

A integral de Riemann

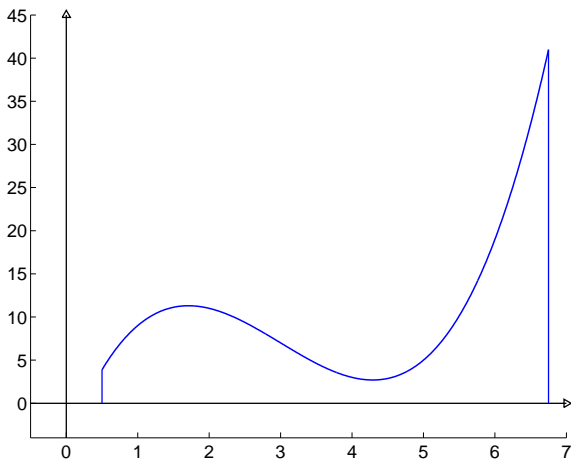
Ademir Alves Ribeiro

Maio / 2007

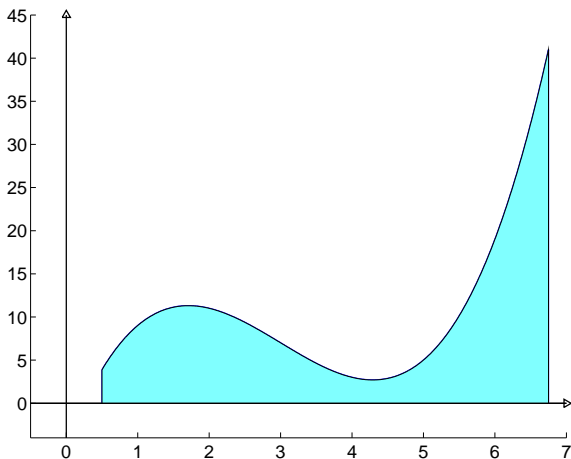


- 1 Integral de Riemann
- 2 Área do círculo
- 3 Comprimento de curva

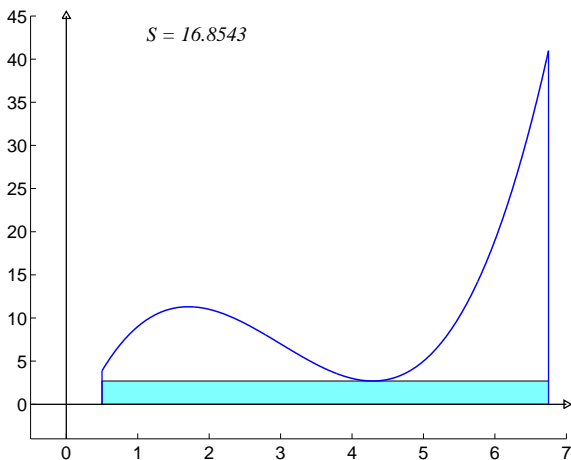
A área sob uma curva



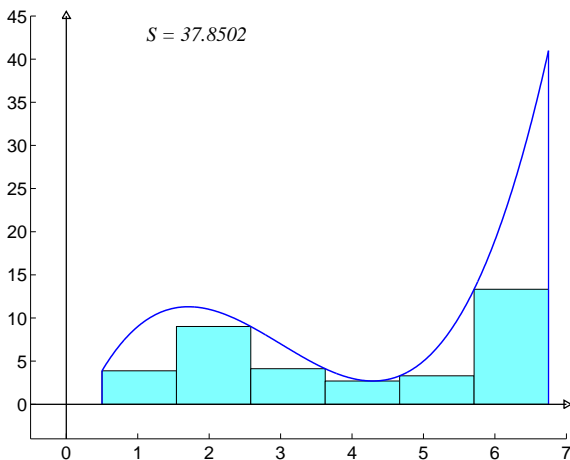
A área sob uma curva



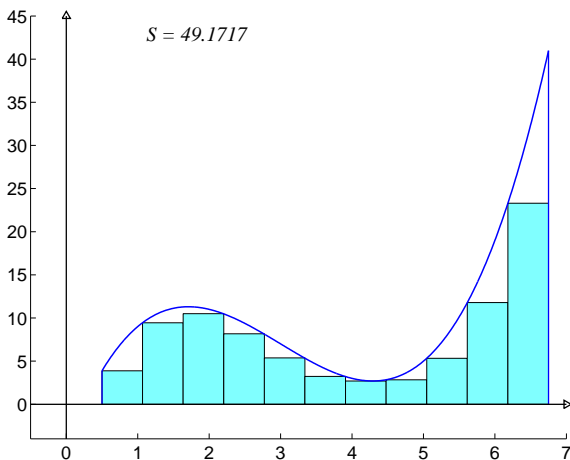
O limite da soma das áreas dos retângulos inscritos



