

# A mesa de bilhar

Ademir Alves Ribeiro

UFPR - 2020

- 1 A subestação de energia
- 2 A mesa de bilhar

# A subestação de energia

## Exercício

Considere os pontos  $A$  e  $B$  no leito de um rio

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$A$ .

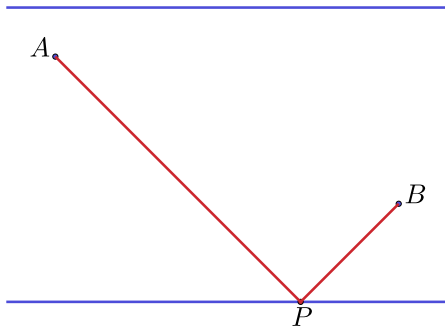
$B$

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# A subestação de energia

## Exercício

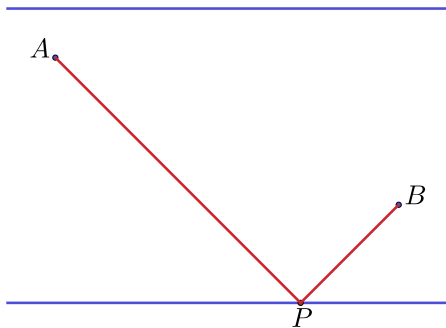
Considere os pontos  $A$  e  $B$  no leito de um rio e o problema de encontrar um ponto  $P$  na margem do rio onde será construída uma subestação de energia para atender os dois pontos.



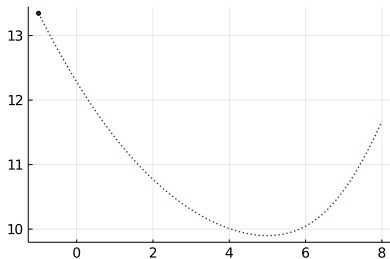
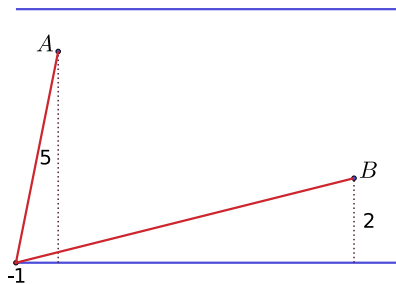
# A subestação de energia

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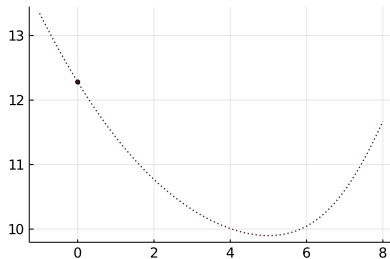
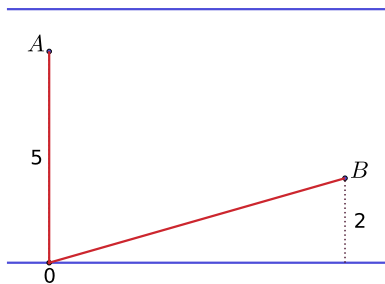
Considere os pontos  $A$  e  $B$  no leito de um rio e o problema de encontrar um ponto  $P$  na margem do rio onde será construída uma subestação de energia para atender os dois pontos. Qual deve ser a localização ótima de  $P$ , no sentido de minimizar a soma das distâncias aos pontos  $A$  e  $B$ ?



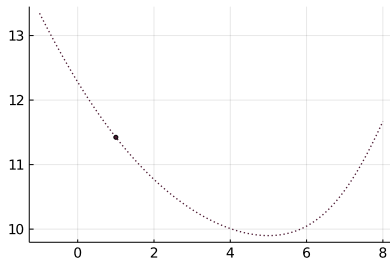
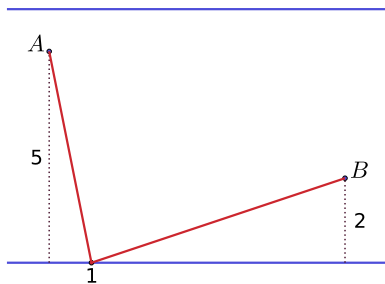
# Algumas configurações



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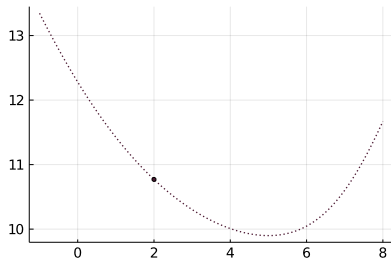
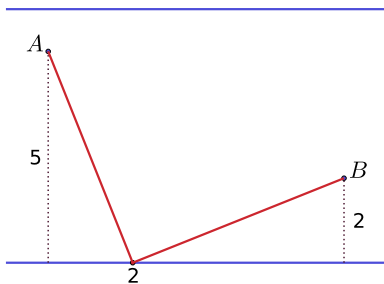


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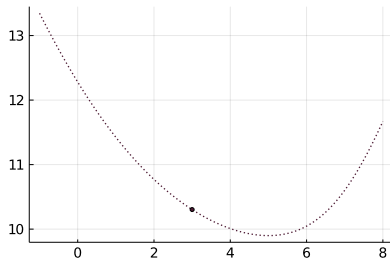
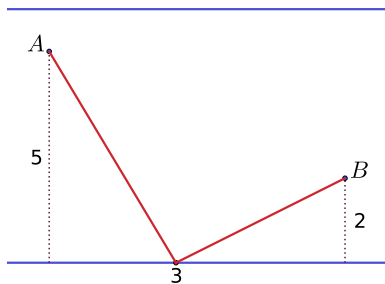




