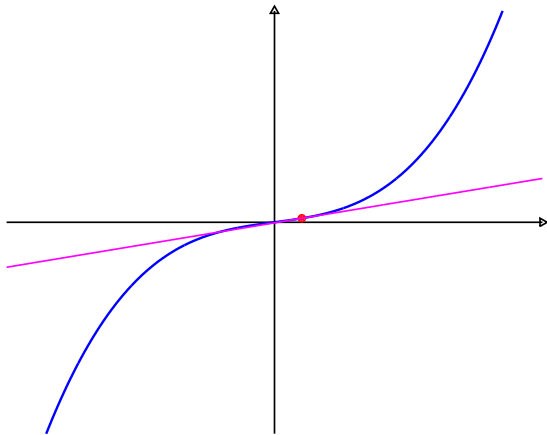
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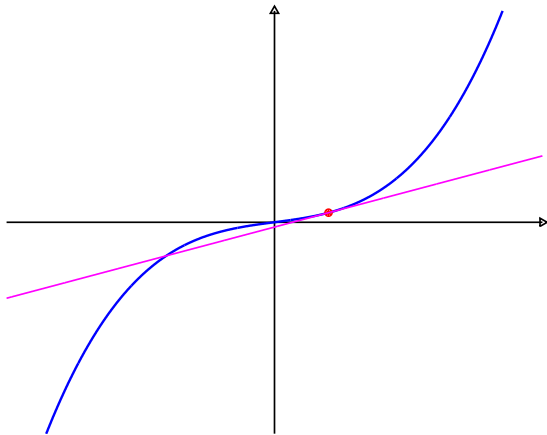
$f'' = 0$ (ponto de inflexão)



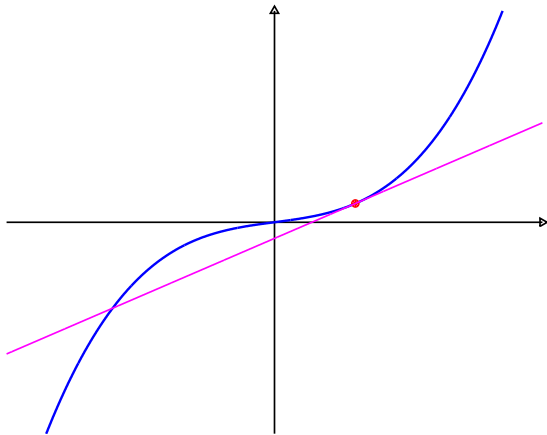
$f'' > 0 \Rightarrow f'$ crescente $\Rightarrow f$ convexa



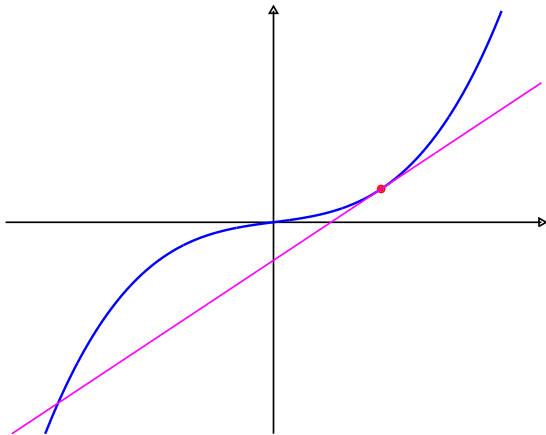
$f'' > 0 \Rightarrow f'$ crescente $\Rightarrow f$ convexa



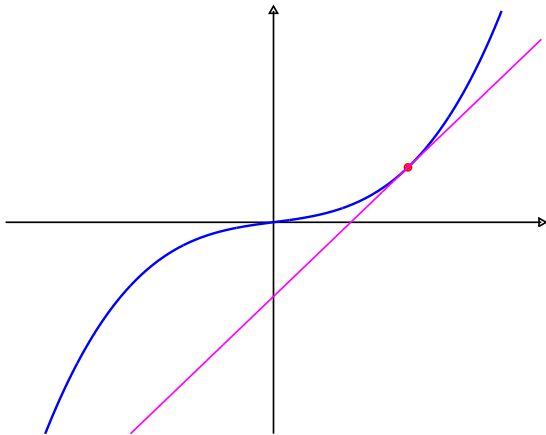
$f'' > 0 \Rightarrow f'$ crescente $\Rightarrow f$ convexa



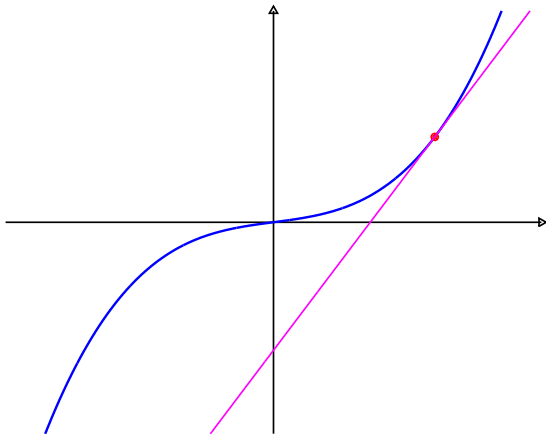
$f'' > 0 \Rightarrow f'$ crescente $\Rightarrow f$ convexa



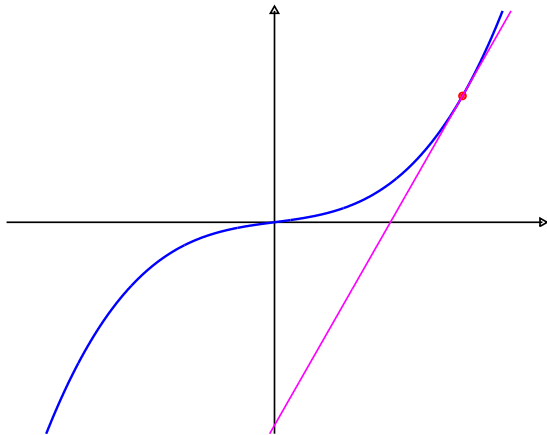
$f'' > 0 \Rightarrow f'$ crescente $\Rightarrow f$ convexa



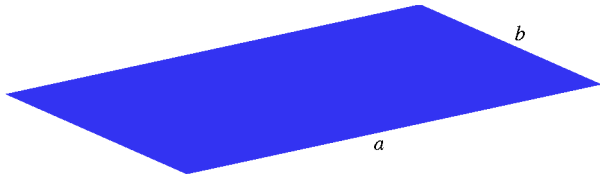
$f'' > 0 \Rightarrow f'$ crescente $\Rightarrow f$ convexa



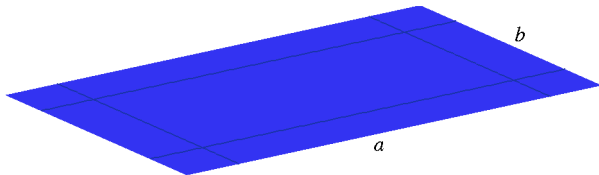
$f'' > 0 \Rightarrow f' \text{ crescente} \Rightarrow f \text{ convexa}$



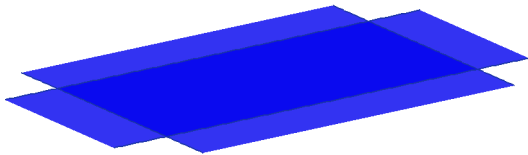
Construir uma caixa de volume máximo.



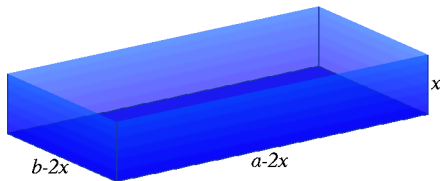
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Cálculo do volume

Volume da caixa

$$V(x) = (a - 2x)(b - 2x)x = 4x^3 - 2(a + b)x^2 + abx$$

Domínio da variável x

$$0 \leq x \leq \frac{b}{2}$$

Derivada

$$V'(x) = 12x^2 - 4(a + b)x + ab$$

Cálculo do volume

Volume da caixa

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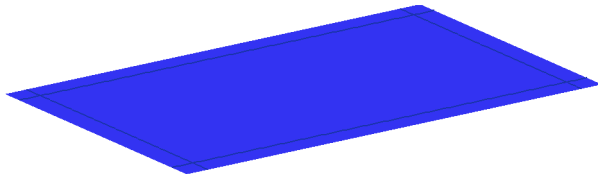
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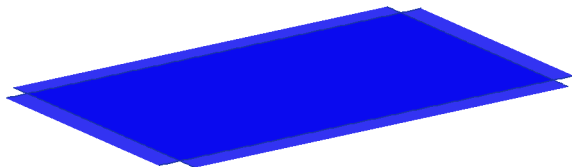
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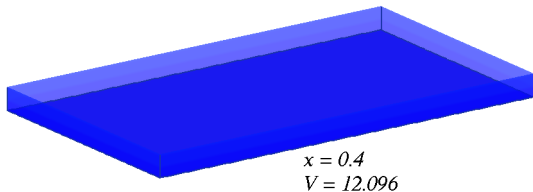
Exemplo com $a = 8$ e $b = 5$



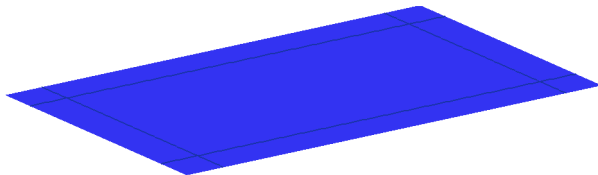
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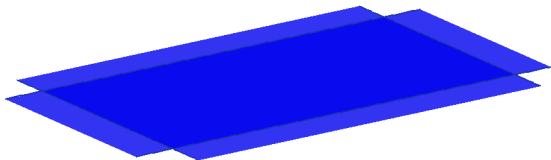
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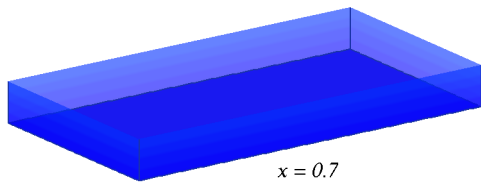
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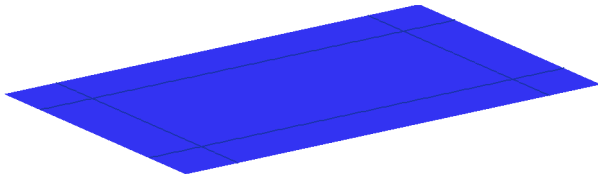