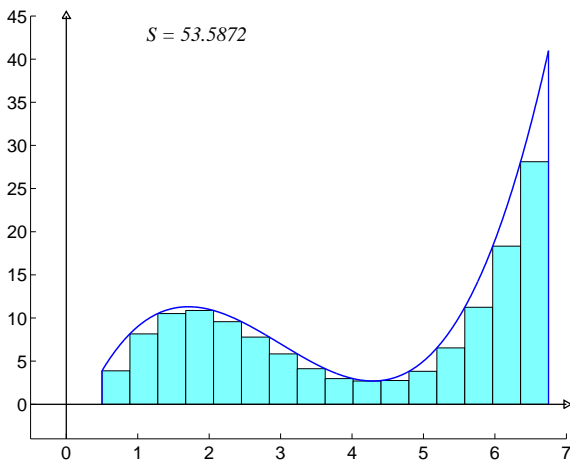
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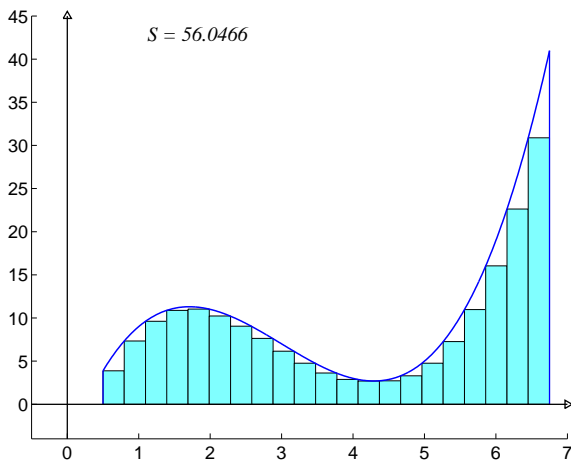
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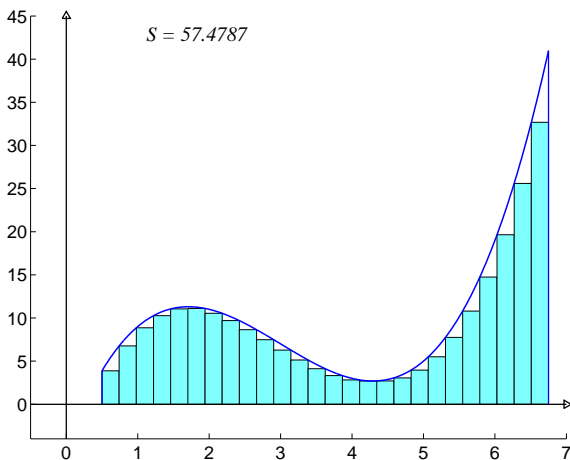
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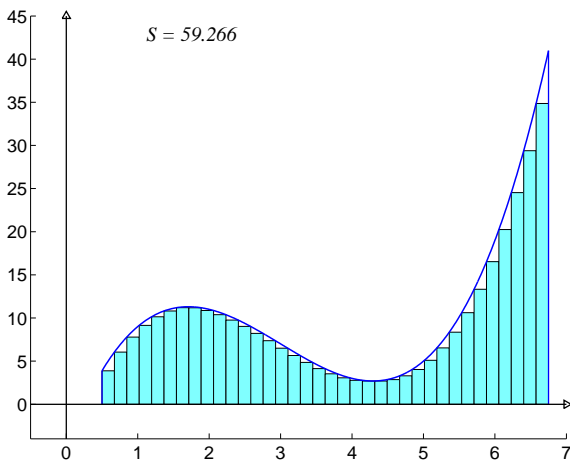
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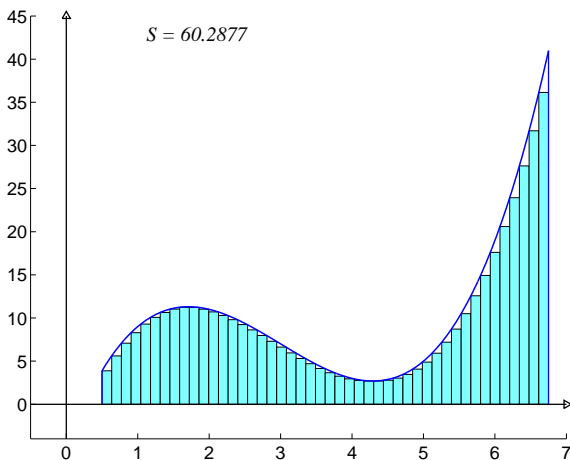
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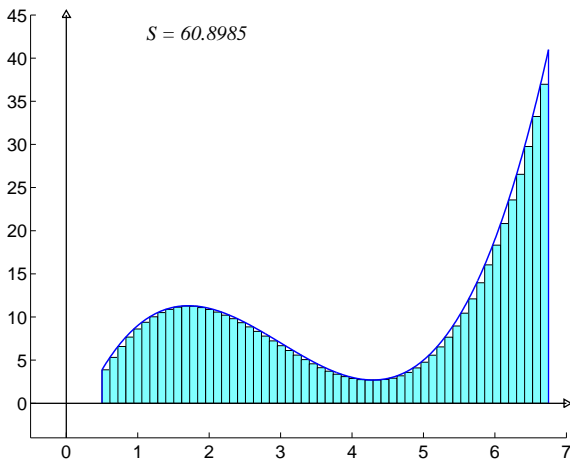
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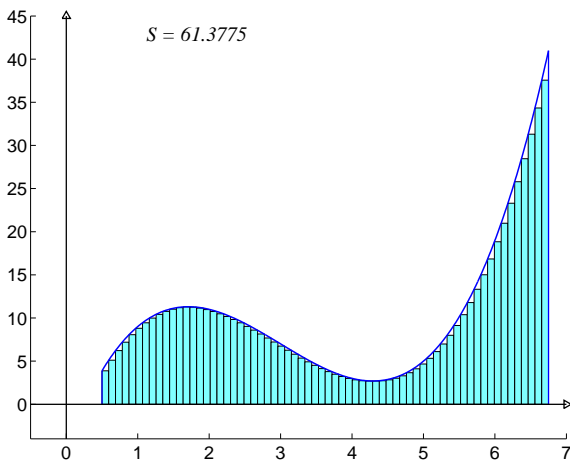
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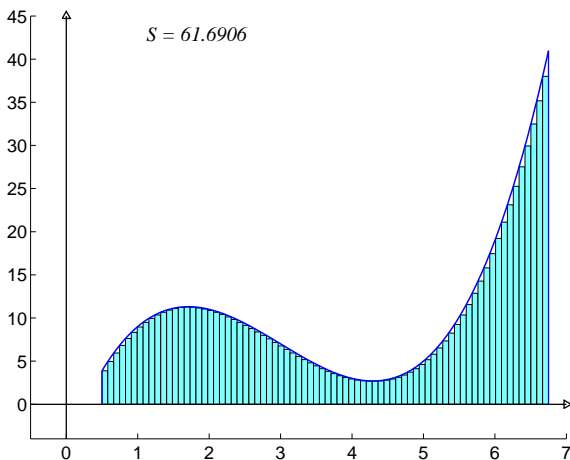
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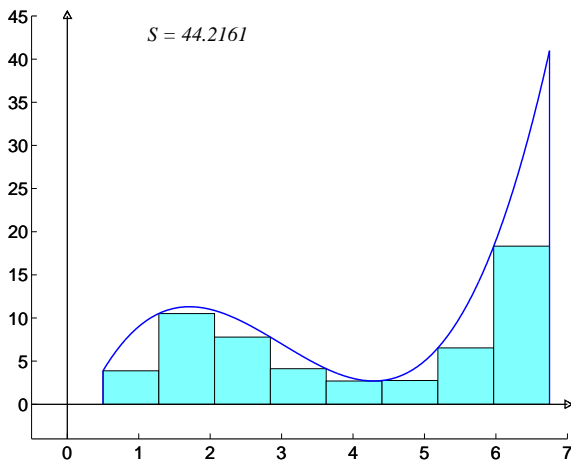
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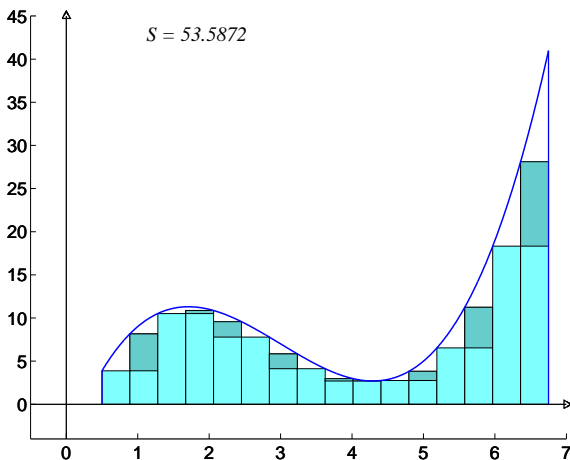
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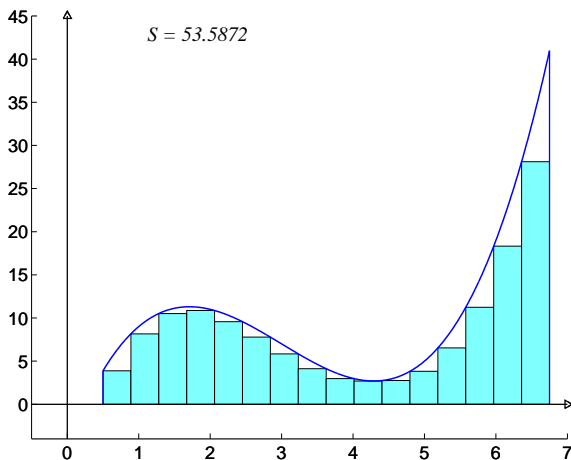
As somas inferiores formam uma sequência crescente