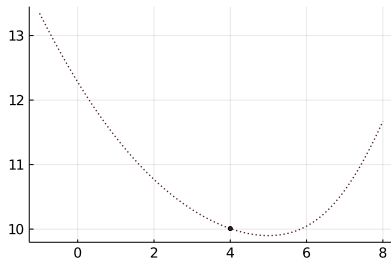
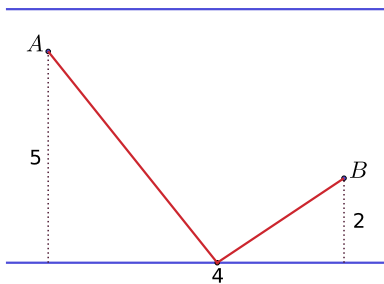
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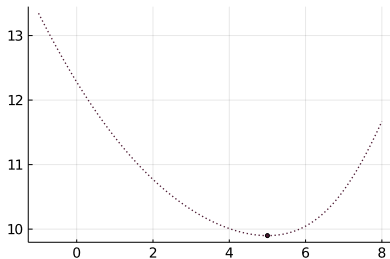
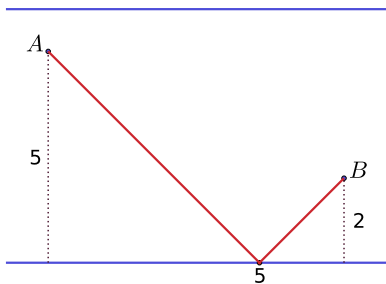
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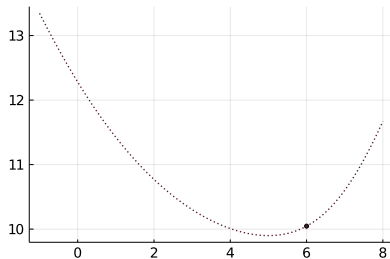
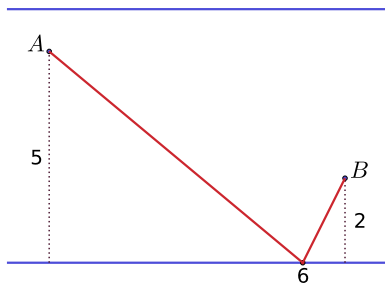
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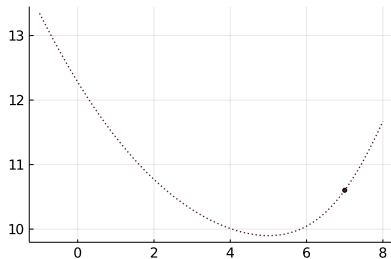
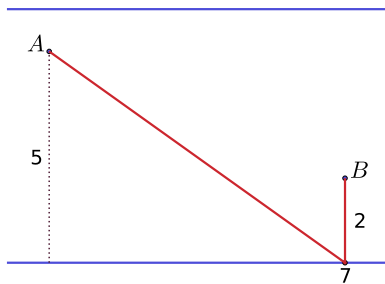
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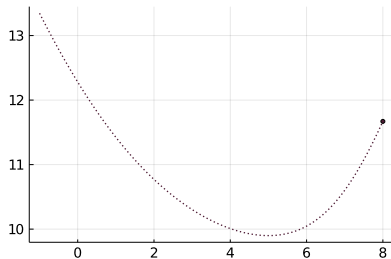
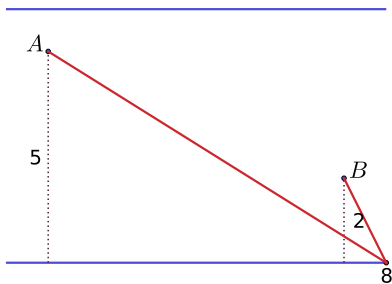
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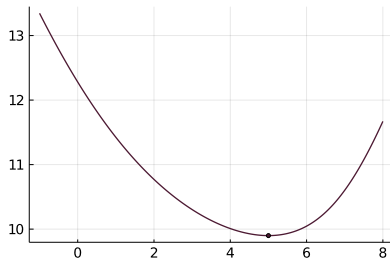
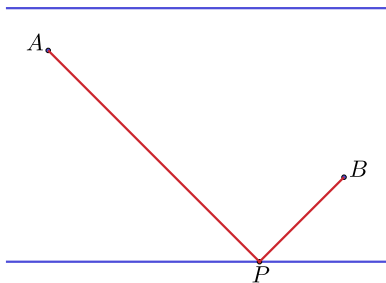
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# Configuração ótima

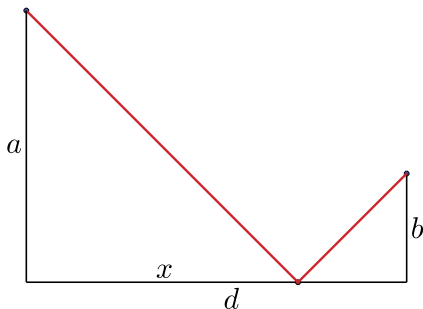




# Formulação matemática (via Cálculo I)

## Problema de otimização

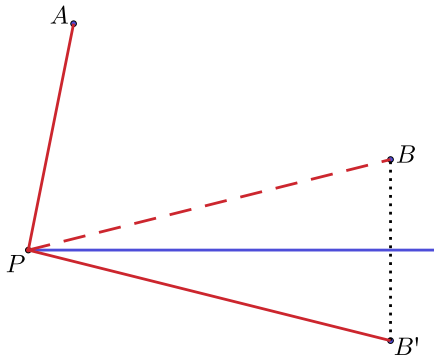
minimizar  $f(x) = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d - x)^2}$