Exemplo com $a = 8$ e $b = 5$

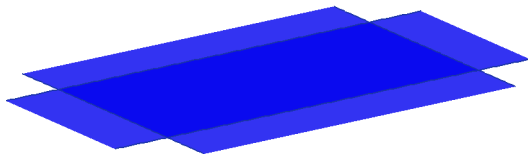


$$x = 0.7$$
$$V = 16.632$$

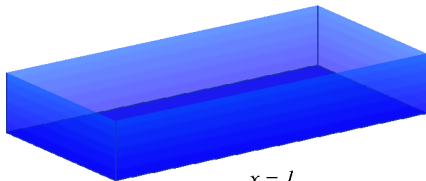
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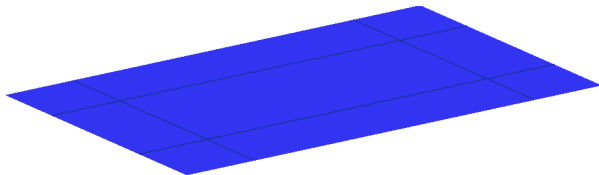


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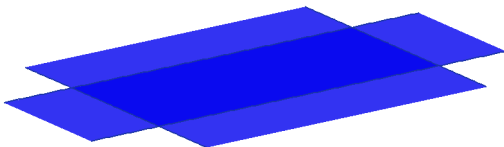


$$x = 1$$
$$V = 18$$

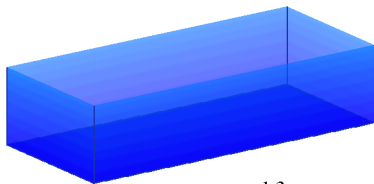
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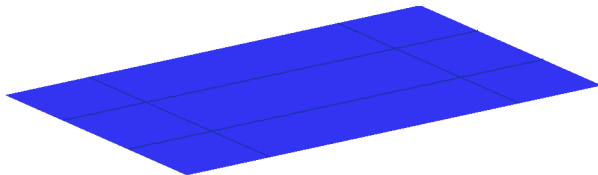


Exemplo com $a = 8$ e $b = 5$

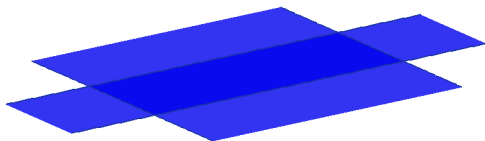


$$x = 1.3$$
$$V = 16.848$$

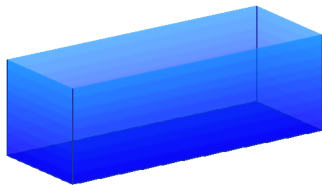
Exemplo com $a = 8$ e $b = 5$



Exemplo com $a = 8$ e $b = 5$



Exemplo com $a = 8$ e $b = 5$



$$x = 1.6$$
$$V = 13.824$$

Exemplo com $a = 8$ e $b = 5$

Volume da caixa

$$V(x) = 4x^3 - 26x^2 + 40x$$

Derivada

$$V'(x) = 12x^2 - 52x + 40$$

Raízes da derivada

$$x_1 = 1 \text{ e } x_2 = \frac{10}{3}$$

Solução

$$x^* = 1 \text{ e } V^* = V(x^*) = 18$$

Exemplo com $a = 8$ e $b = 5$

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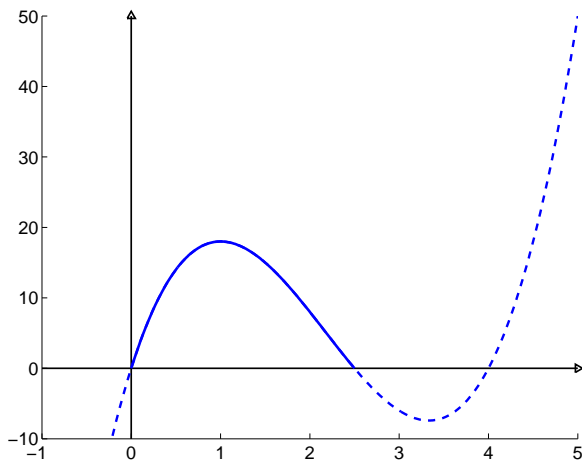
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$$x_1 = 1 \text{ e } x_2 = \frac{10}{3}$$

Solução

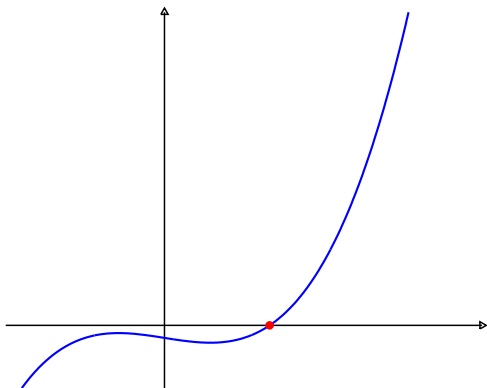
$$x^* = 1 \text{ e } V^* = V(x^*) = 18$$

Exemplo com $a = 8$ e $b = 5$

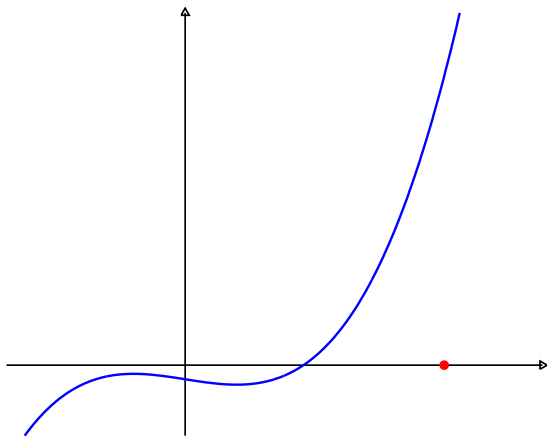


O Método de Newton

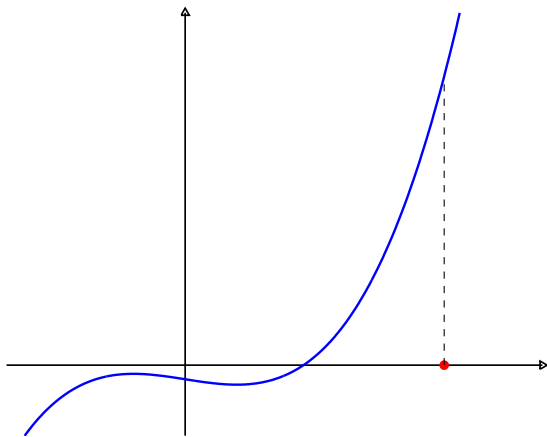
O problema aqui consiste em obter uma raiz da equação $f(x) = 0$.



