

Coordenadas polares - Parte 1

Ademir Alves Ribeiro

2021

<https://youtu.be/ruHL3WGZIjQ>



Representação de objetos no plano

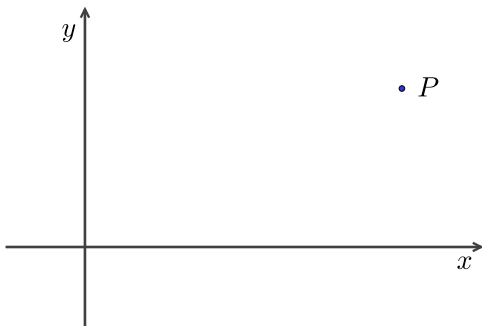
• P

- $P = ?$

- P

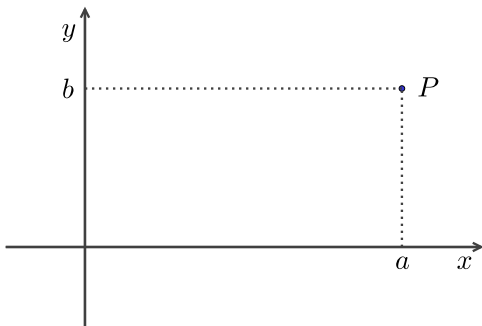
Representação de objetos no plano

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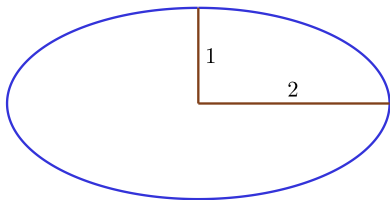


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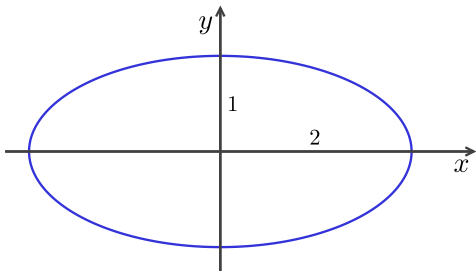
- $P = (a, b)$.



- Elipse \mathcal{E} : ?



- Elipse $\mathcal{E} : x^2 + 4y^2 - 4 = 0$.



O sistema polar

• P

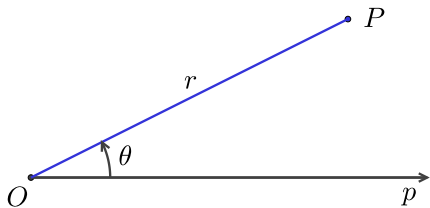
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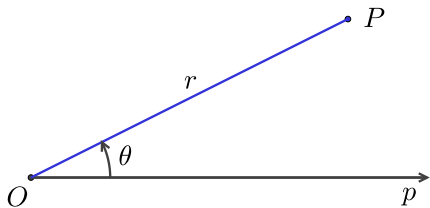


- $P = (r, \theta)$;



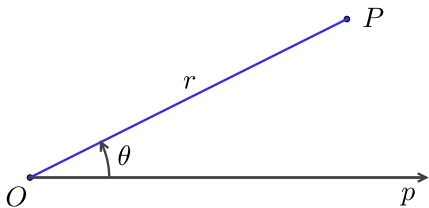
O sistema polar

- $P = (r, \theta)$;
- $r \geq 0, \theta \in [0, 2\pi)$;



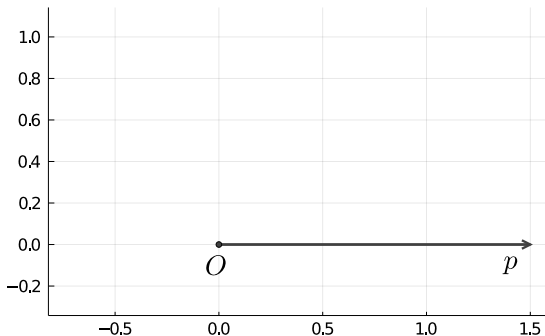
O sistema polar

- $P = (r, \theta)$;
- $r \geq 0$, $\theta \in [0, 2\pi)$;
- Pode ser conveniente considerar $\theta \in X$ para outros conjuntos X ;



