O baricentro

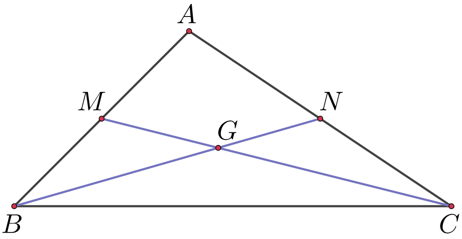
Ademir Alves Ribeiro

UFPR - 2020

O baricentro

Exercício 1

Considere o triângulo abaixo, onde M e N são os pontos médios dos lados AB e AC, respectivamente. Mostre que o ponto G divide os segmentos BN e CM na razão $2/3$.



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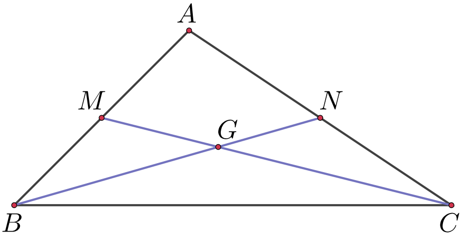

- $\overrightarrow{BG} = s\overrightarrow{BN}$

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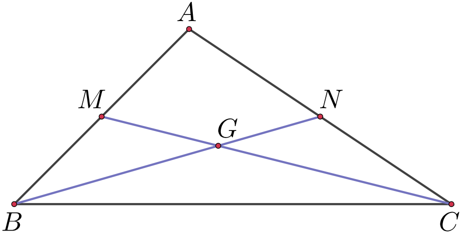
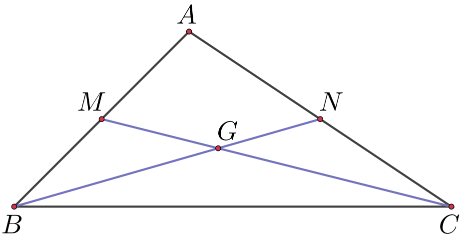


- $\overrightarrow{BG} = s\overrightarrow{BN} = s(\overrightarrow{BA} + 1/2\overrightarrow{AC})$

O baricentro



- $\overrightarrow{BG} = s\overrightarrow{BN} = s(\overrightarrow{BA} + 1/2\overrightarrow{AC}) = s(-\overrightarrow{AB} + 1/2\overrightarrow{AC});$

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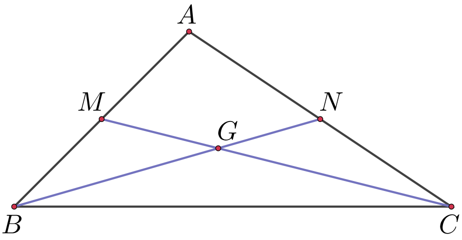

- $\overrightarrow{BG} = s\overrightarrow{BN} = s(\overrightarrow{BA} + 1/2\overrightarrow{AC}) = s(-\overrightarrow{AB} + 1/2\overrightarrow{AC})$;
- $\overrightarrow{CG} = t\overrightarrow{CM}$

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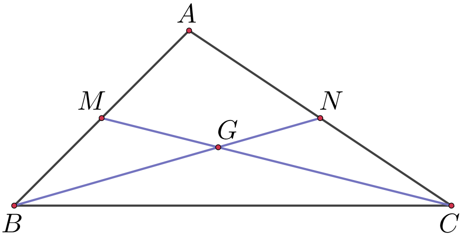

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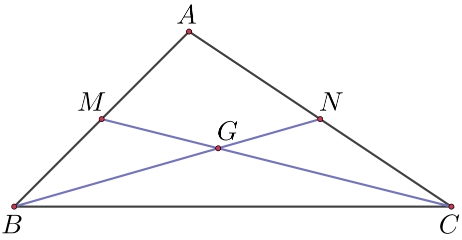

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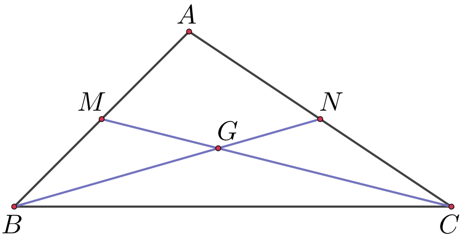