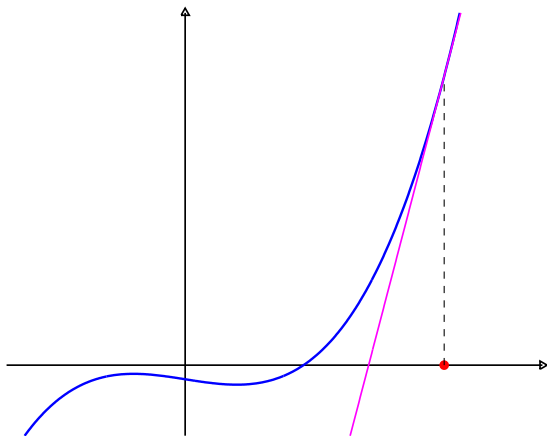
Exemplo 1



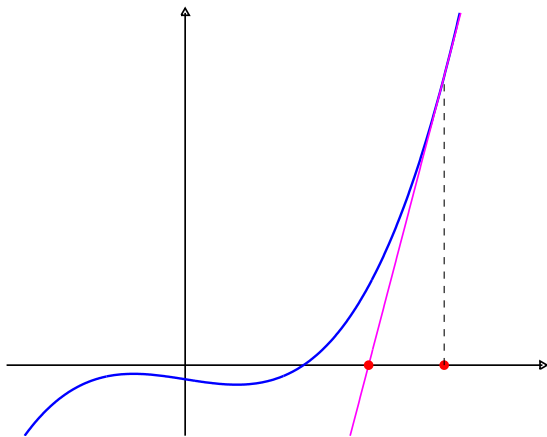
Exemplo 1



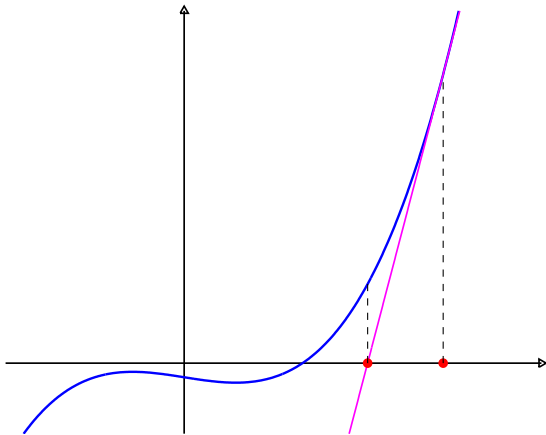
Exemplo 1



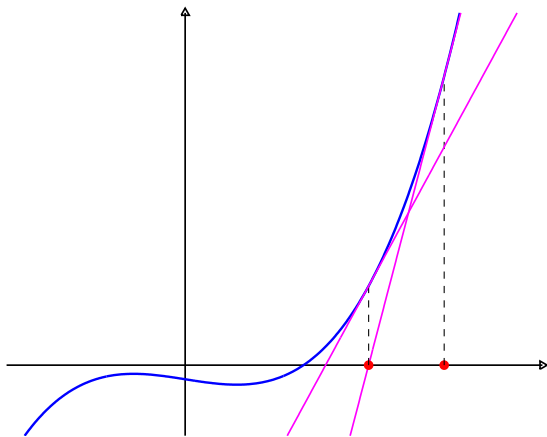
Exemplo 1



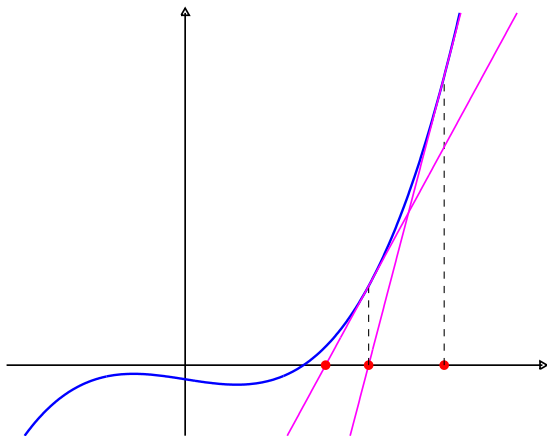
Exemplo 1



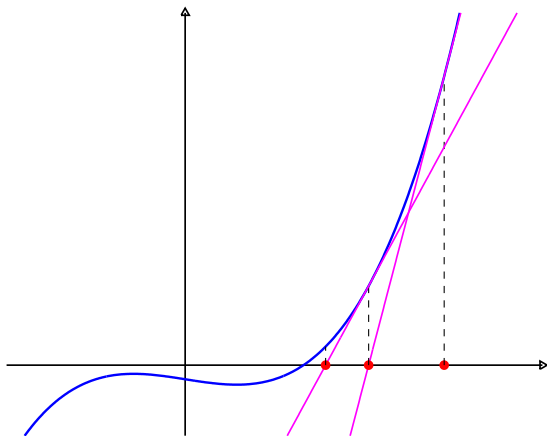
Exemplo 1



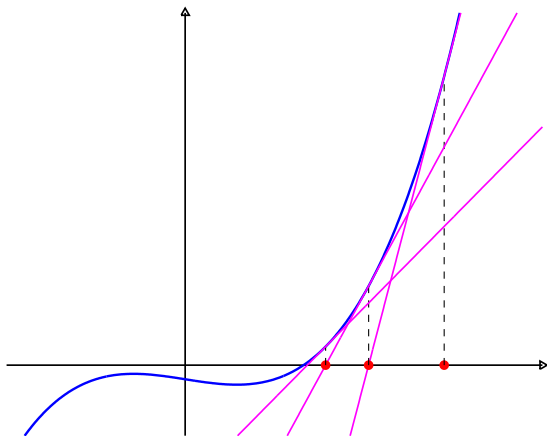
Exemplo 1



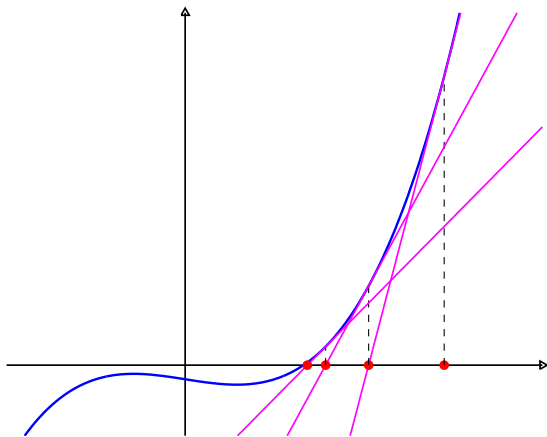
Exemplo 1



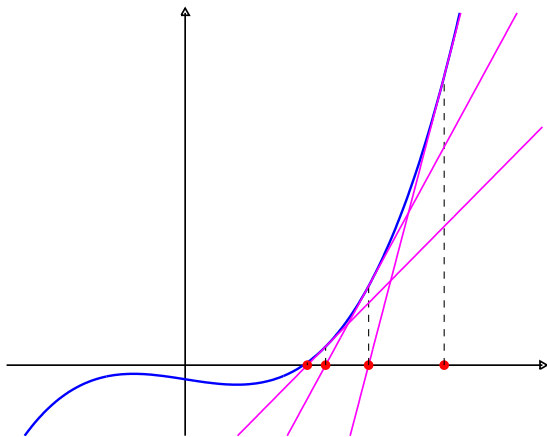
Exemplo 1



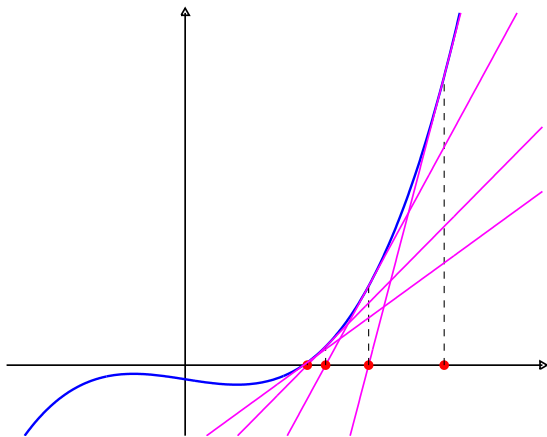
Exemplo 1



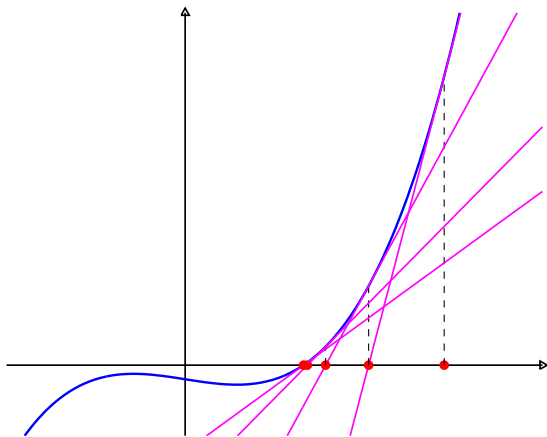
Exemplo 1



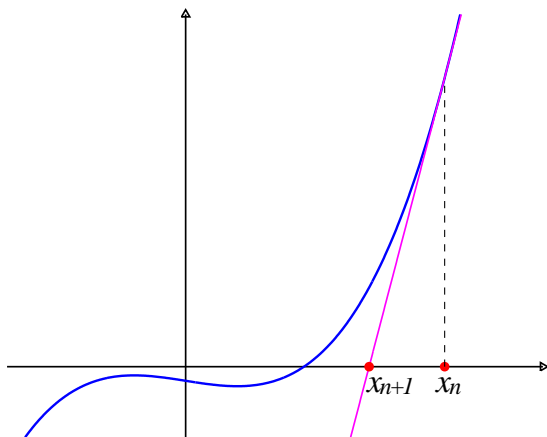
Exemplo 1



Exemplo 1



Cálculo do passo



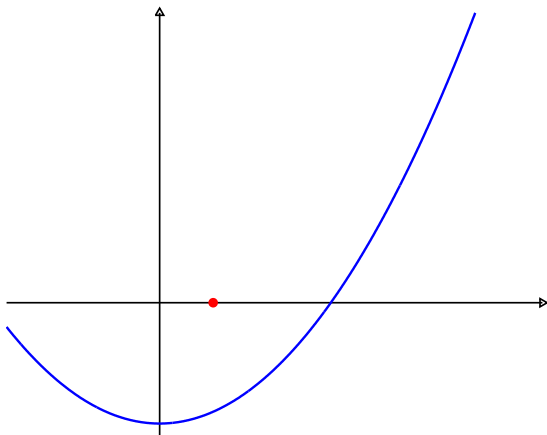
Reta tangente

$$y - f(x_n) = f'(x_n)(x - x_n)$$

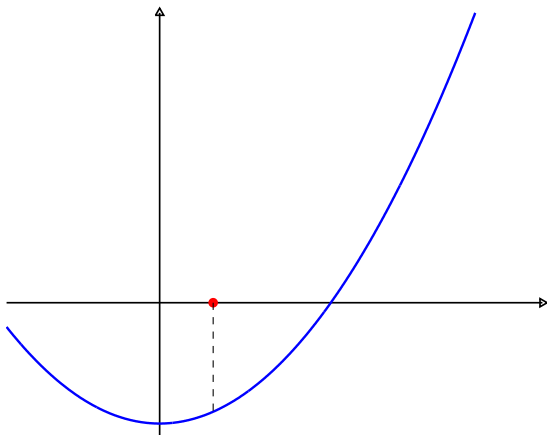
Novo ponto ($y = 0$)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