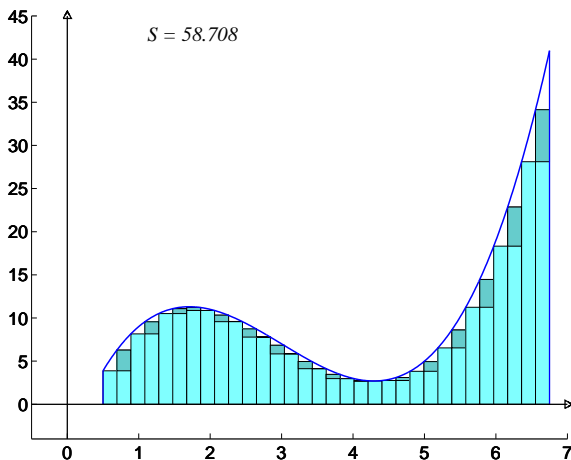
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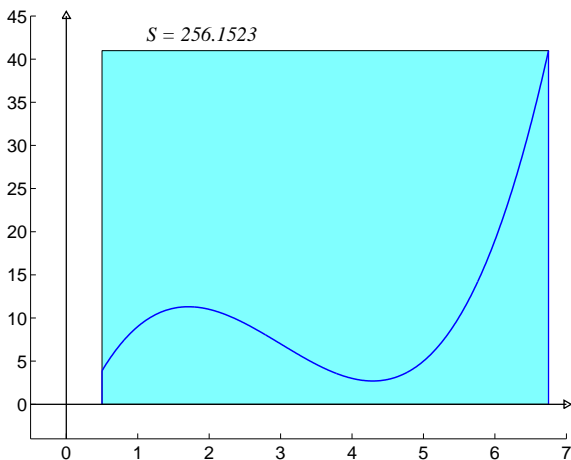
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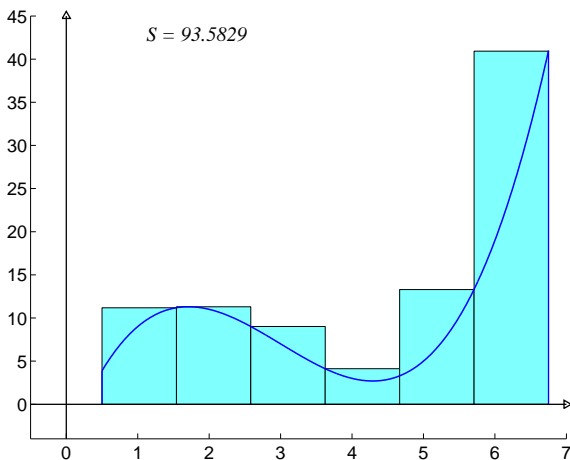
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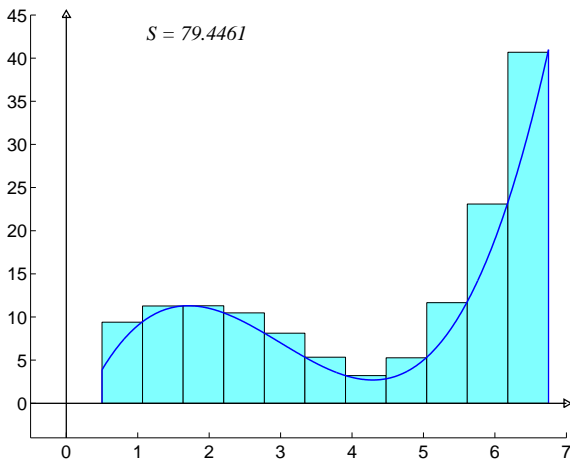
O limite da soma das áreas dos retângulos circunscritos



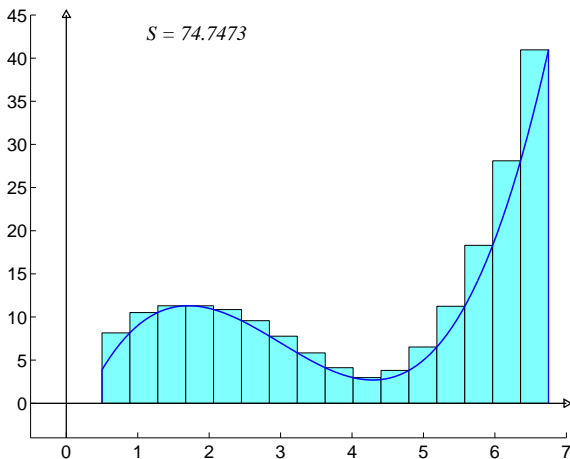
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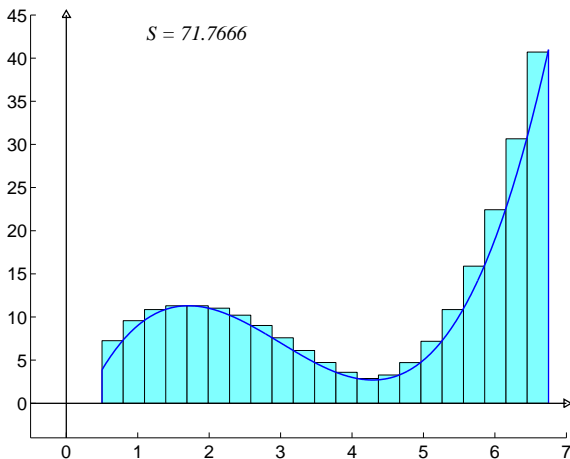
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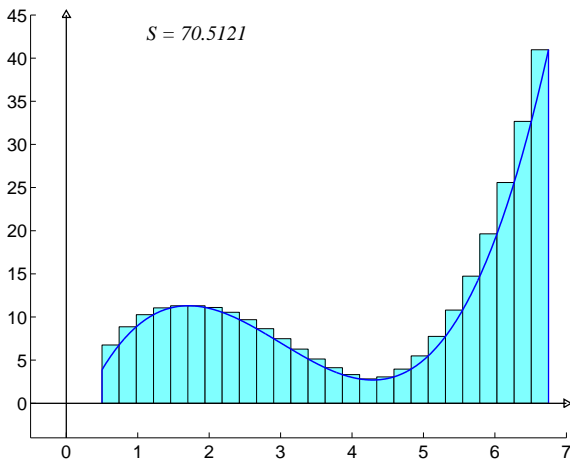
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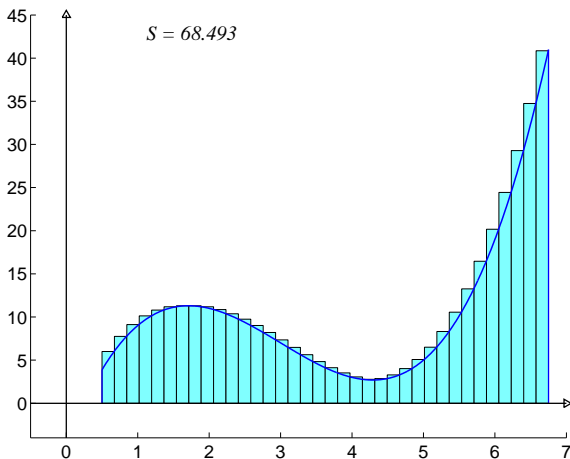
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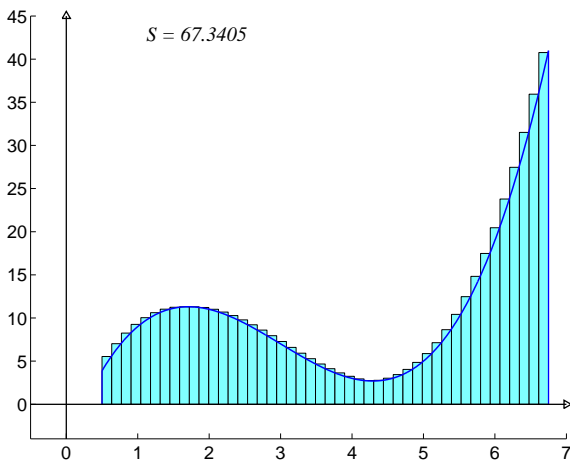
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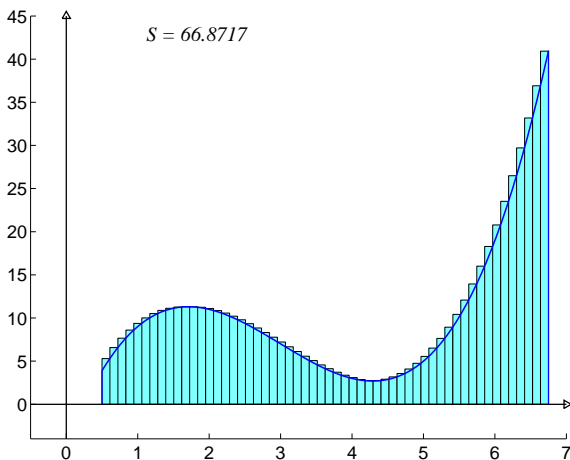
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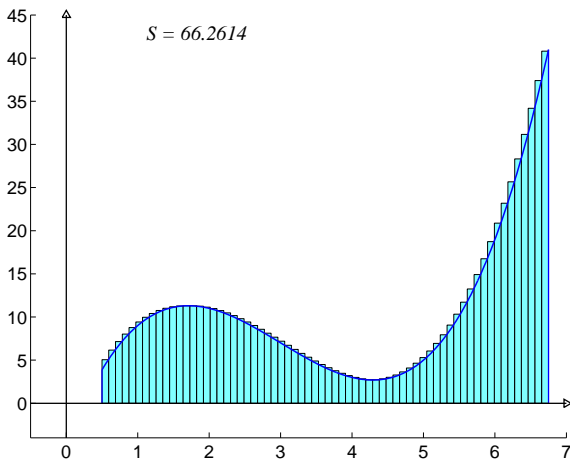
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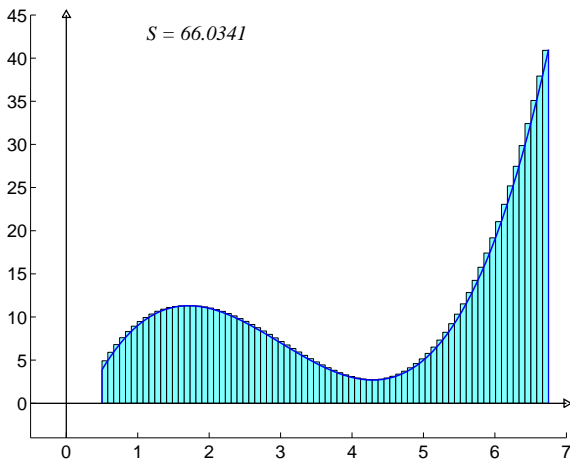
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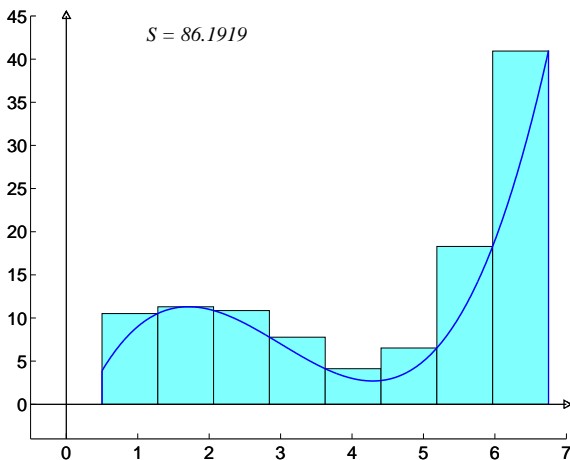
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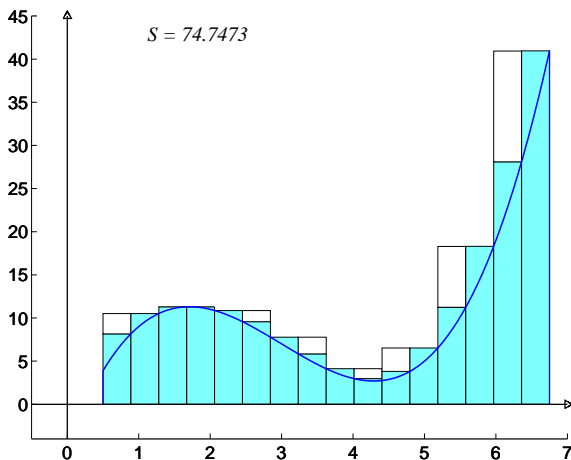
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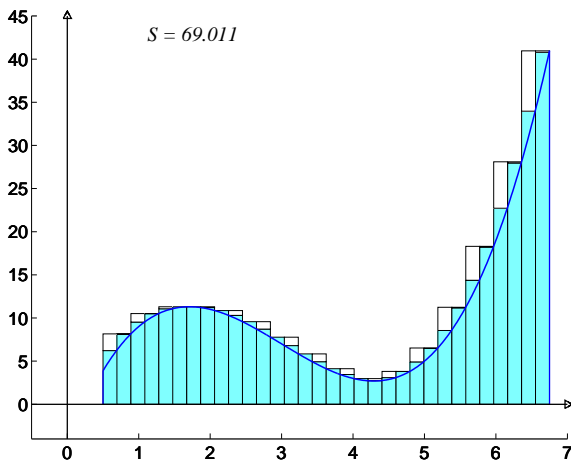
As somas superiores formam uma sequência decrescente