## Abordagem usando geometria analítica

Usando uma reflexão

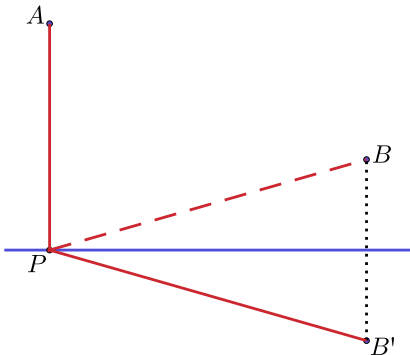
$$\text{dist}(P,A) + \text{dist}(P,B) = \text{dist}(A,P) + \text{dist}(P,B').$$



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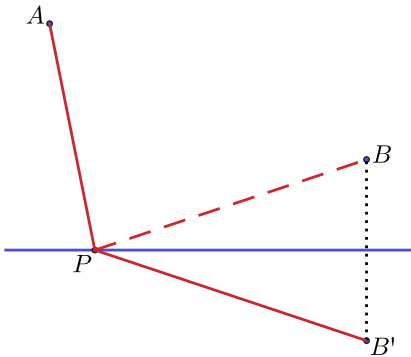
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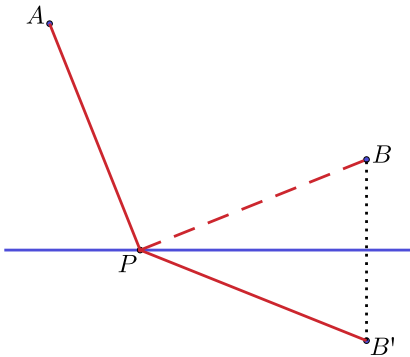
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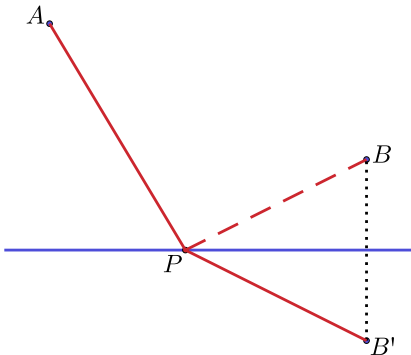
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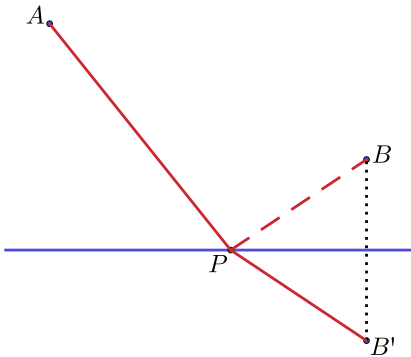
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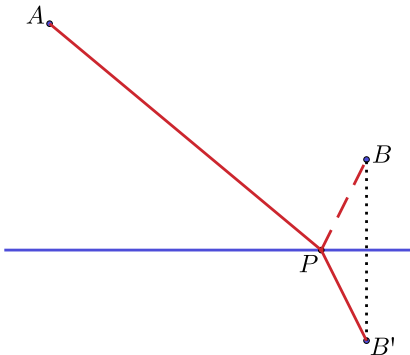
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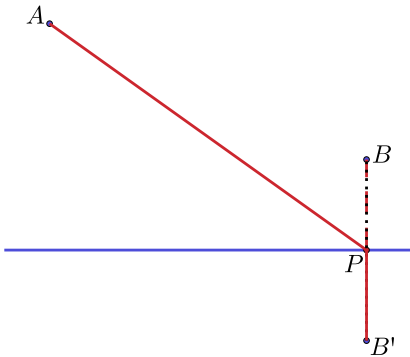




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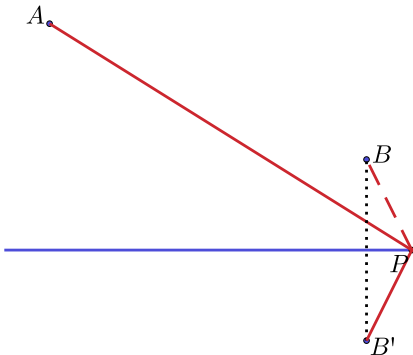
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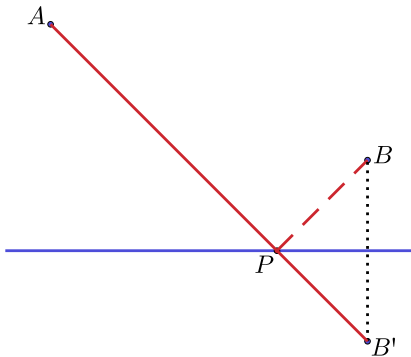
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## Configuração ótima

### Fato

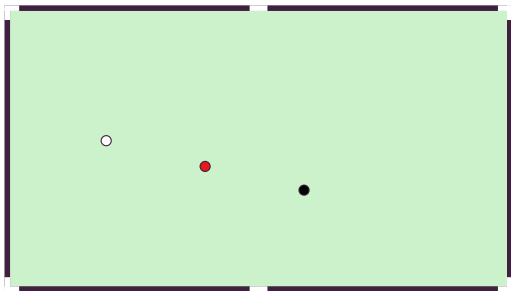
A soma mínima ocorre quando  $A$ ,  $P$  e  $B'$  estão alinhados.



# A mesa de bilhar

## Exercício

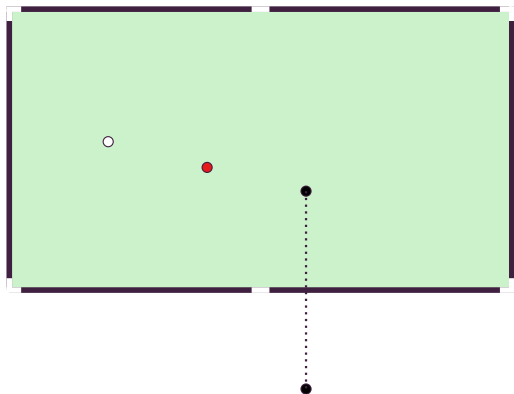
Em qual ponto da mesa mirar a bola branca para atingir a bola preta?