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- $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$

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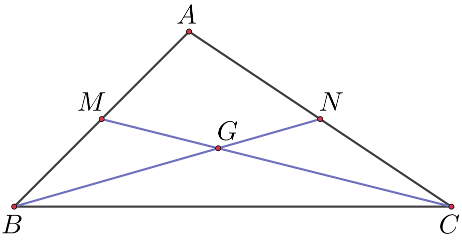

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- $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC} = \overrightarrow{BG} + \overrightarrow{GC}$

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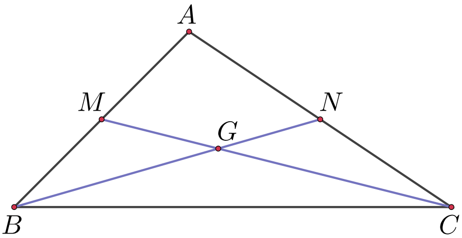

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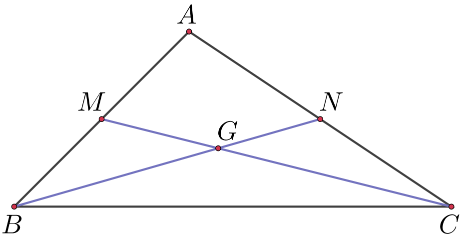

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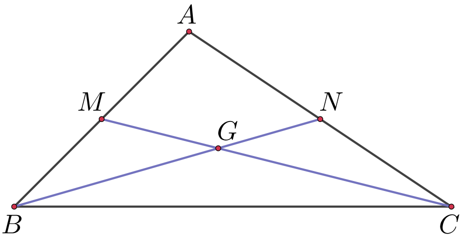

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- $(1 - s/2 - t)\overrightarrow{AC} =$

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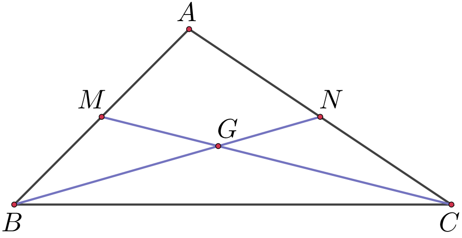

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- $(1 - s/2 - t)\overrightarrow{AC} = (1 - s - t/2)\overrightarrow{AB}$;

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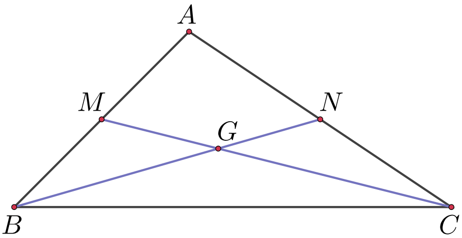

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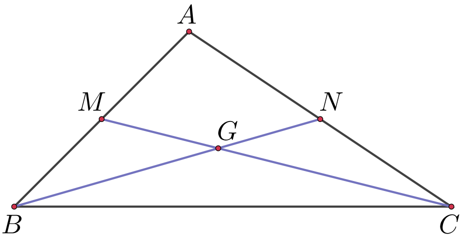
- $(1 - s/2 - t)\overrightarrow{AC} = (1 - s - t/2)\overrightarrow{AB};$
- $1 - s/2 - t = 0$

O baricentro



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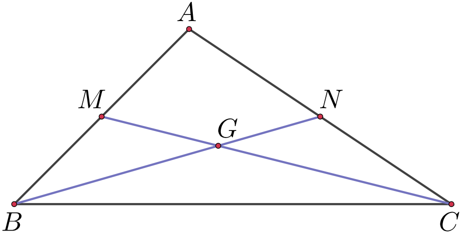

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- $s/2 + t = s + t/2$

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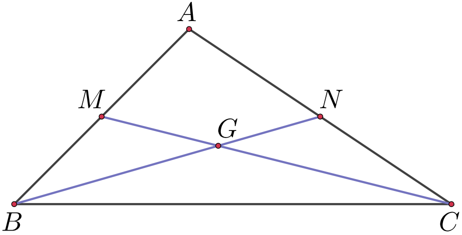