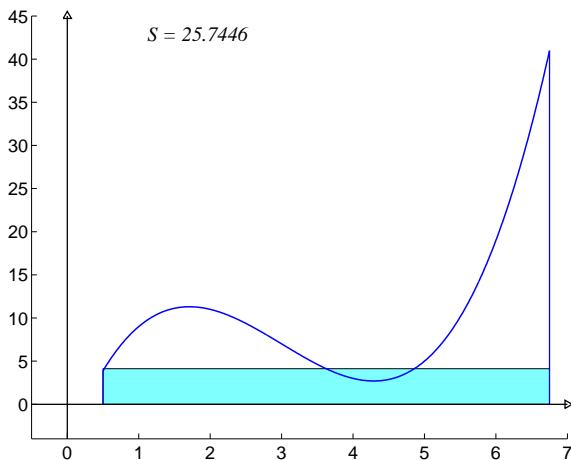
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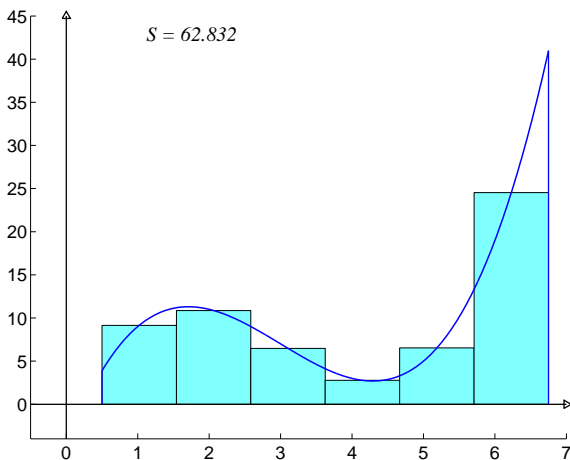
As somas superiores formam uma sequência decrescente



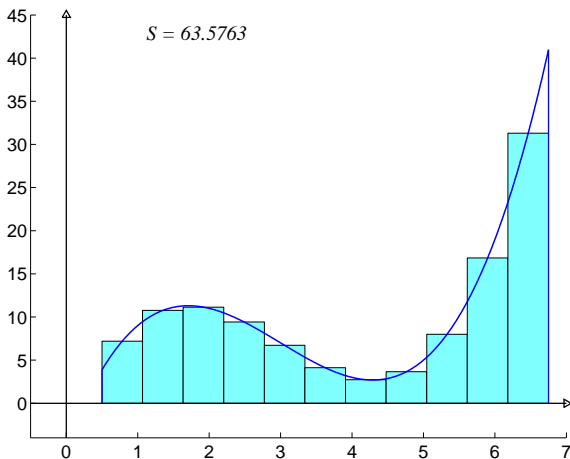
O limite da soma das áreas de retângulos