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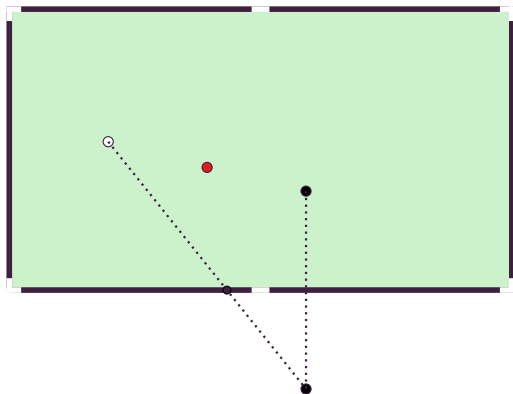
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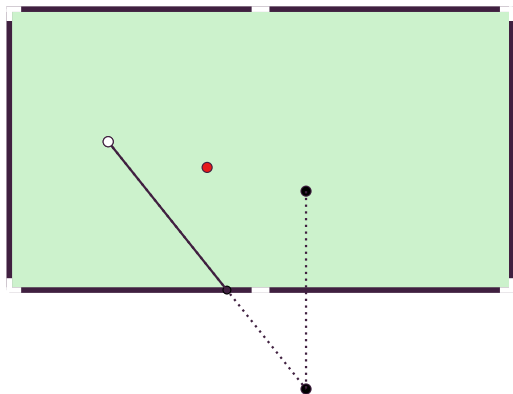
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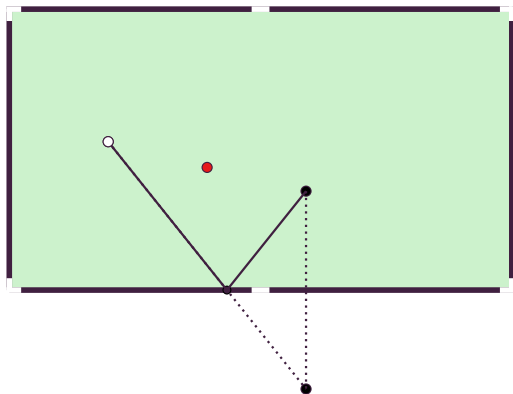
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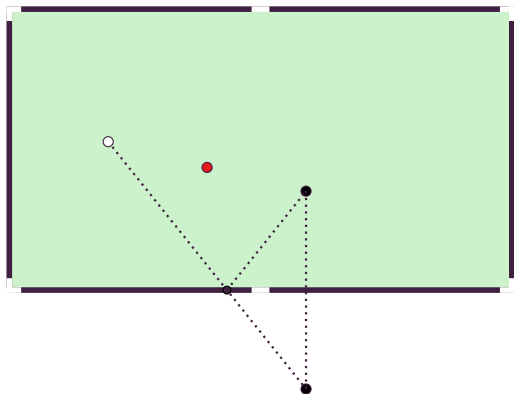