Exercício 1

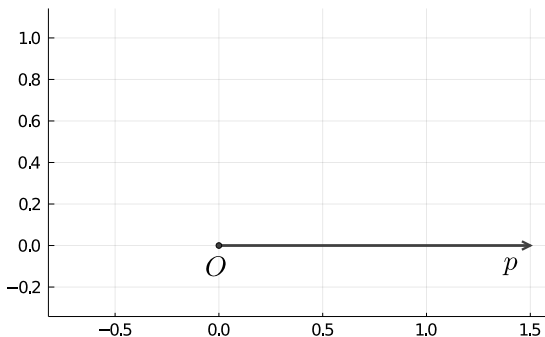
Considere um sistema polar (O, p) .



O sistema polar

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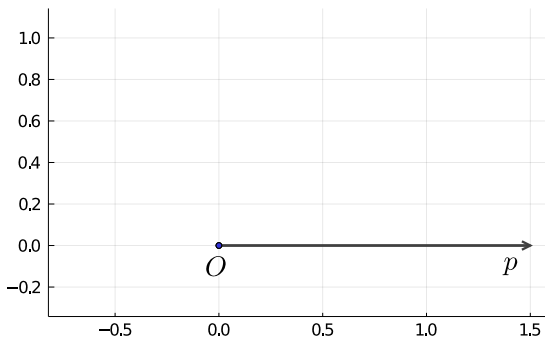
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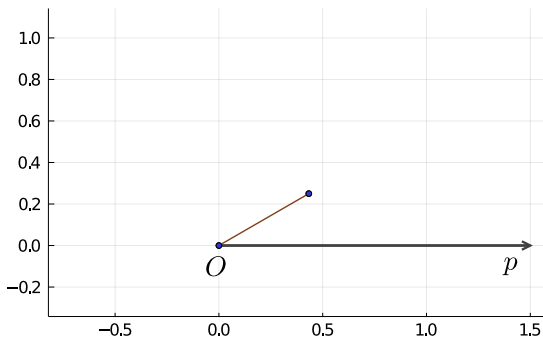
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O sistema polar