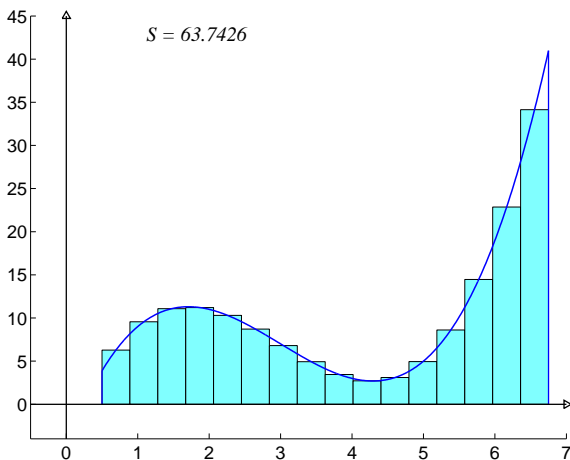
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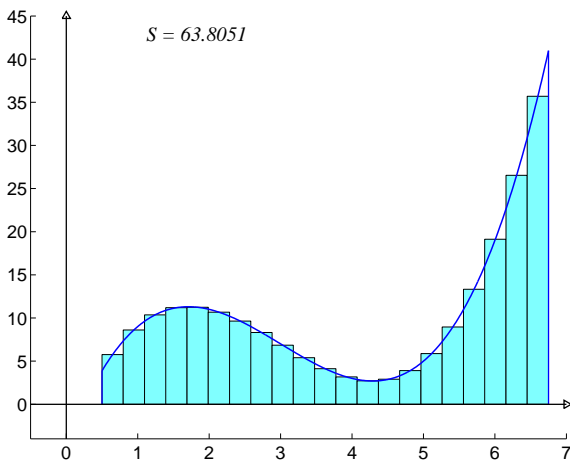
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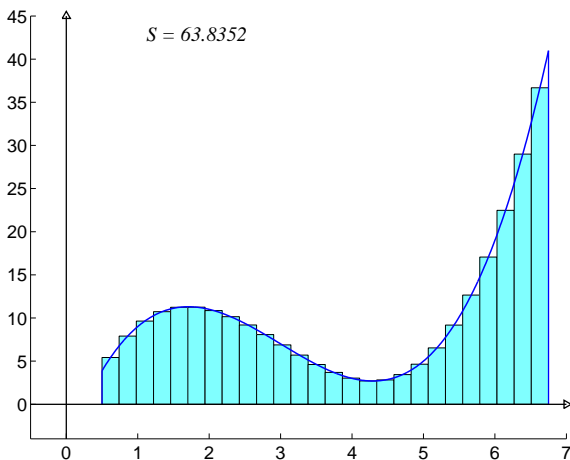
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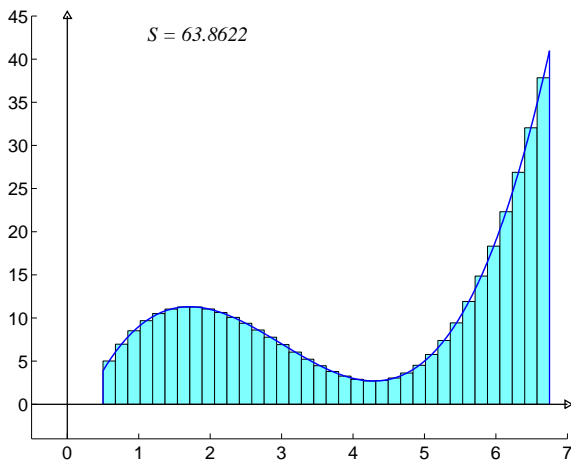
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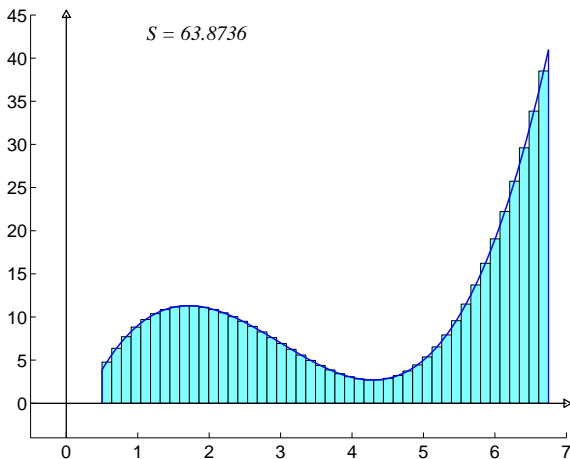
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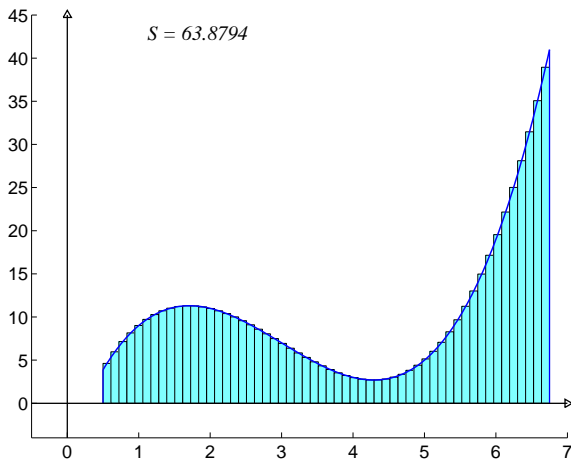
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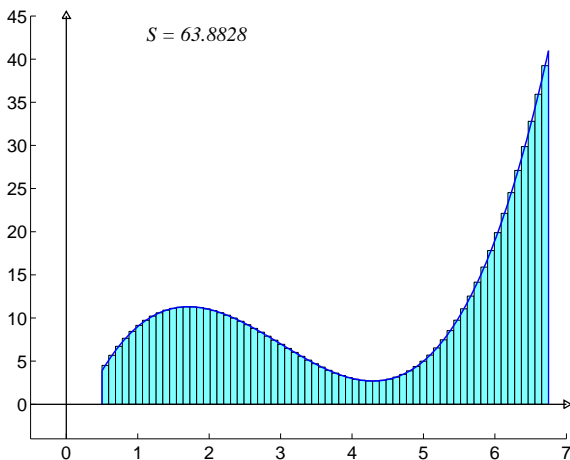
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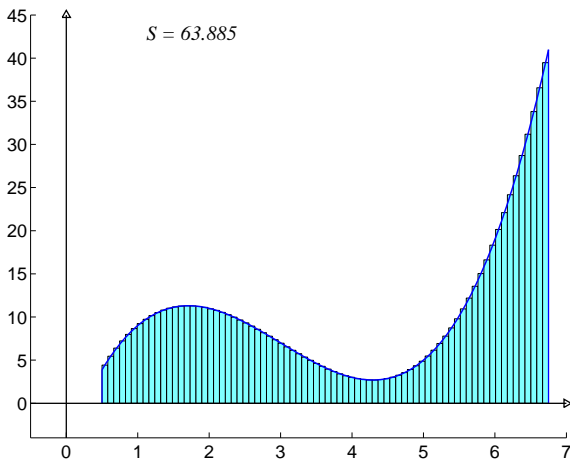
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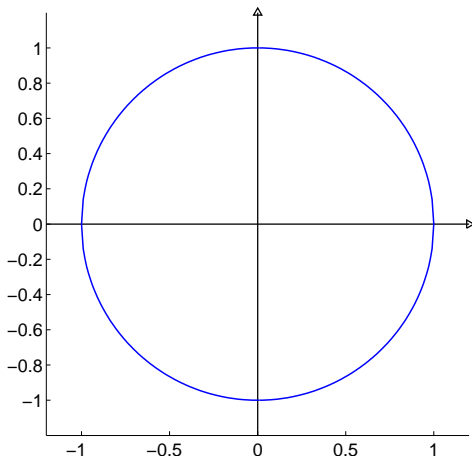
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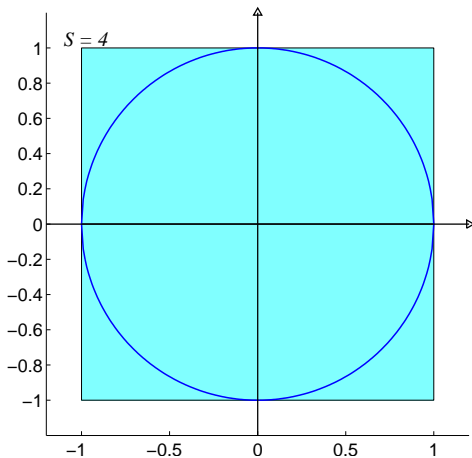
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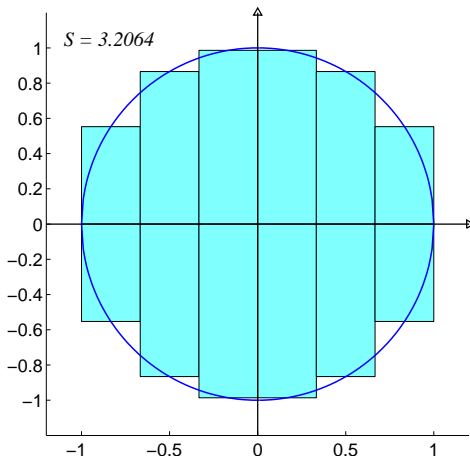
Aproximações da área do círculo