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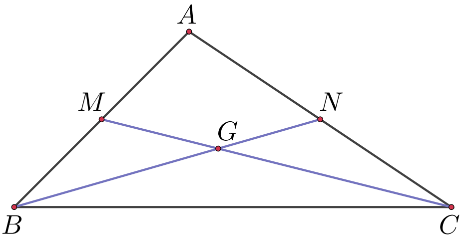
- $1 - s/2 - t = 0$ e $1 - s - t/2 = 0;$
- $s/2 + t = s + t/2 \Rightarrow s = t$

O baricentro



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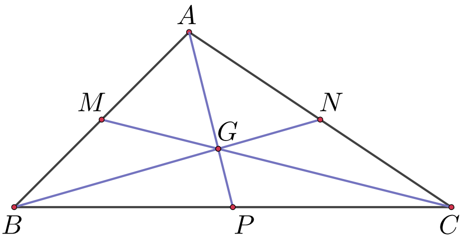
- $1 - s/2 - t = 0$ e $1 - s - t/2 = 0$;
- $s/2 + t = s + t/2 \Rightarrow s = t = 2/3$.

Exercício 2

Considere o triângulo abaixo, onde M, N e P são os pontos médios dos lados AB, AC e BC, respectivamente. Mostre que as medianas AP, BN e CM se intesectam no mesmo ponto.



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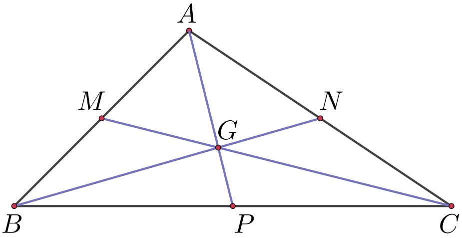

- Basta aplicar o Exercício 1 para as medianas AP e BN;

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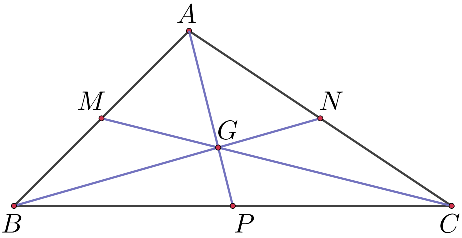

- Basta aplicar o Exercício 1 para as medianas AP e BN;
- O ponto de interseção está no segmento BN na razão $2/3$;

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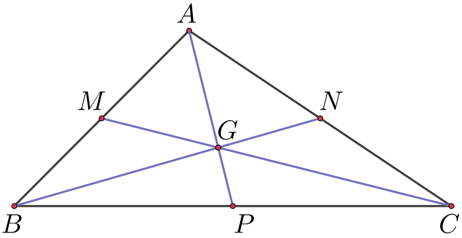

- Basta aplicar o Exercício 1 para as medianas AP e BN;
- O ponto de interseção está no segmento BN na razão $2/3$;
- Portanto, é o mesmo ponto de interseção entre BN e CM.

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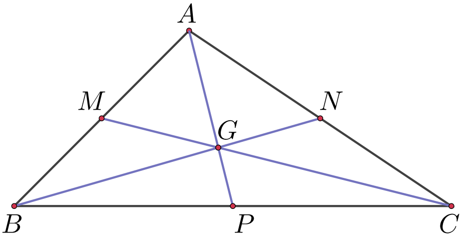
Exercício 3

Considere o triângulo abaixo, onde M, N e P são os pontos médios dos lados AB, AC e BC, respectivamente. Seja G o ponto de interseção das medianas AP, BN e CM (baricentro). Mostre que $\overrightarrow{AG} + \overrightarrow{BG} + \overrightarrow{CG} = 0$.