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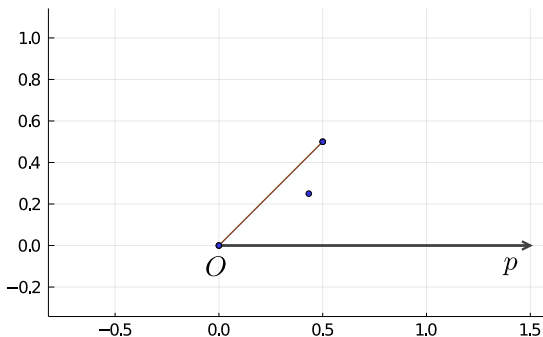
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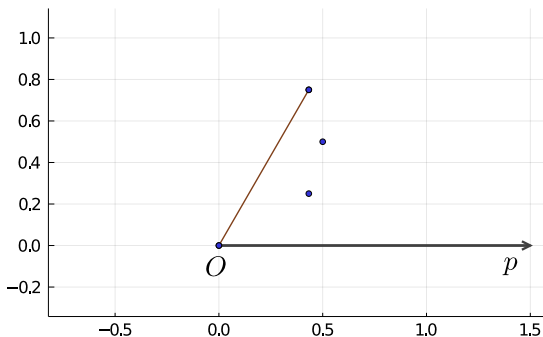
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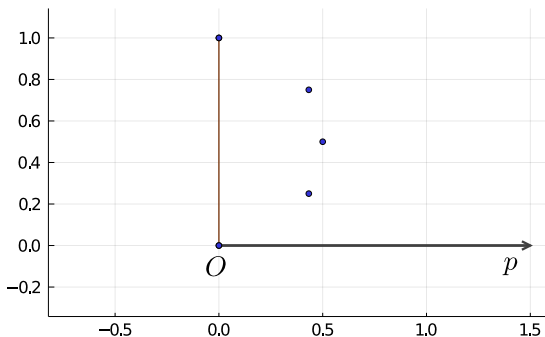
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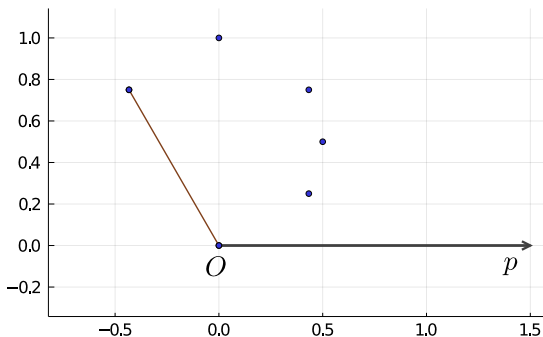
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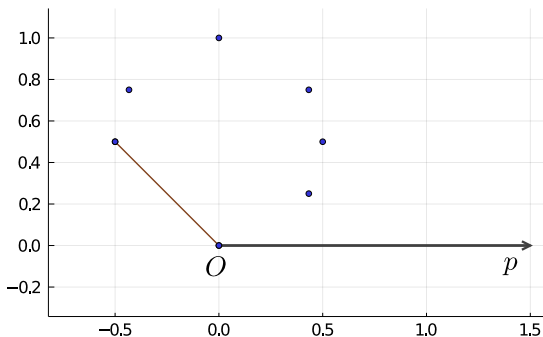
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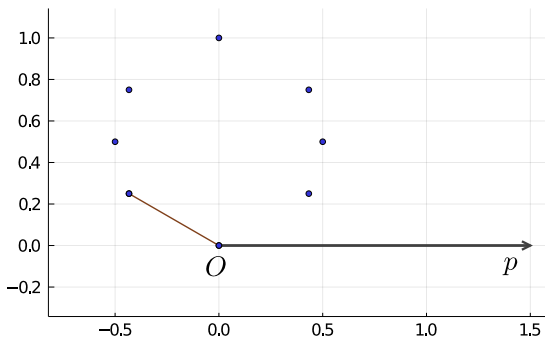


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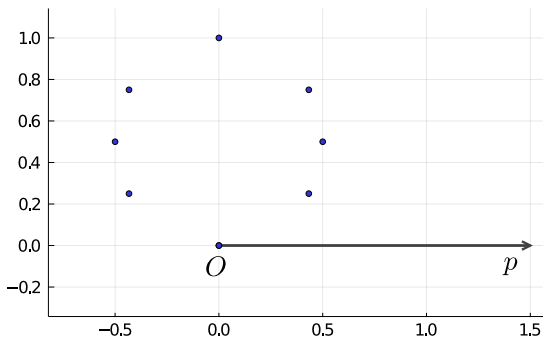
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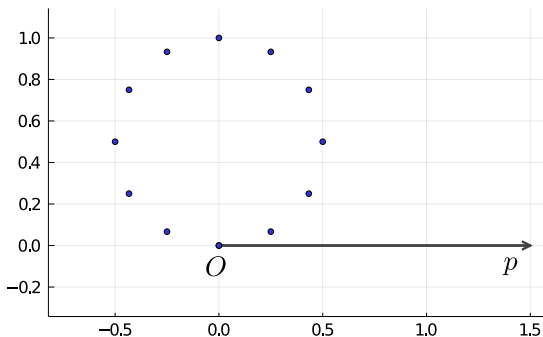
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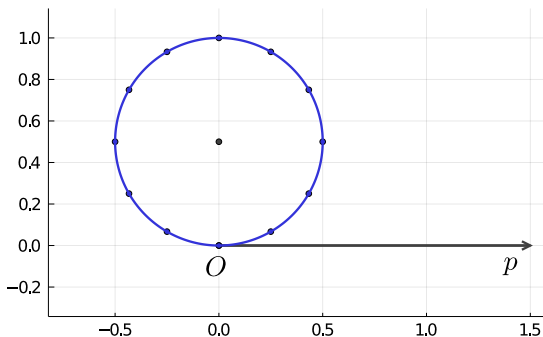
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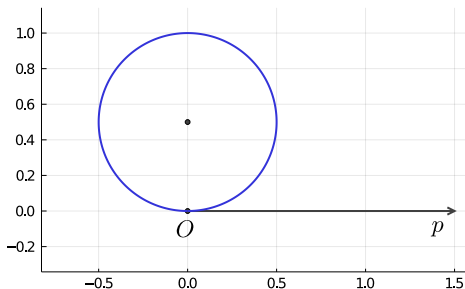
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Equações no sistema polar