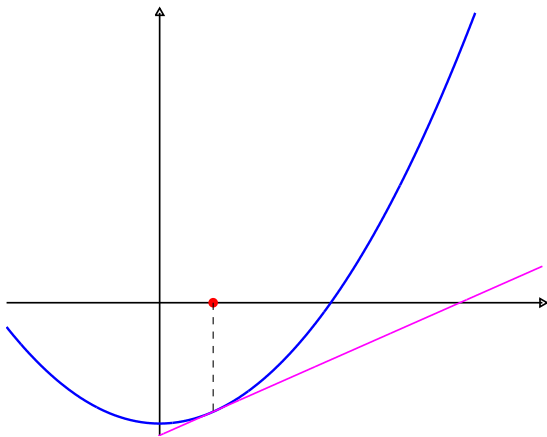
Exemplo 2



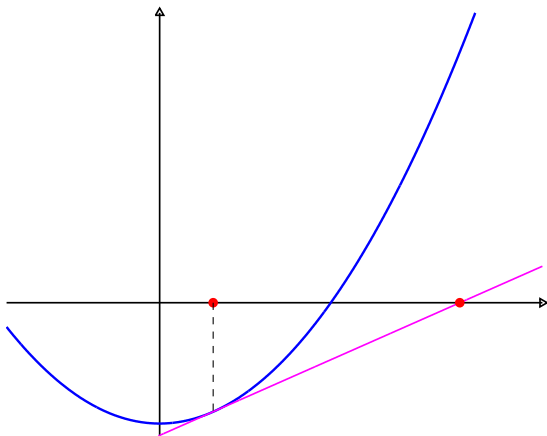
Exemplo 2



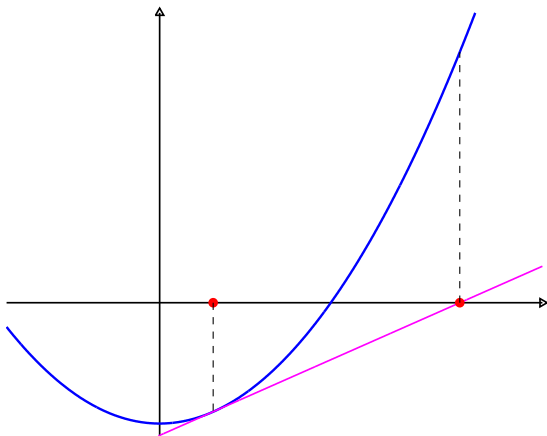
Exemplo 2



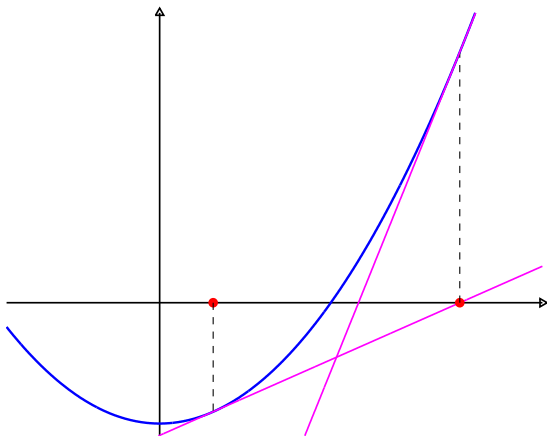
Exemplo 2



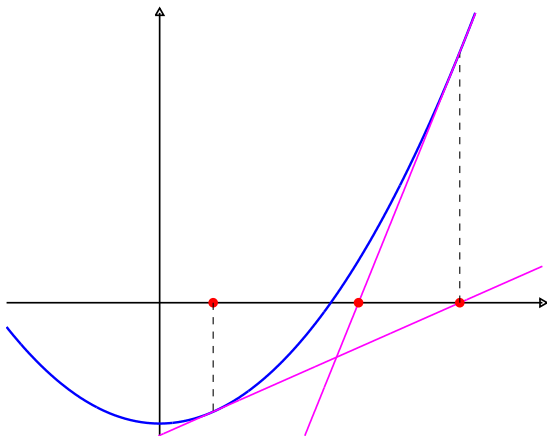
Exemplo 2



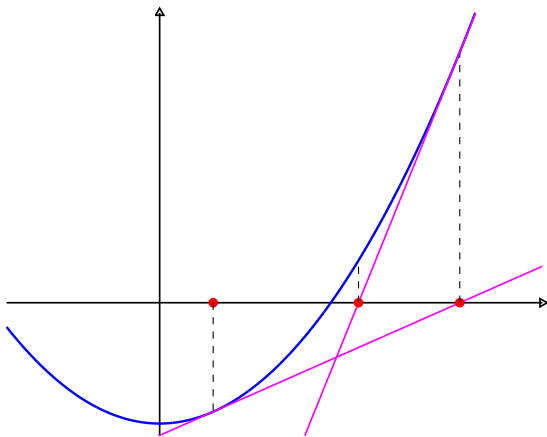
Exemplo 2



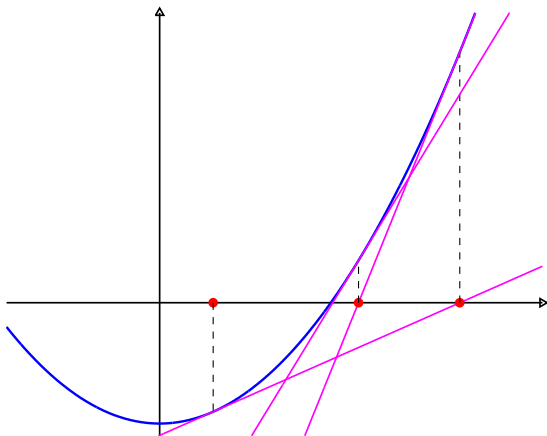
Exemplo 2



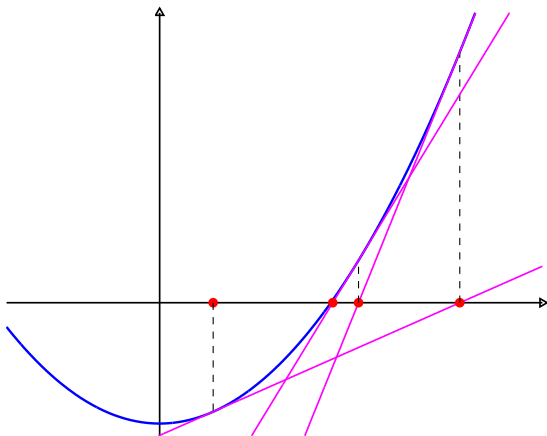
Exemplo 2



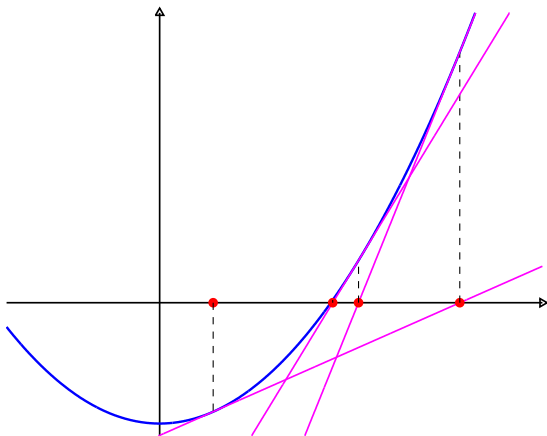
Exemplo 2



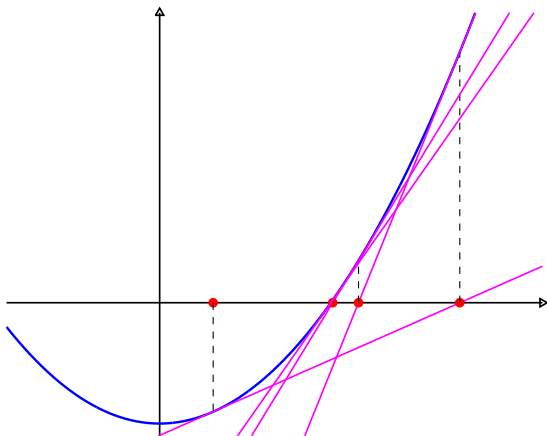
Exemplo 2



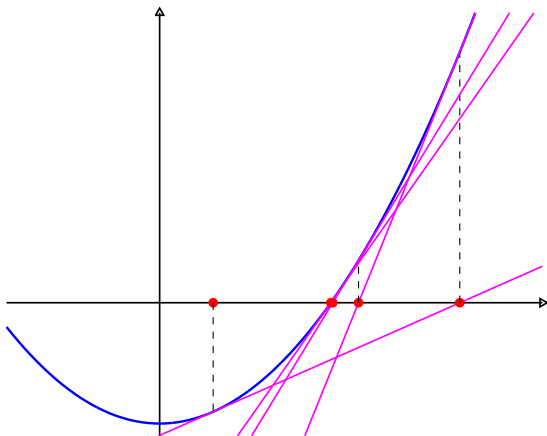
Exemplo 2



Exemplo 2



Exemplo 2

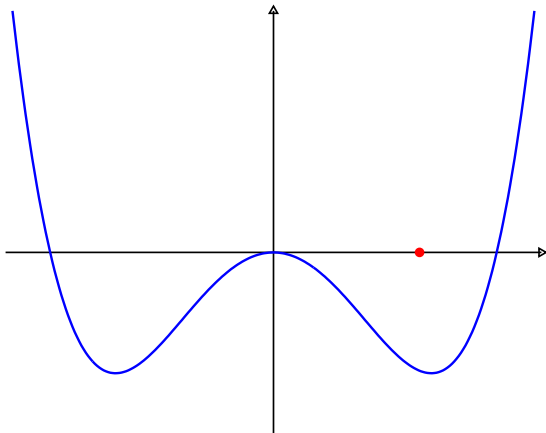


O novo ponto neste caso

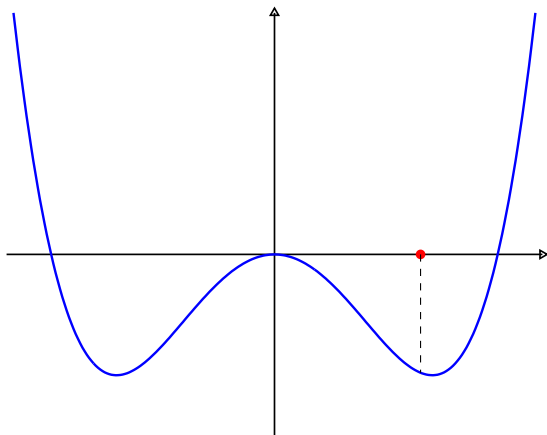
$$f(x) = x^2 - a$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - a}{2x_n} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right)$$

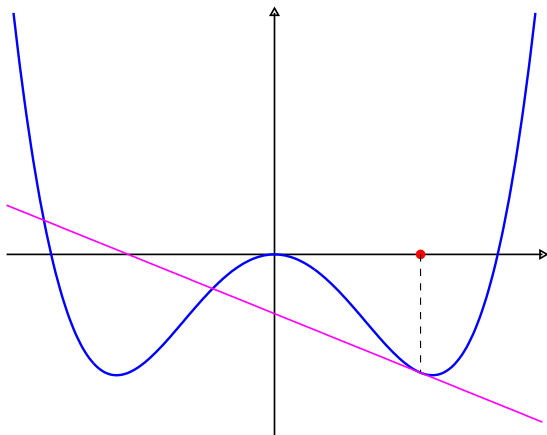
Exemplo 3: o método pode não convergir!



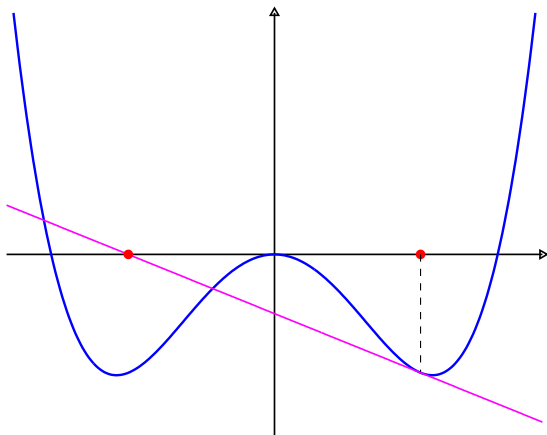
Exemplo 3: o método pode não convergir!



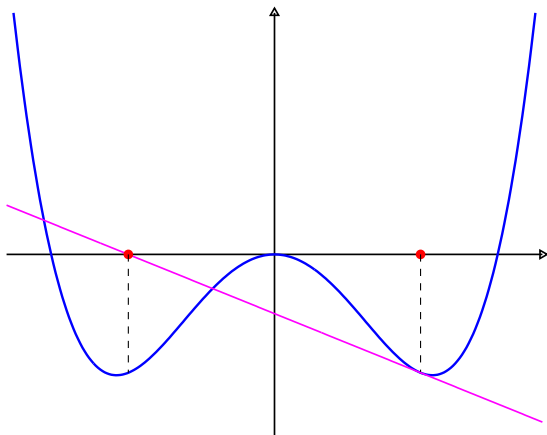
Exemplo 3: o método pode não convergir!



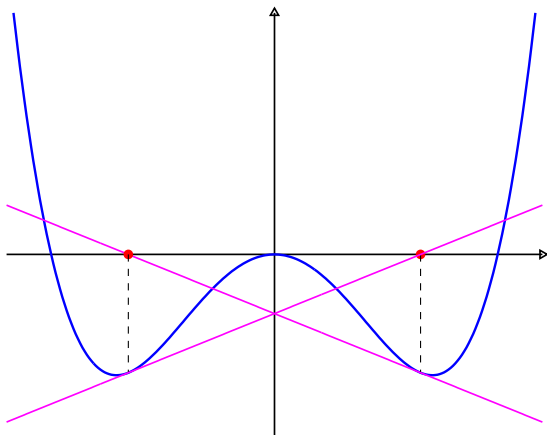
Exemplo 3: o método pode não convergir!



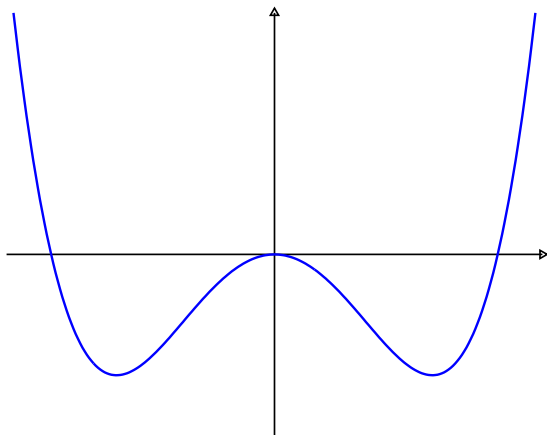
Exemplo 3: o método pode não convergir!