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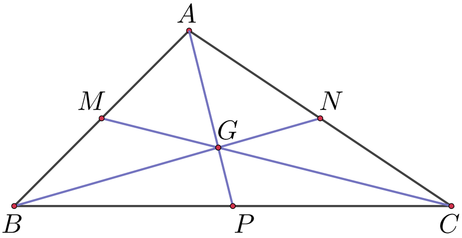

- Note que $2\overrightarrow{AP}=(\overrightarrow{AB}+\overrightarrow{BP})+(\overrightarrow{AC}+\overrightarrow{CP})$

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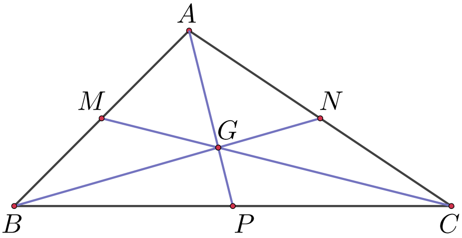

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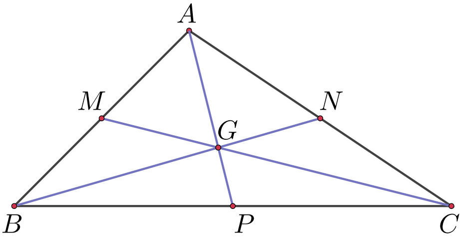

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- Analogamente, $2\overrightarrow{BN}=\overrightarrow{BA}+\overrightarrow{BC}$ e $2\overrightarrow{CM}=\overrightarrow{CA}+\overrightarrow{CB}$;

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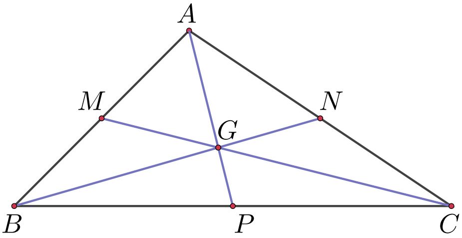

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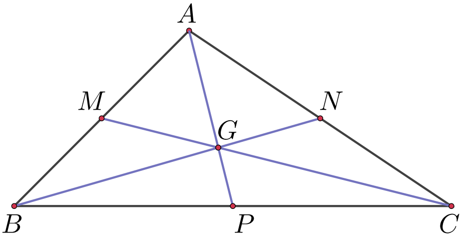

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- Analogamente, $2\overrightarrow{BN} = \overrightarrow{BA} + \overrightarrow{BC}$ e $2\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{CB}$;
- Portanto, $\overrightarrow{AG} + \overrightarrow{BG} + \overrightarrow{CG} = \frac{2}{3}(\overrightarrow{AP} + \overrightarrow{BN} + \overrightarrow{CM})$

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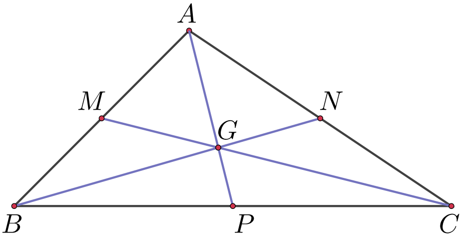

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- Analogamente, $2\overrightarrow{BN}=\overrightarrow{BA}+\overrightarrow{BC}$ e $2\overrightarrow{CM}=\overrightarrow{CA}+\overrightarrow{CB}$;
- Portanto, $\overrightarrow{AG}+\overrightarrow{BG}+\overrightarrow{CG}=\frac{2}{3}(\overrightarrow{AP}+\overrightarrow{BN}+\overrightarrow{CM})=0$.

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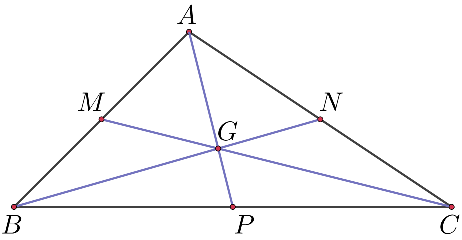
Conclusão

Considere o triângulo abaixo, onde M, N e P são os pontos médios dos lados AB, AC e BC, respectivamente. Seja G o ponto de interseção das medianas AP, BN e CM (baricentro). Temos que $G = \dfrac{A+B+C}{3}$.



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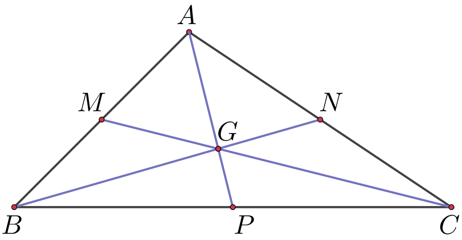

- Note que $3G - (A+B+C) = \overrightarrow{AG} + \overrightarrow{BG} + \overrightarrow{CG} = 0$.