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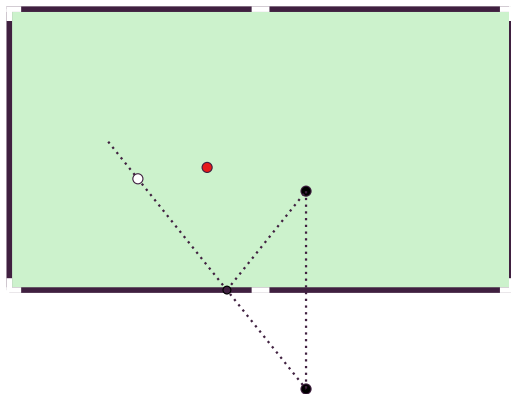
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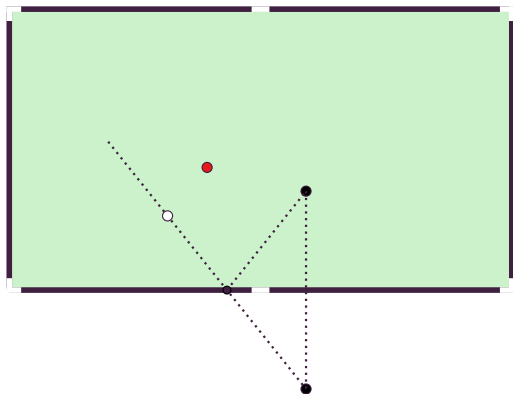
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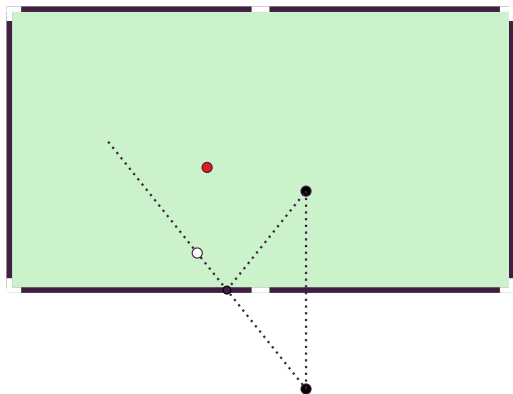
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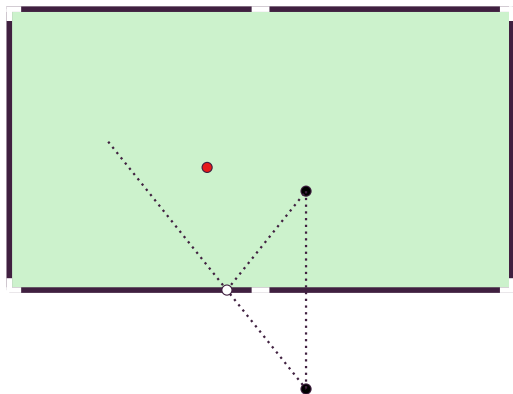
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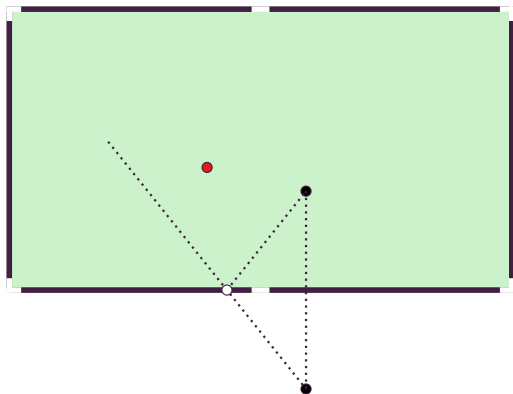
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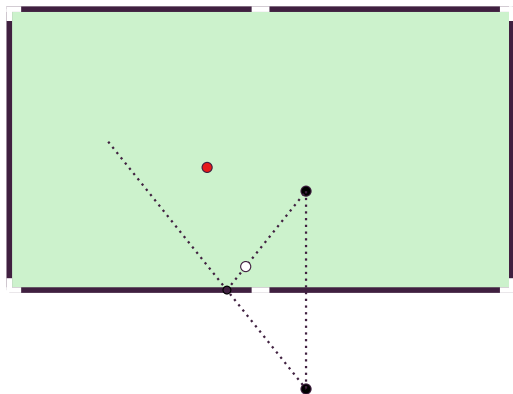
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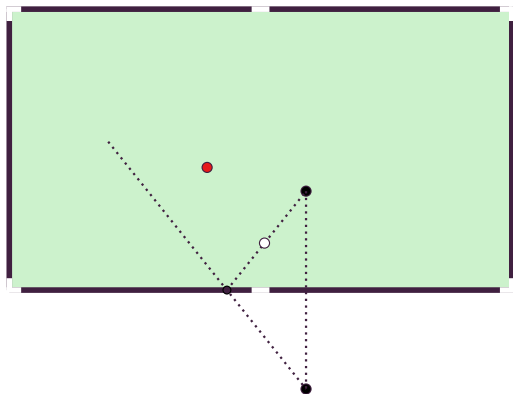
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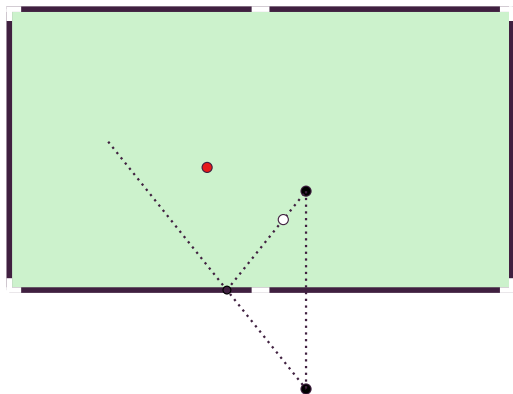