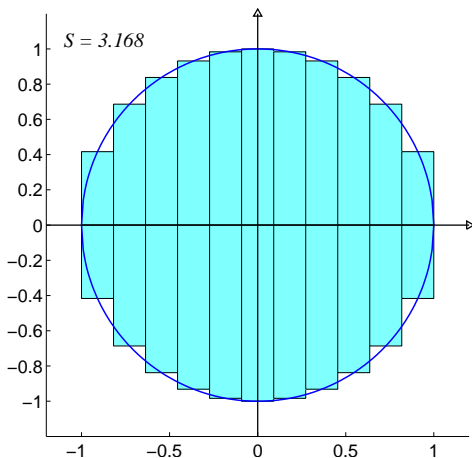
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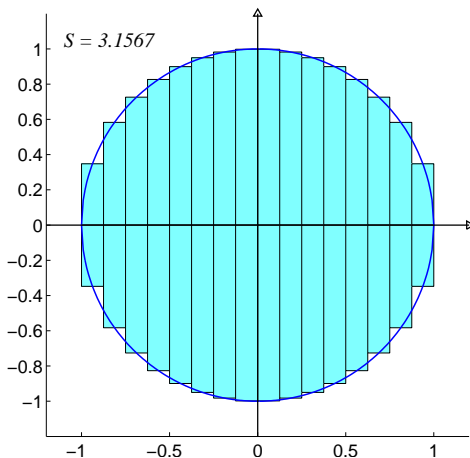
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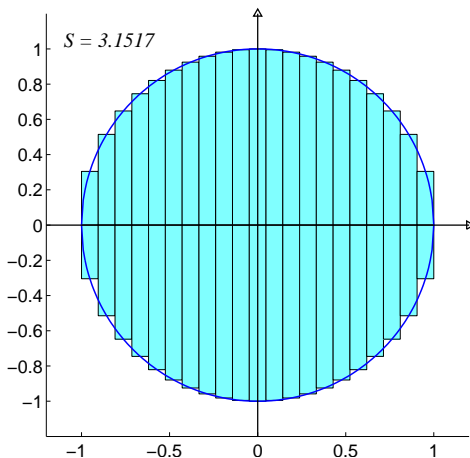
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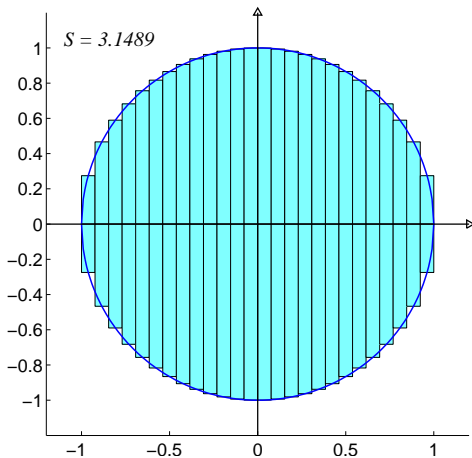
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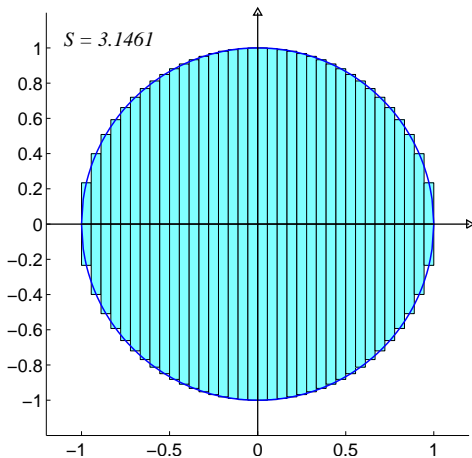
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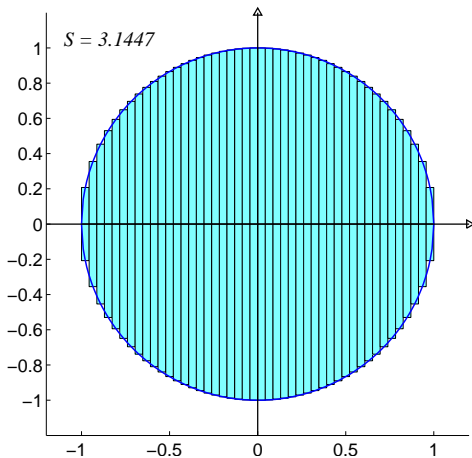
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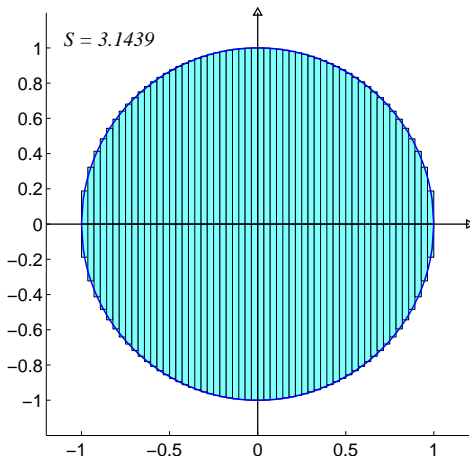
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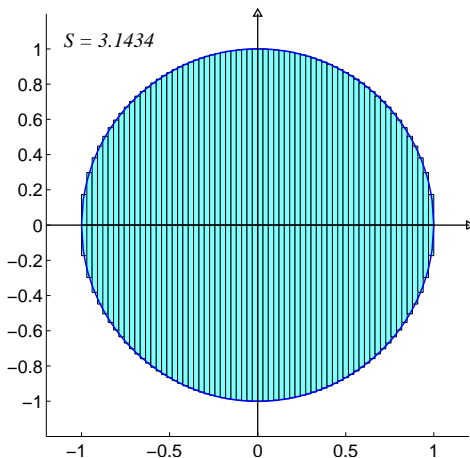
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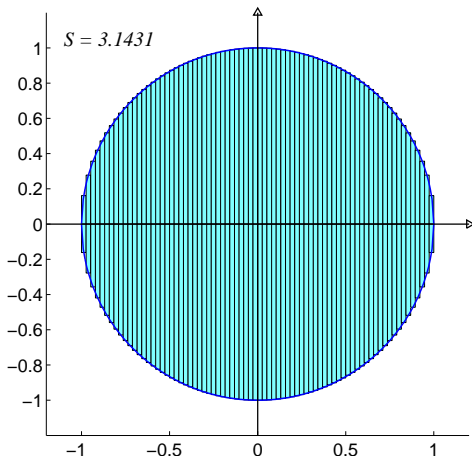
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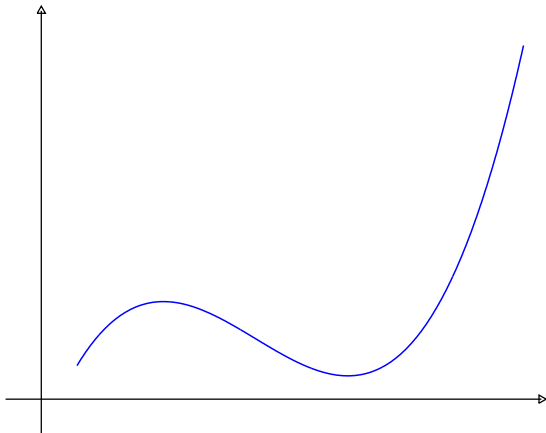
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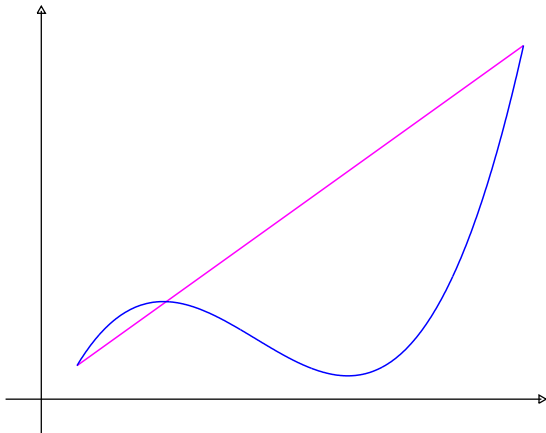
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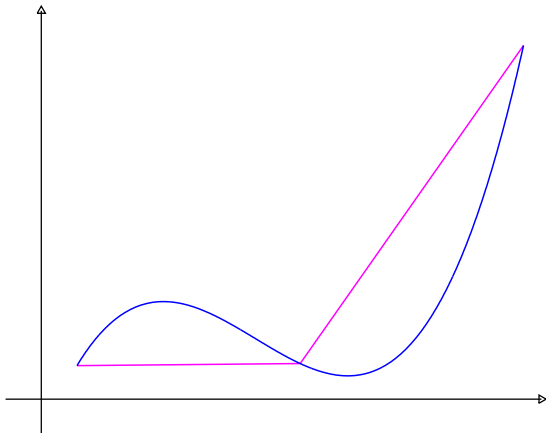
O comprimento de uma curva