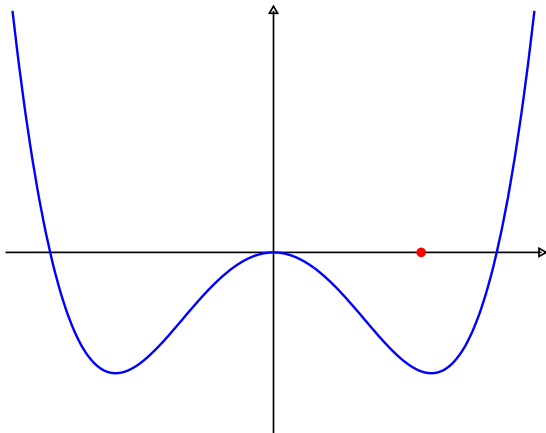
Exemplo 3: o método pode não convergir!



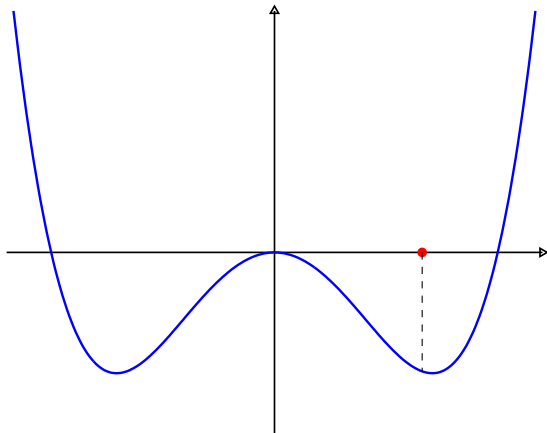
Exemplo 3: o método pode convergir para outro ponto!



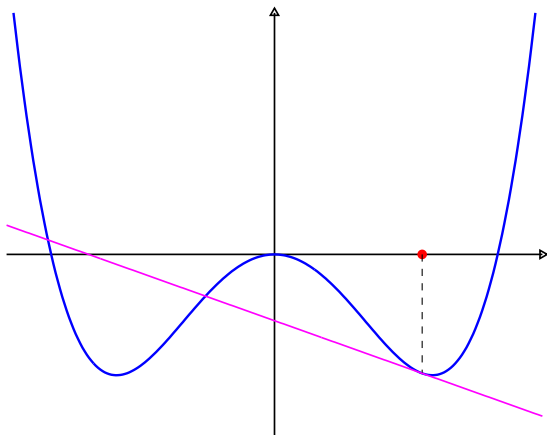
Exemplo 3: o método pode convergir para outro ponto!



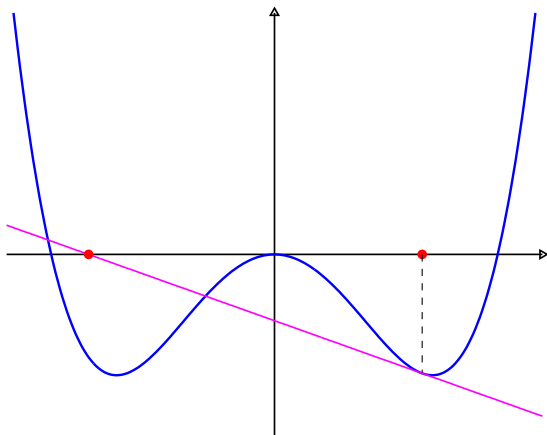
Exemplo 3: o método pode convergir para outro ponto!



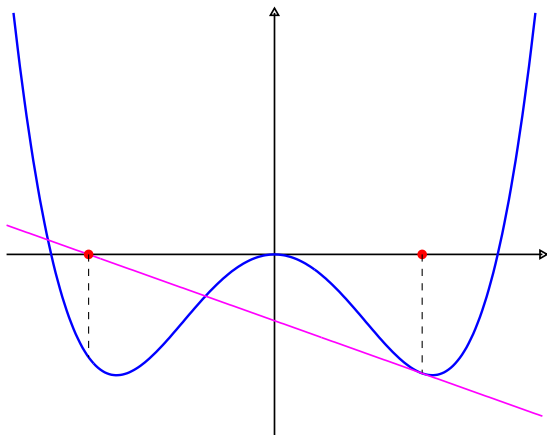
Exemplo 3: o método pode convergir para outro ponto!



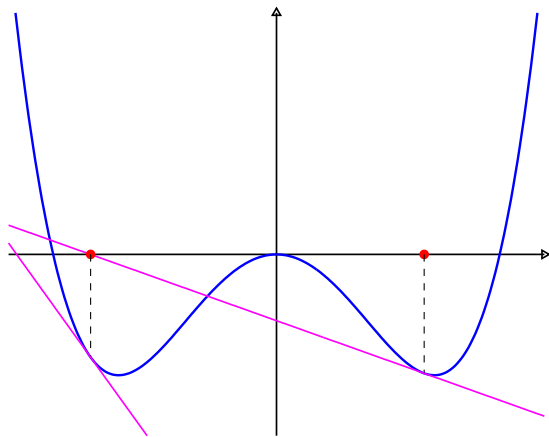
Exemplo 3: o método pode convergir para outro ponto!



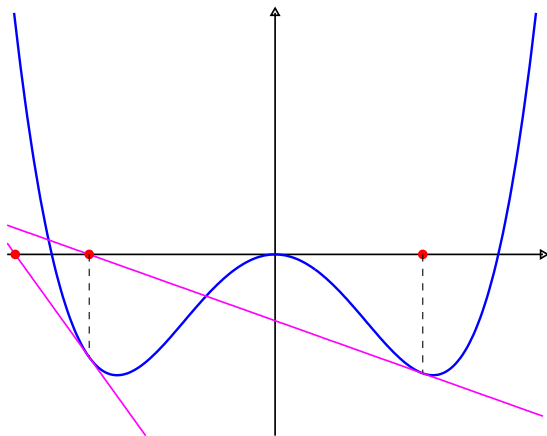
Exemplo 3: o método pode convergir para outro ponto!



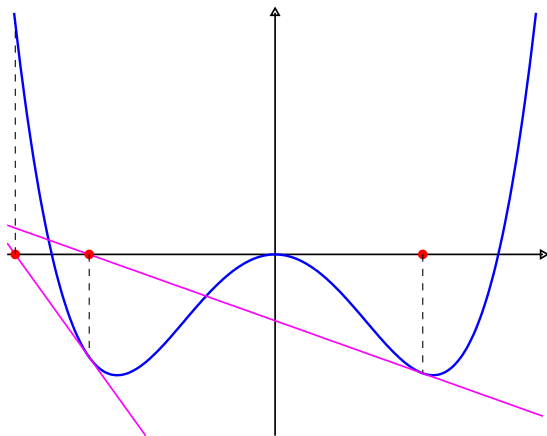
Exemplo 3: o método pode convergir para outro ponto!



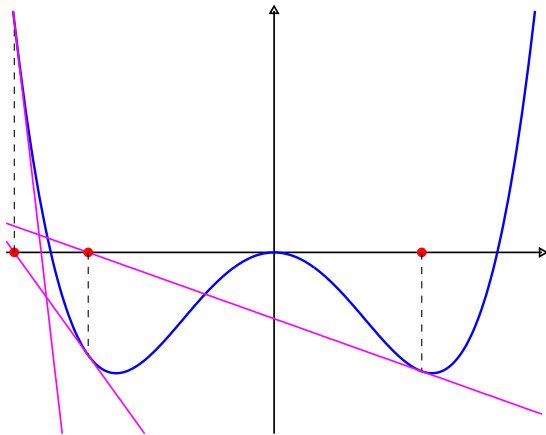
Exemplo 3: o método pode convergir para outro ponto!



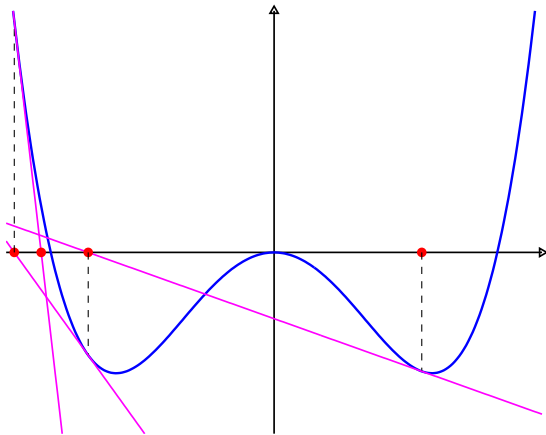
Exemplo 3: o método pode convergir para outro ponto!



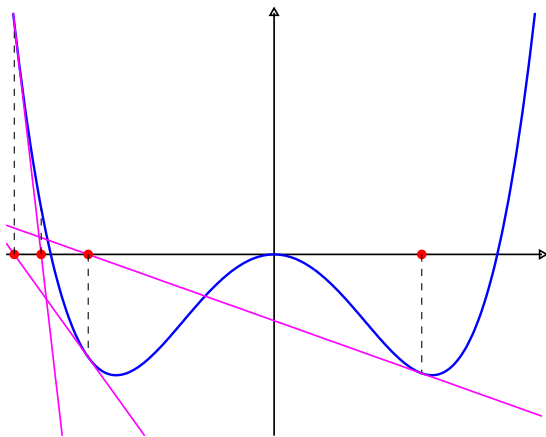
Exemplo 3: o método pode convergir para outro ponto!



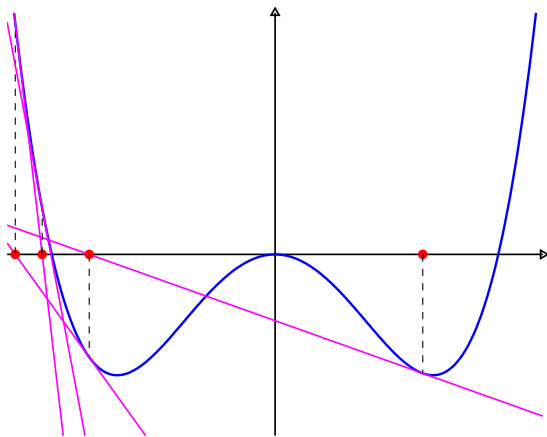
Exemplo 3: o método pode convergir para outro ponto!



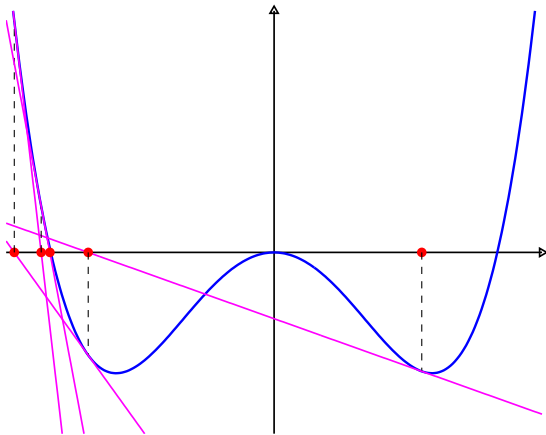
Exemplo 3: o método pode convergir para outro ponto!