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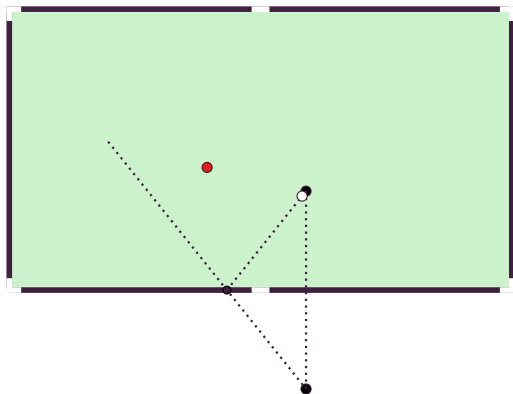
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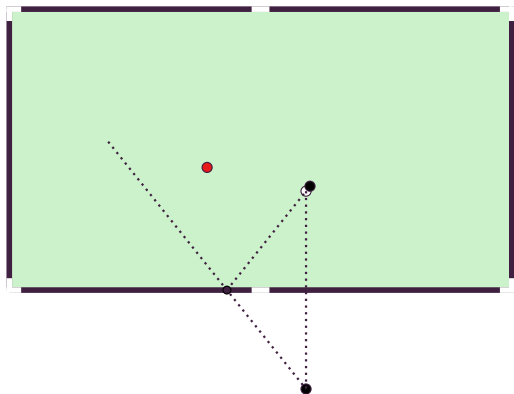
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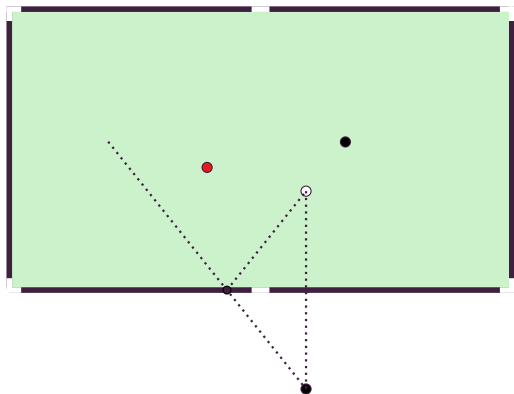
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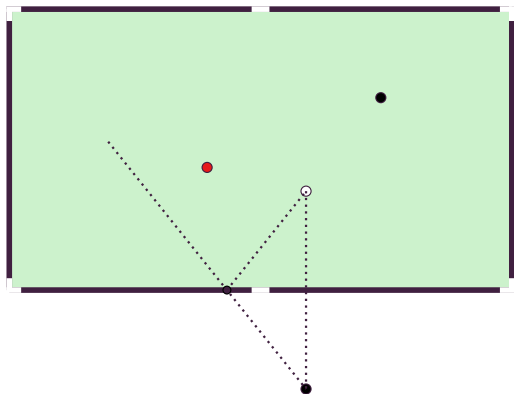
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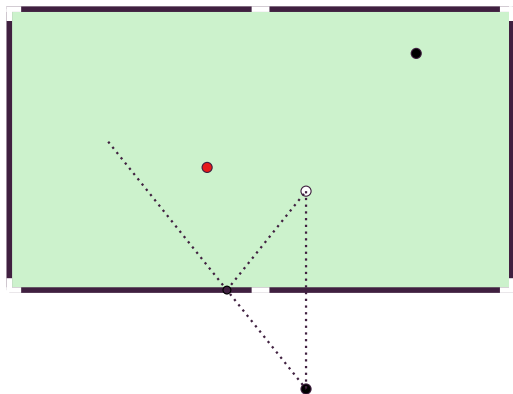
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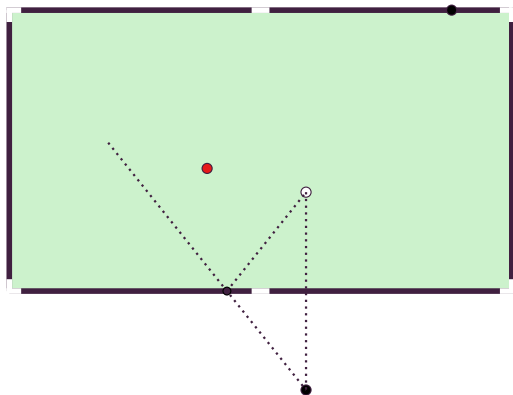
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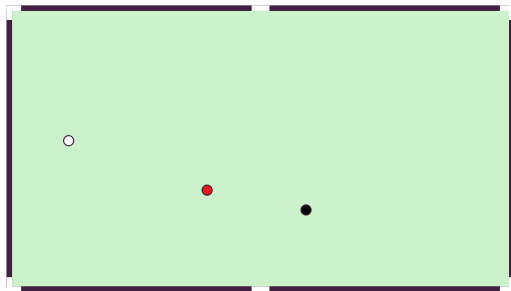
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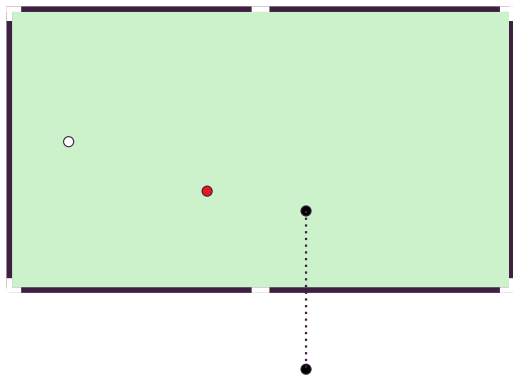




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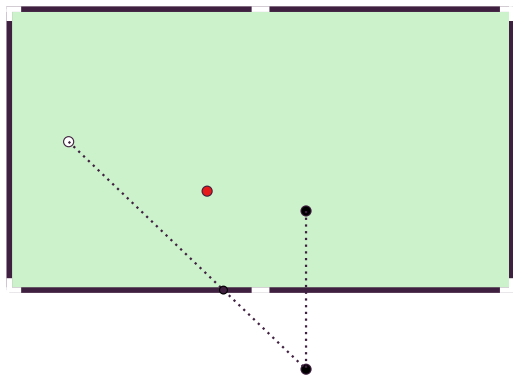
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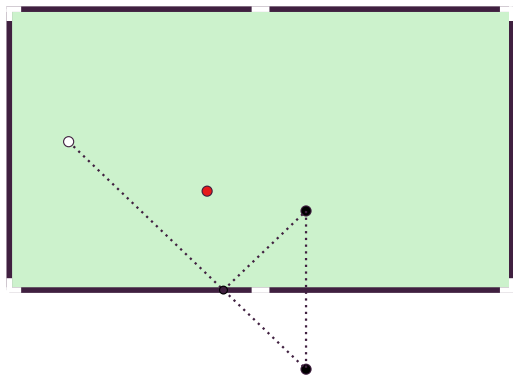
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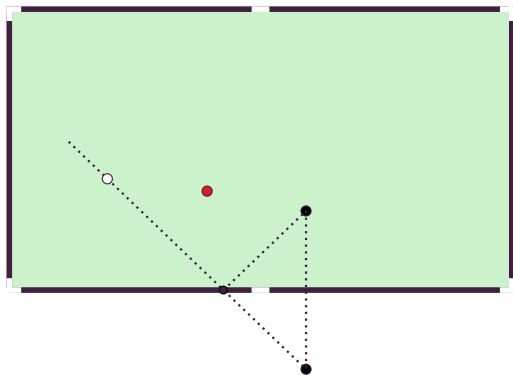
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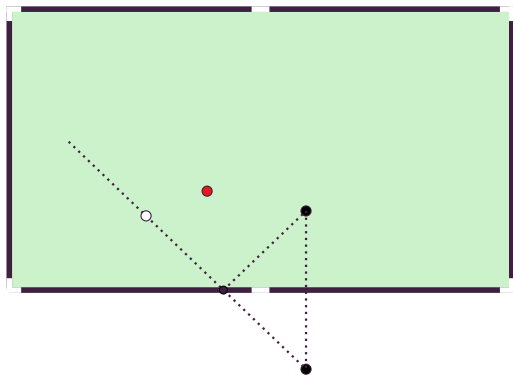
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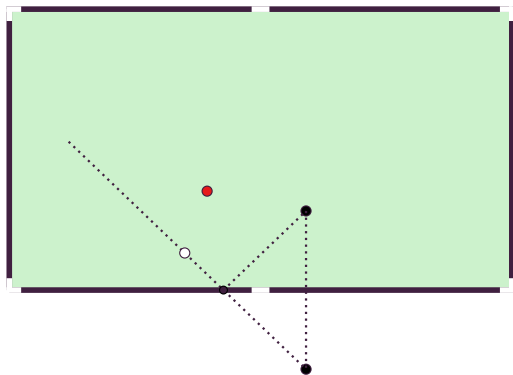
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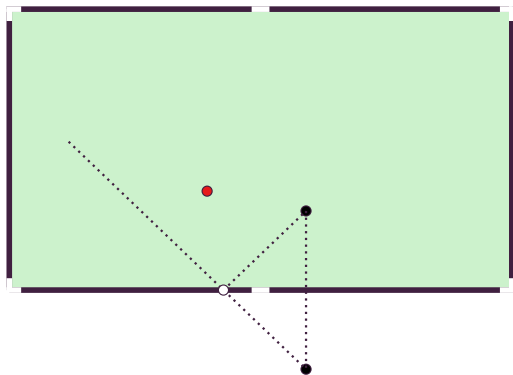
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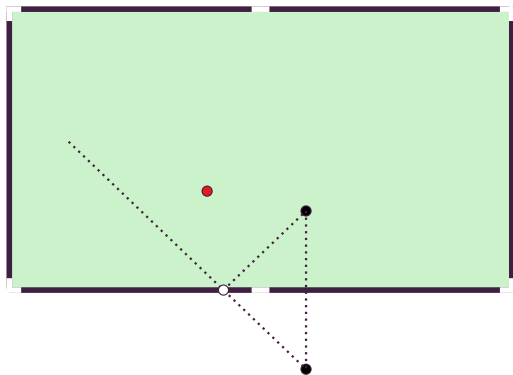
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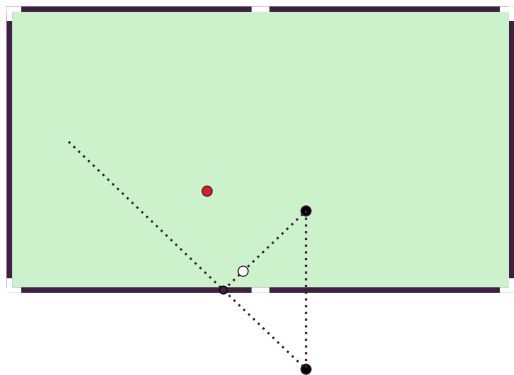