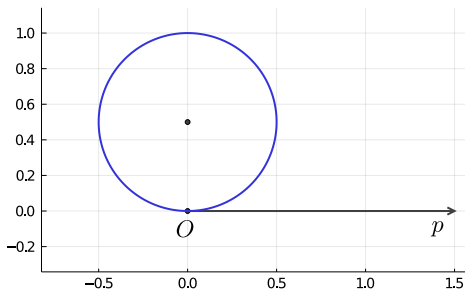
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Considere um sistema polar (O, p) e a circunferência de raio $1/2$, centrada no ponto de coordenadas polares $(1/2, \pi/2)$.



Exercício 2

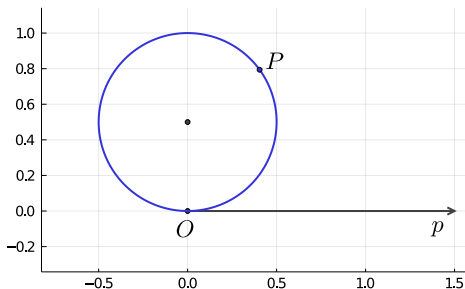
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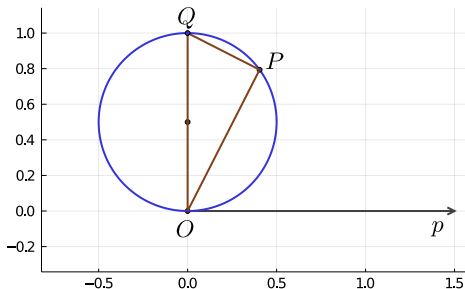


Equações no sistema polar

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- Tome $P = (r, \theta) \in C$;
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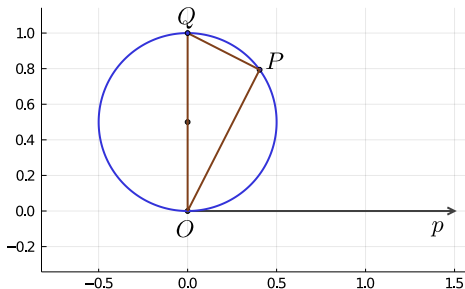


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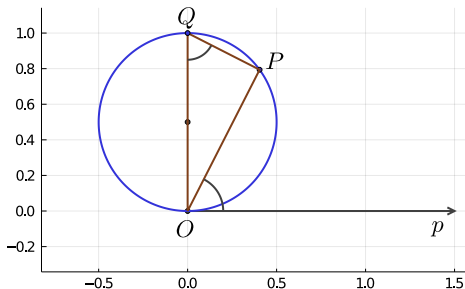