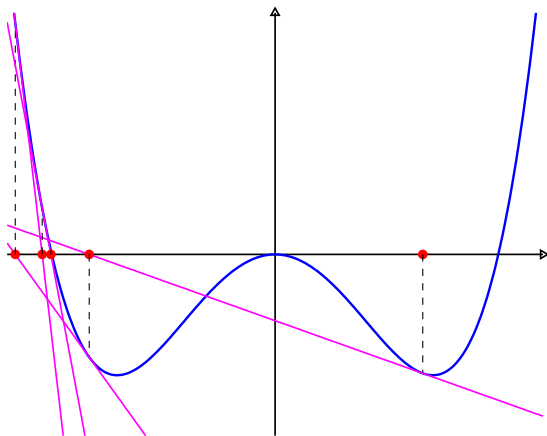
Exemplo 3: o método pode convergir para outro ponto!



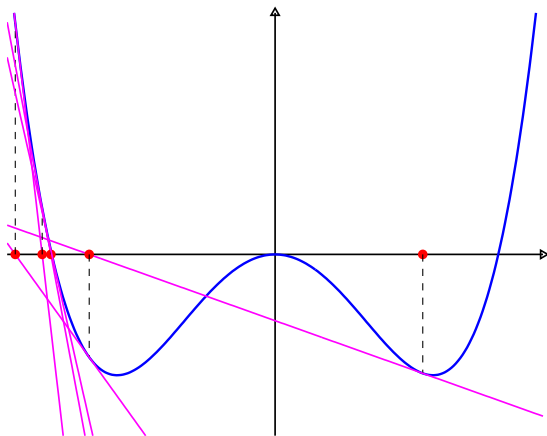
Exemplo 3: o método pode convergir para outro ponto!



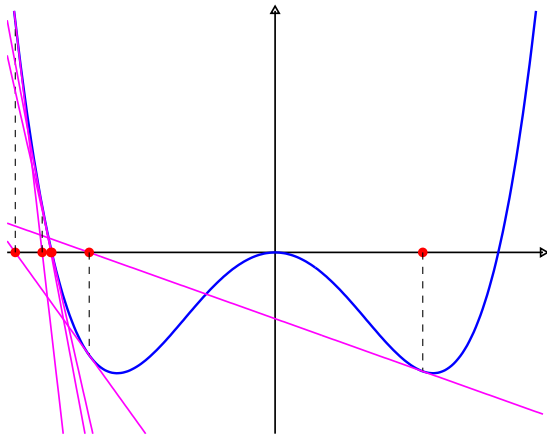
Exemplo 3: o método pode convergir para outro ponto!



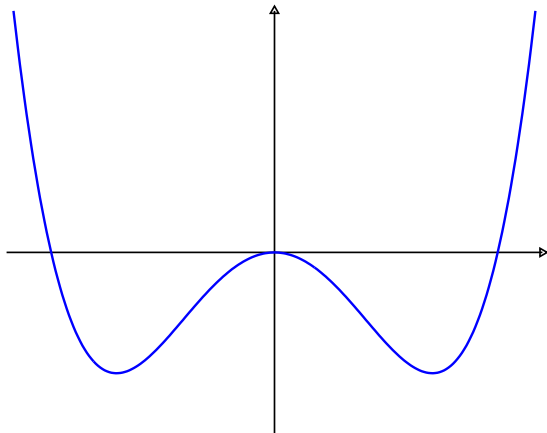
Exemplo 3: o método pode convergir para outro ponto!



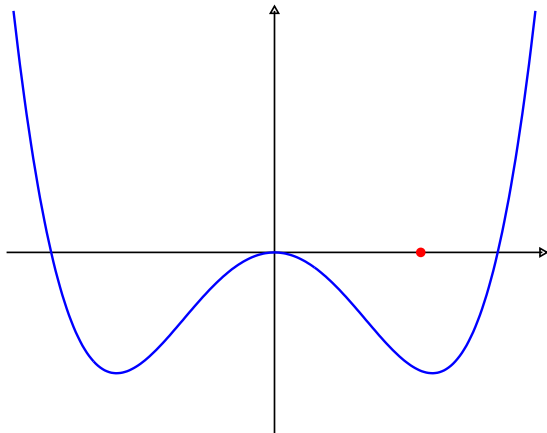
Exemplo 3: o método pode convergir para outro ponto!



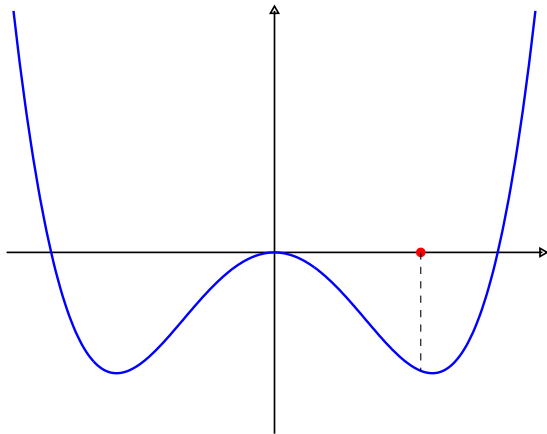
Exemplo 3: o método converge para o ponto esperado!



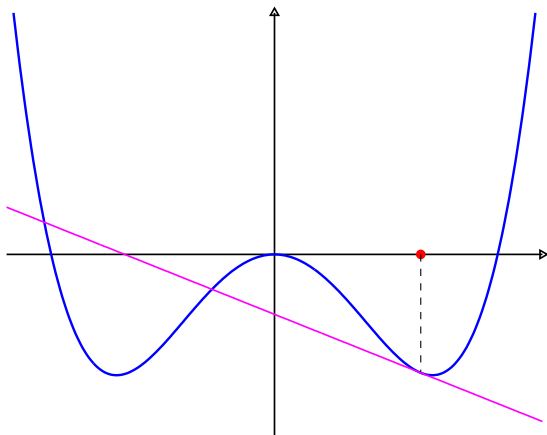
Exemplo 3: o método converge para o ponto esperado!



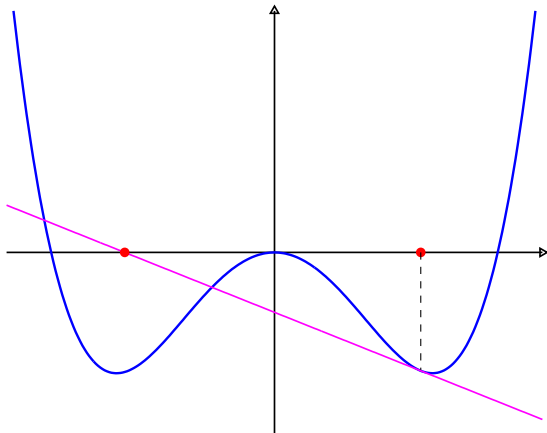
Exemplo 3: o método converge para o ponto esperado!



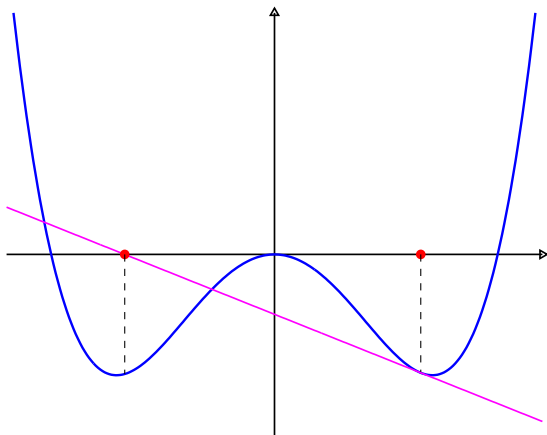
Exemplo 3: o método converge para o ponto esperado!



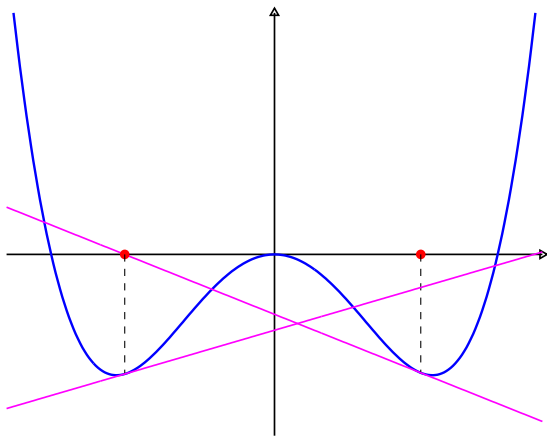
Exemplo 3: o método converge para o ponto esperado!



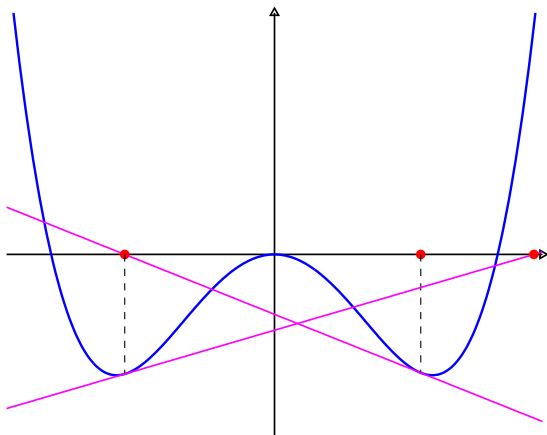
Exemplo 3: o método converge para o ponto esperado!



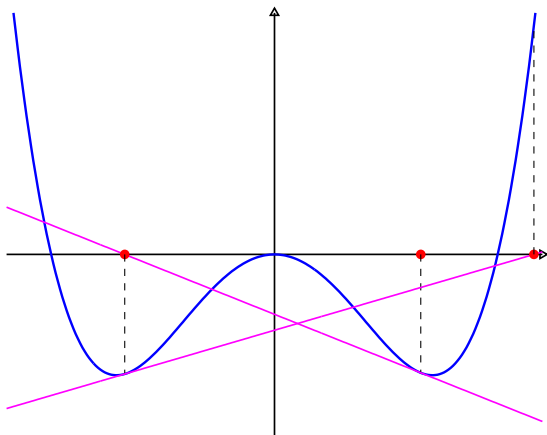
Exemplo 3: o método converge para o ponto esperado!



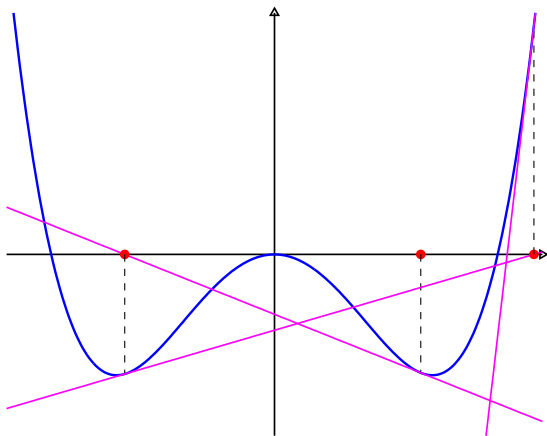
Exemplo 3: o método converge para o ponto esperado!