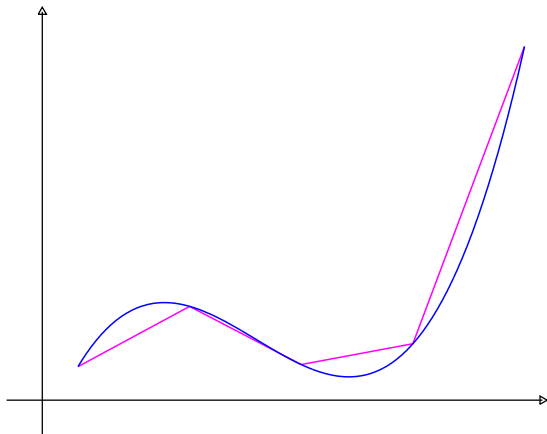
O limite dos comprimentos da poligonal



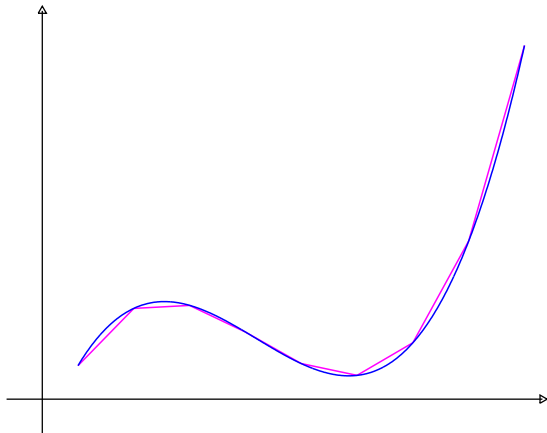
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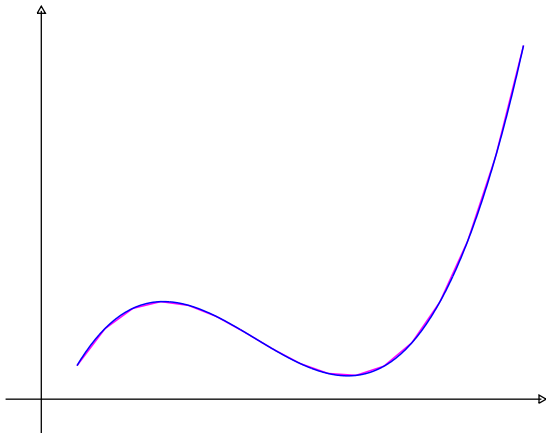
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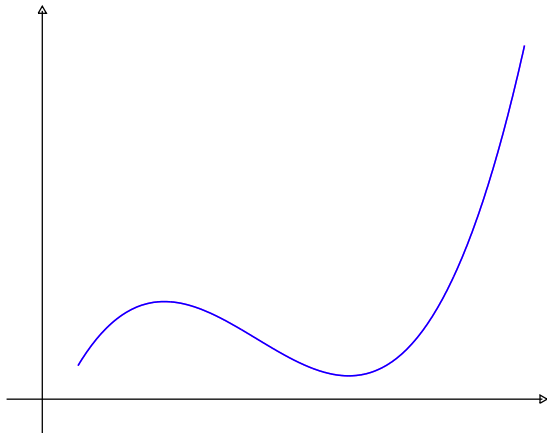
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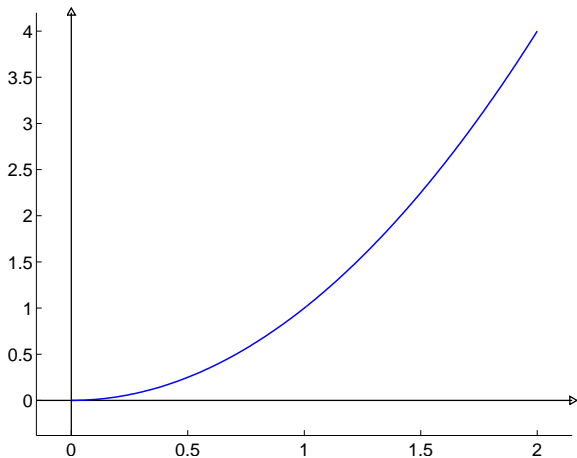
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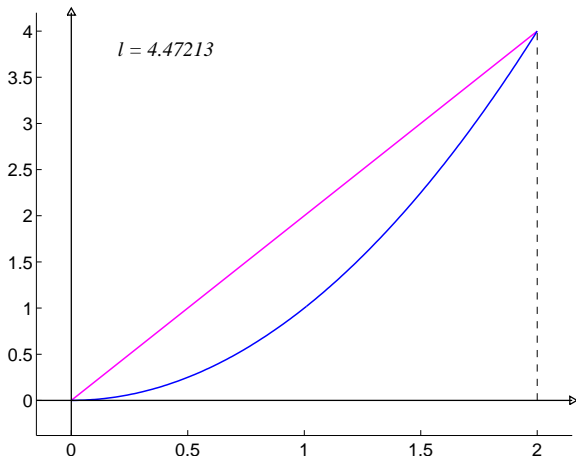
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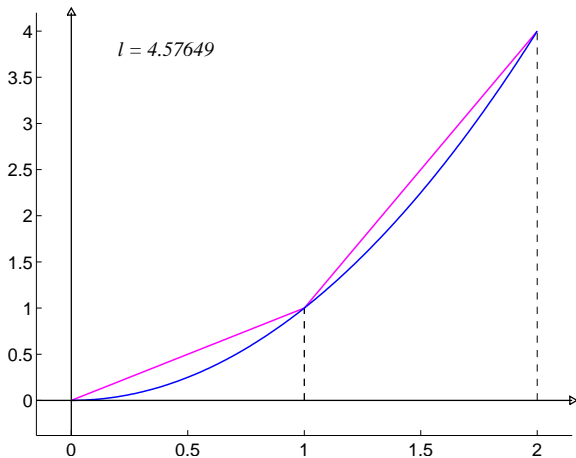
Exemplo 1: $f(x) = x^2, x \in [0, 2]$