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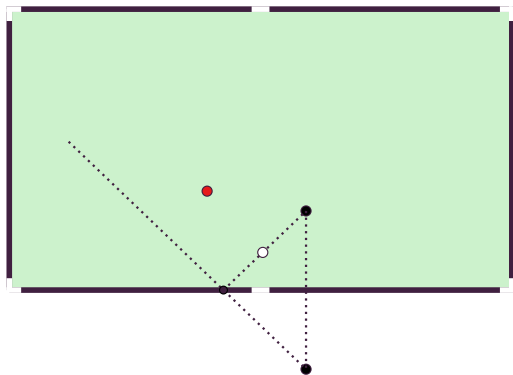
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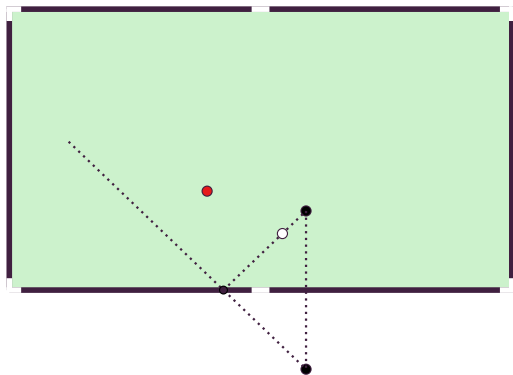
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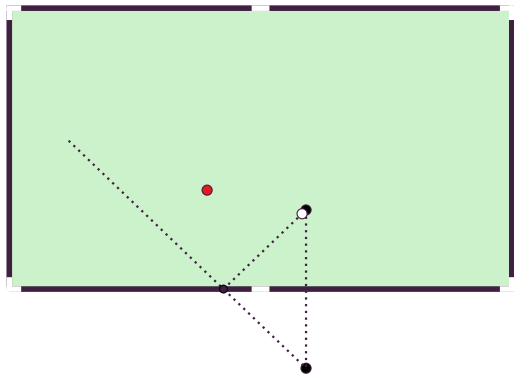
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# A mesa de bilhar

## Exercício

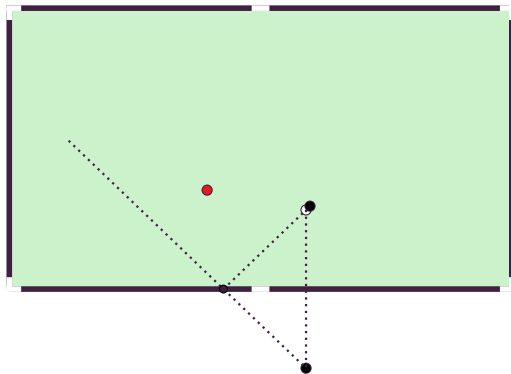
Em qual ponto da mesa mirar a bola branca para atingir a bola preta?