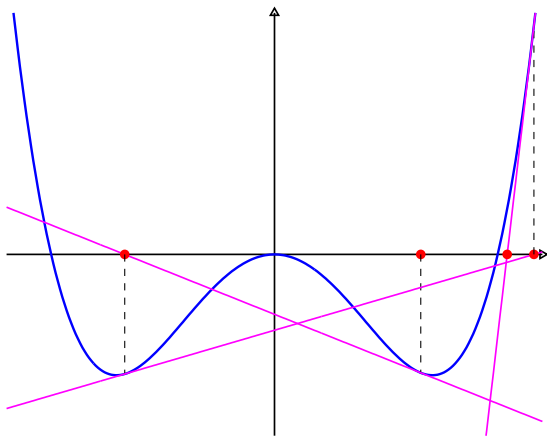
Exemplo 3: o método converge para o ponto esperado!



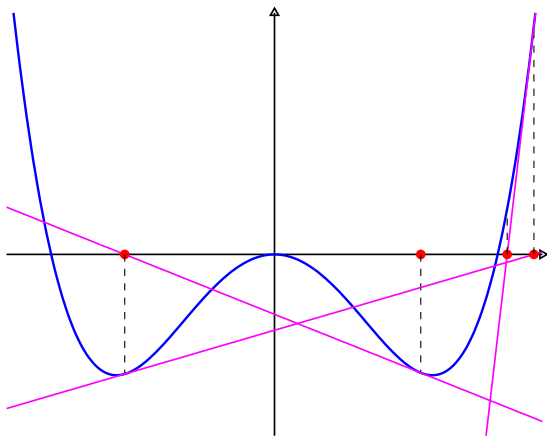
Exemplo 3: o método converge para o ponto esperado!



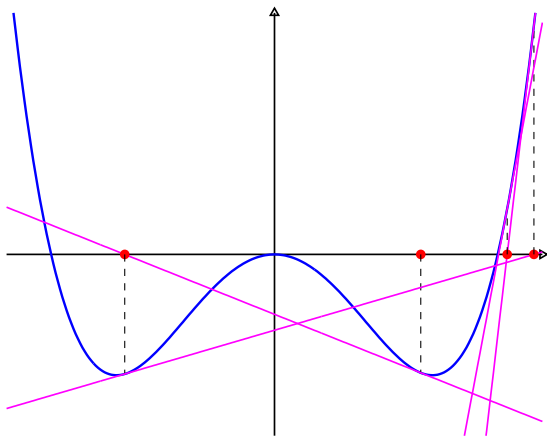
Exemplo 3: o método converge para o ponto esperado!



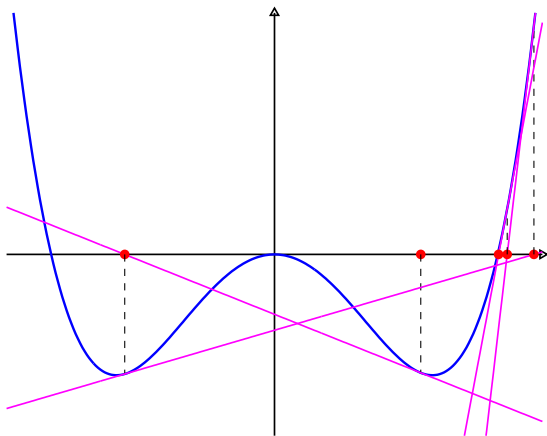
Exemplo 3: o método converge para o ponto esperado!



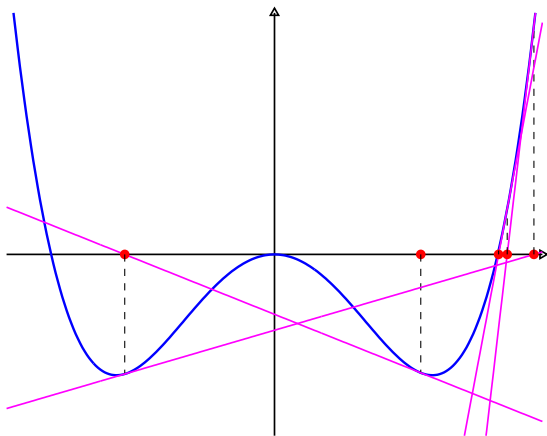
Exemplo 3: o método converge para o ponto esperado!



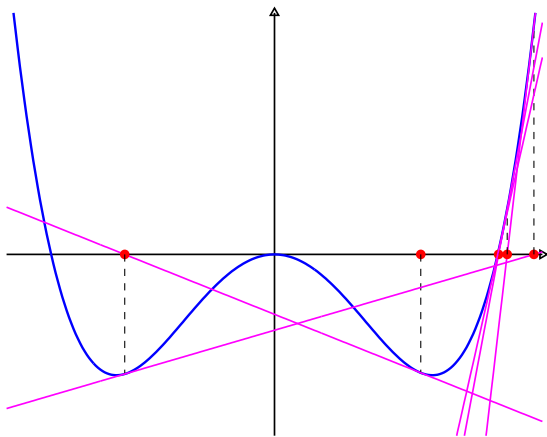
Exemplo 3: o método converge para o ponto esperado!



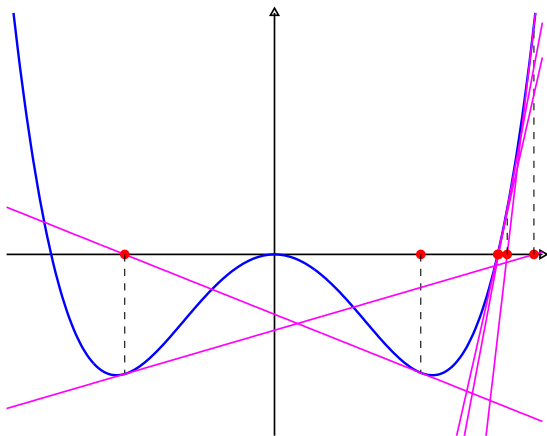
Exemplo 3: o método converge para o ponto esperado!



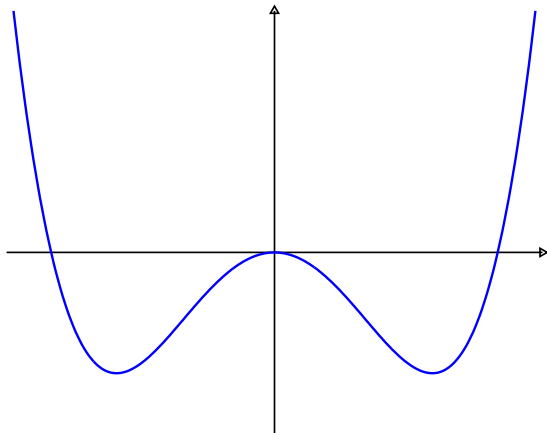
Exemplo 3: o método converge para o ponto esperado!



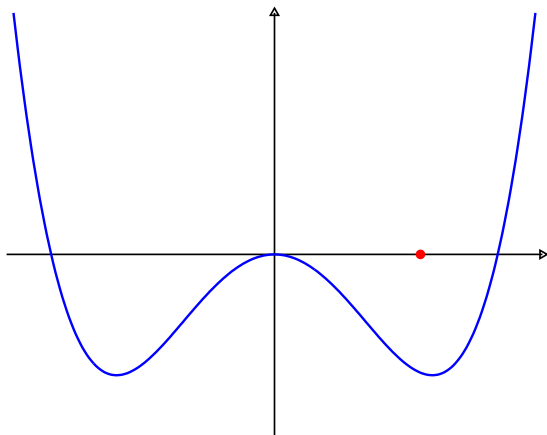
Exemplo 3: o método converge para o ponto esperado!



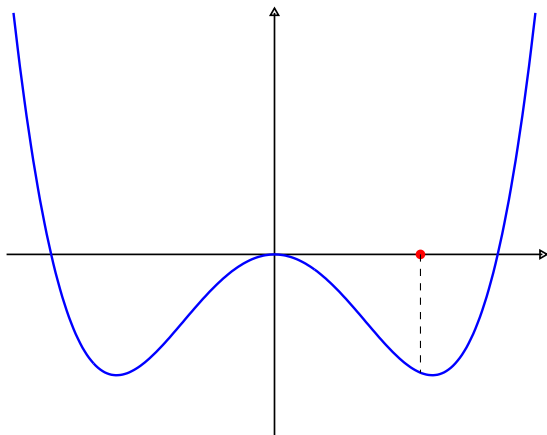
Exemplo 3: mais um ponto inicial!



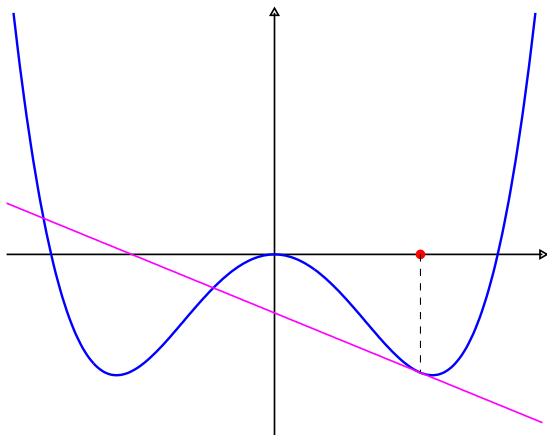
Exemplo 3: mais um ponto inicial!



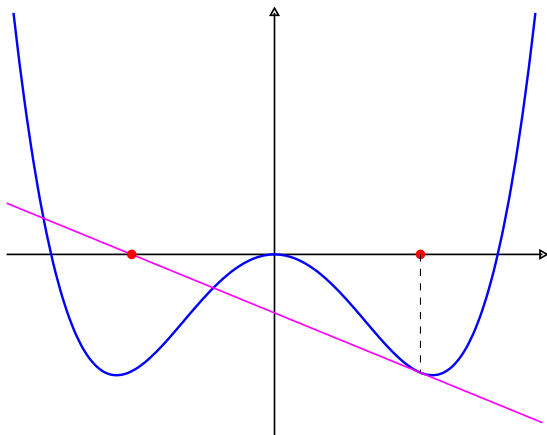
Exemplo 3: mais um ponto inicial!



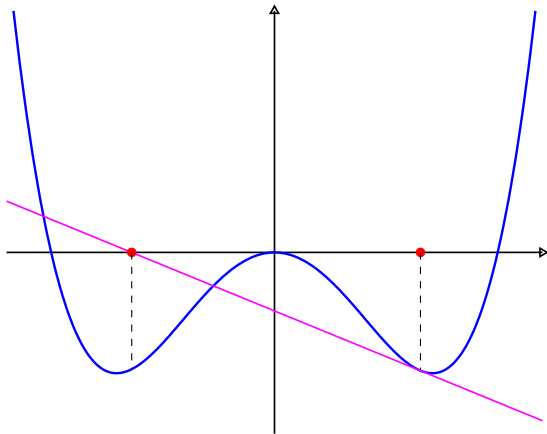
Exemplo 3: mais um ponto inicial!