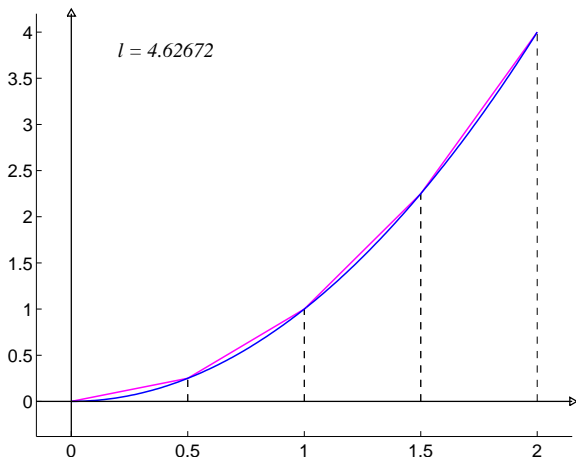
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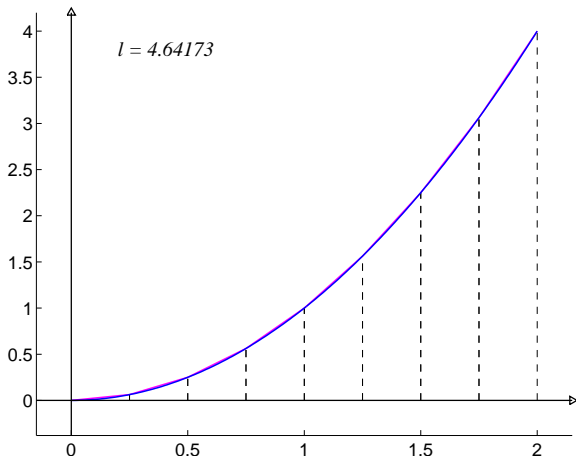
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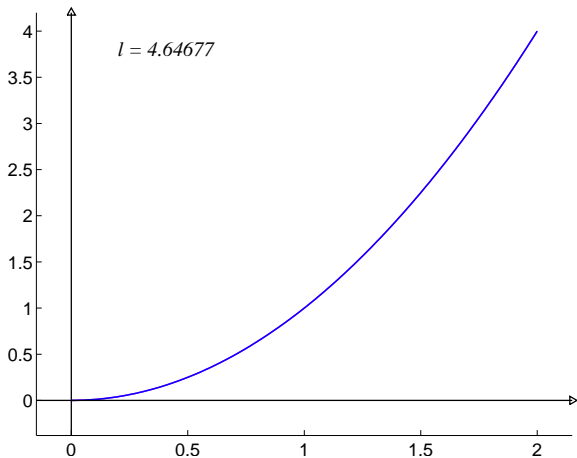
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O comprimento exato da curva é dado por

$$L = \int_0^2 \sqrt{1 + [f'(x)]^2} dx = \int_0^2 \sqrt{1 + 4x^2} dx$$

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Fazendo $u = 2x$, obtemos

$$L = \frac{1}{2} \int_0^4 \sqrt{1 + u^2} du = \frac{1}{4} \left[u\sqrt{1 + u^2} + \ln \left(u + \sqrt{1 + u^2} \right) \right] \Big|_0^4$$

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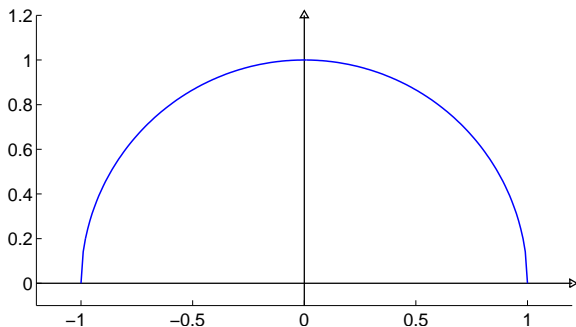
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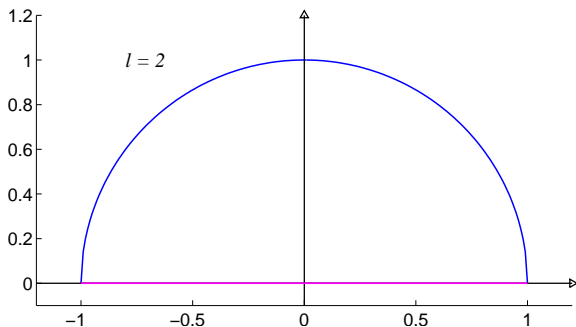
Um cálculo simples nos fornece

$$L = \frac{1}{4} \left[4\sqrt{17} + \ln \left(4 + \sqrt{17} \right) \right] \approx 4.64678$$

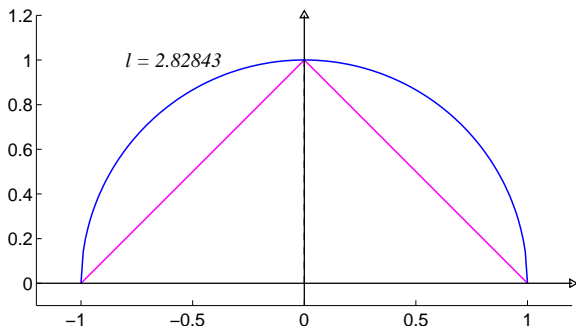
Exemplo 2: a semi-circunferência



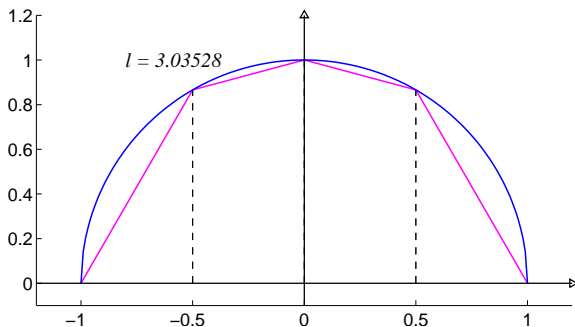
Exemplo 2: a semi-circunferência



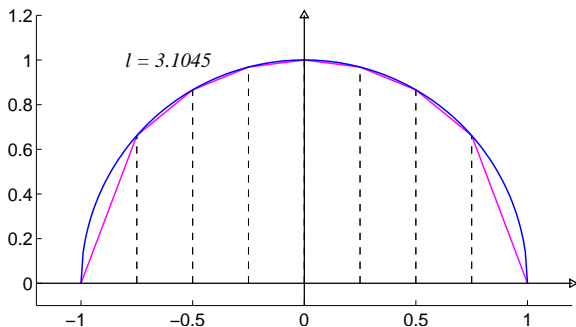
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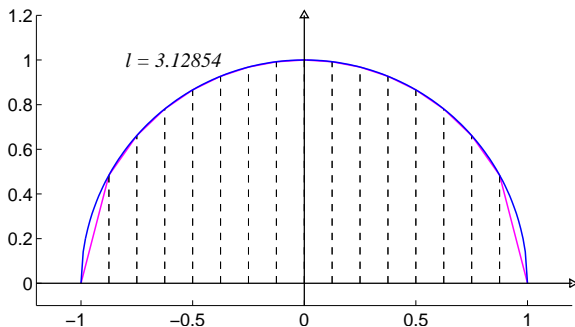
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