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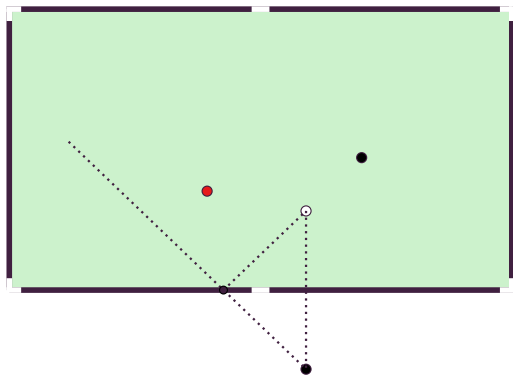
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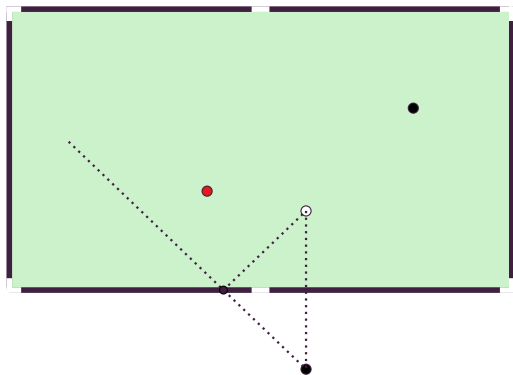
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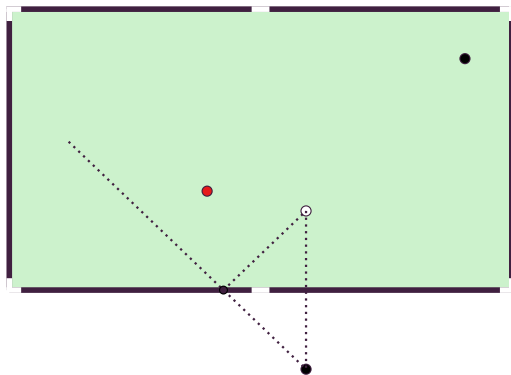
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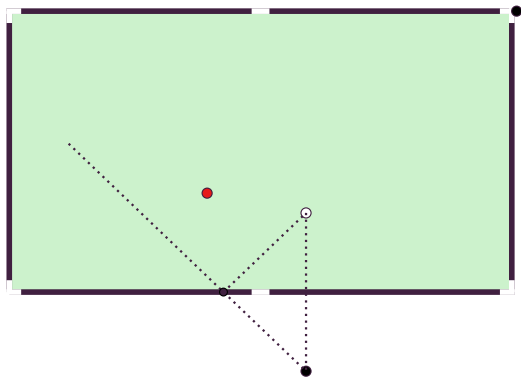




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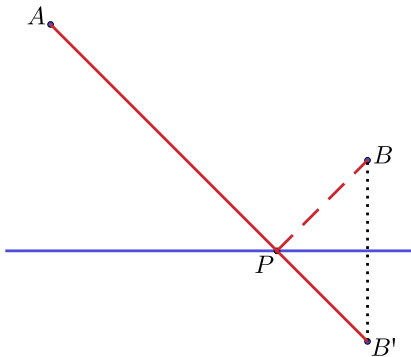
Em qual ponto da mesa mirar a bola branca para atingir a bola preta?



## Resolução do problema

### Exercício

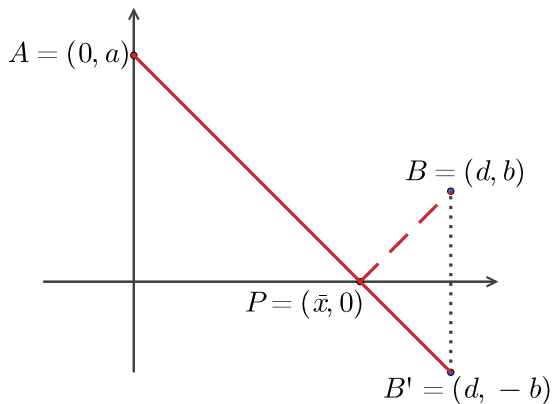
Obter a reta que passa por  $A$  e  $B'$  e intersectar com a reta horizontal.



## Resolução do problema

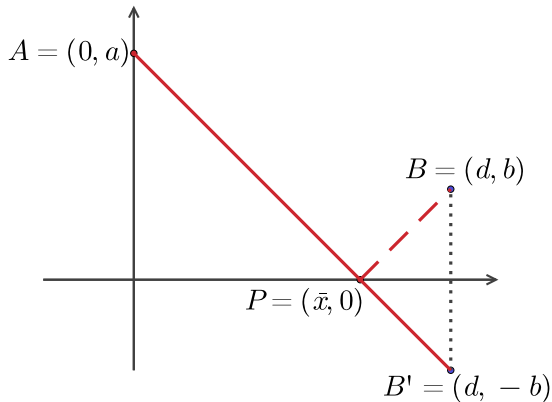
### Exercício

Obter a reta que passa por  $A$  e  $B'$  e intersectar com o eixo das abscissas.



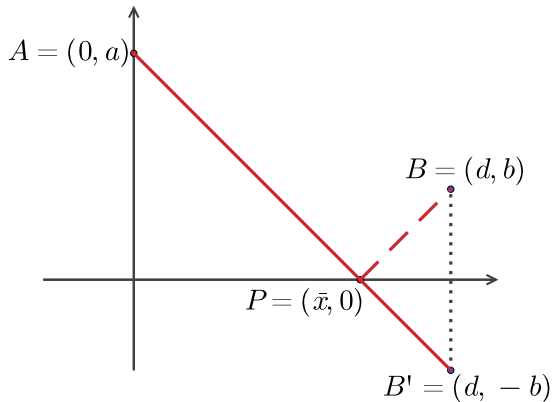
## Resolução do problema

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- Reta:  $y - a = m(x - 0) \Rightarrow y = a - \frac{a+b}{d}x$ ;



## Resolução do problema

- Reta:  $y - a = m(x - 0) \Rightarrow y = a - \frac{a+b}{d}x$ ;
- Solução:  $\bar{x} = \frac{ad}{a+b}$ .