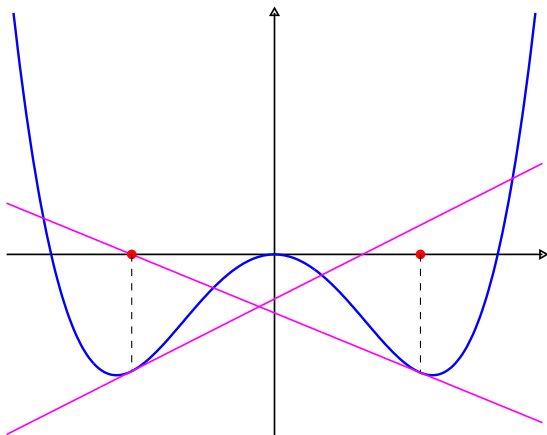
Exemplo 3: mais um ponto inicial!



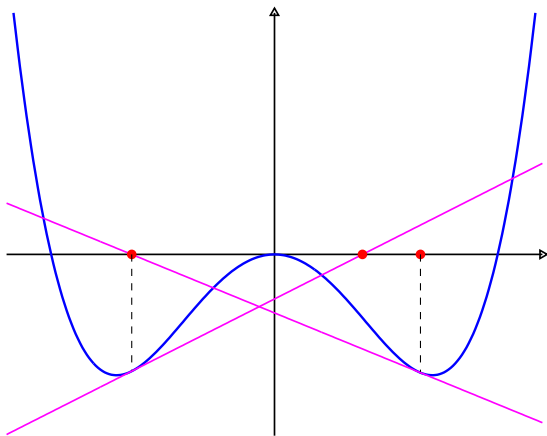
Exemplo 3: mais um ponto inicial!



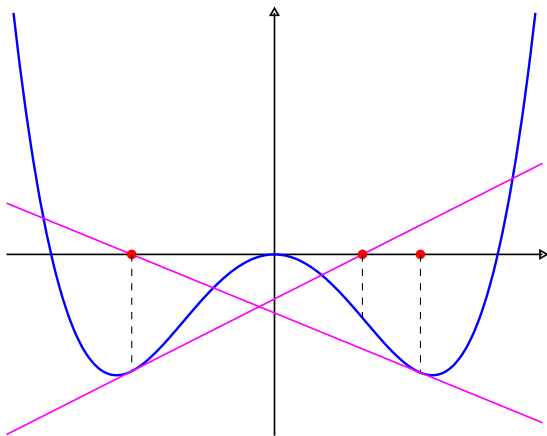
Exemplo 3: mais um ponto inicial!



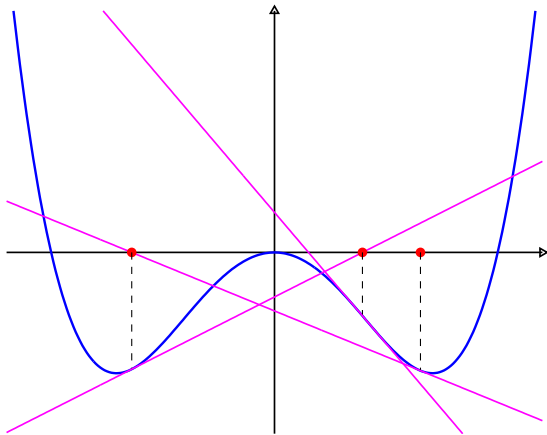
Exemplo 3: mais um ponto inicial!



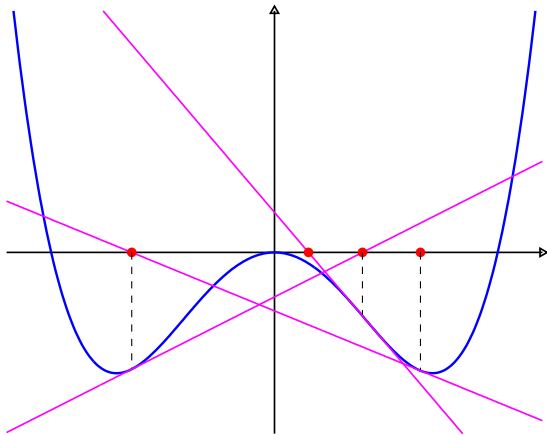
Exemplo 3: mais um ponto inicial!



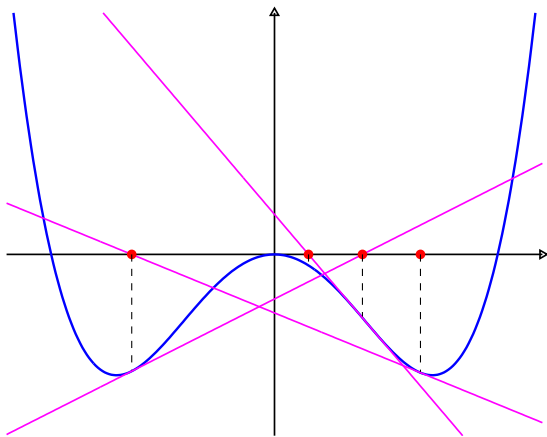
Exemplo 3: mais um ponto inicial!



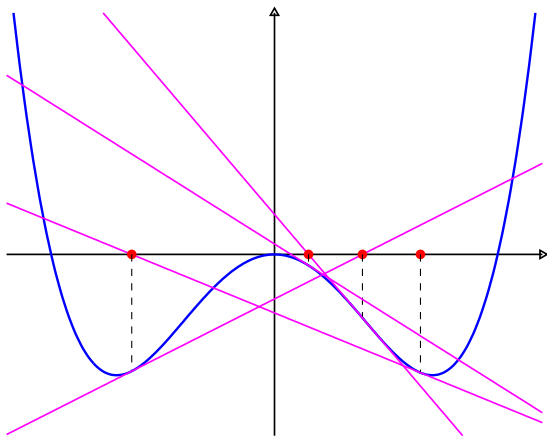
Exemplo 3: mais um ponto inicial!



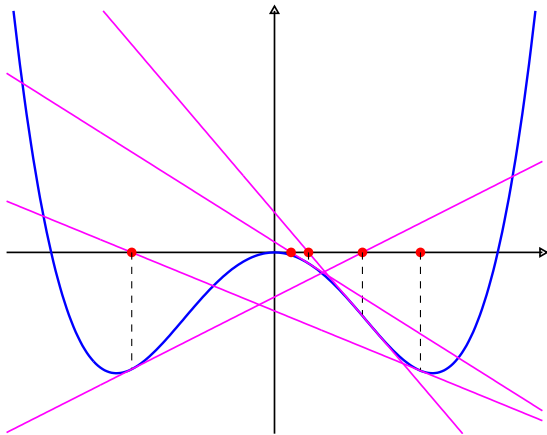
Exemplo 3: mais um ponto inicial!



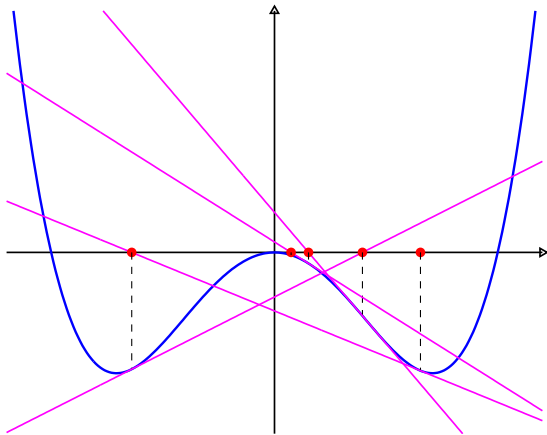
Exemplo 3: mais um ponto inicial!



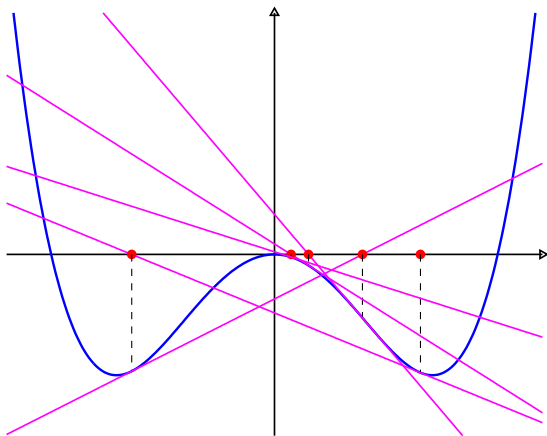
Exemplo 3: mais um ponto inicial!



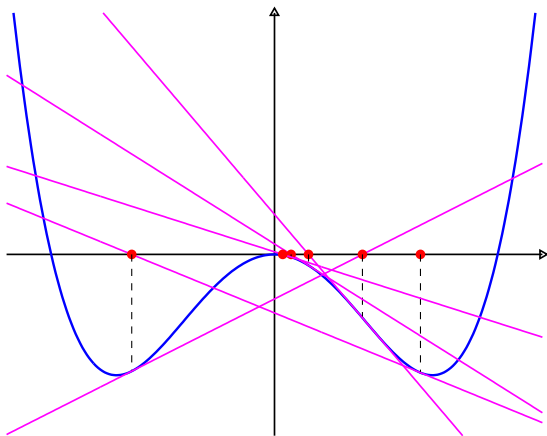
Exemplo 3: mais um ponto inicial!



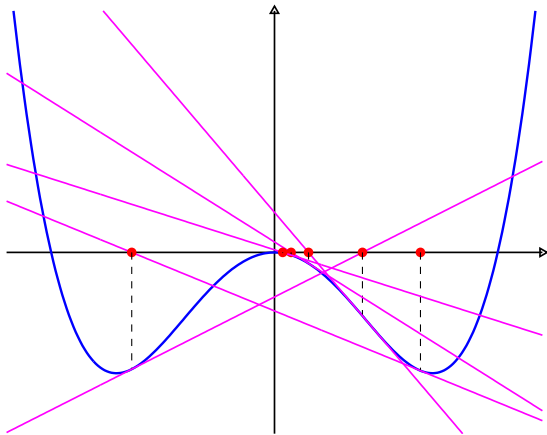
Exemplo 3: mais um ponto inicial!



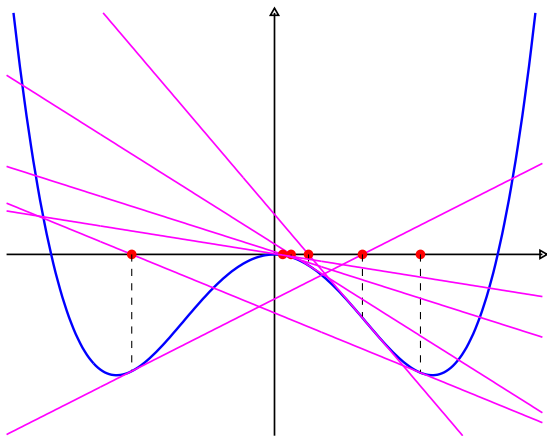
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