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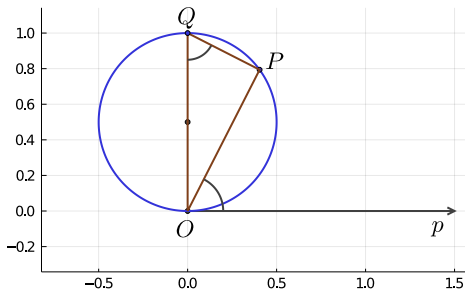


Equações no sistema polar

Exercício 2

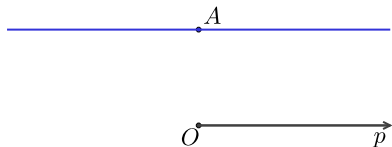
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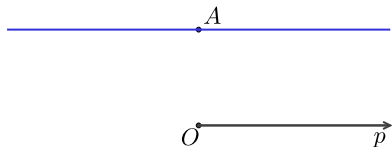
Exercício 3

Considere um sistema polar (O, p) e a reta paralela ao eixo polar que passa pelo ponto de coordenadas polares $A = (1, \pi/2)$.



Exercício 3

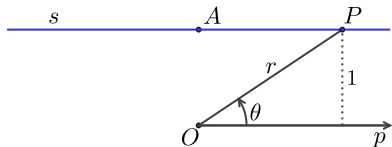
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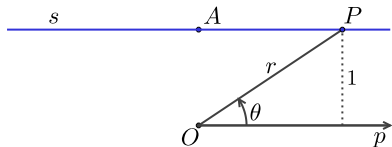
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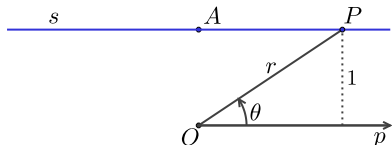
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- Equação da reta s : $r \sin \theta = 1$.

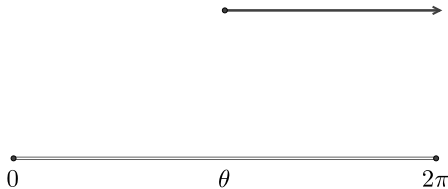


Equação de uma borboleta

$$r = e^{\sin\theta} - 2\cos(4\theta) + \sin^5\left(\frac{2\theta - \pi}{24}\right).$$

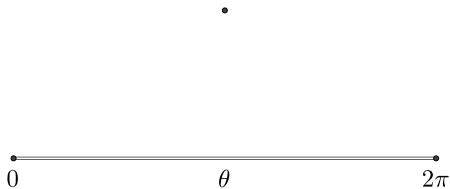
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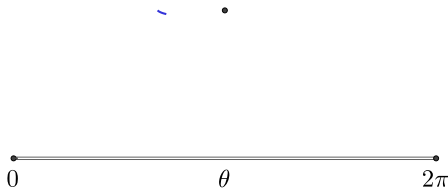
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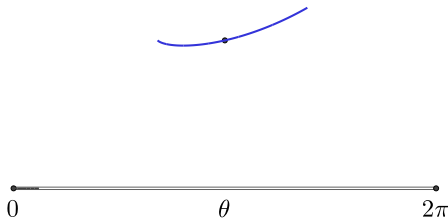
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Equações no sistema polar

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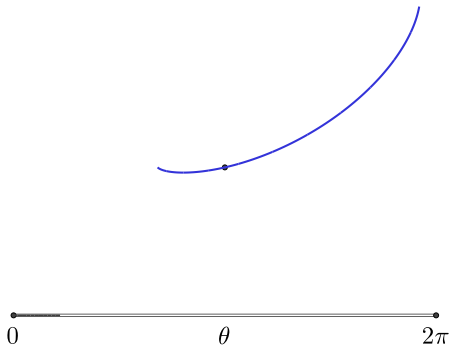
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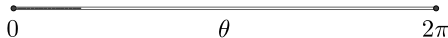
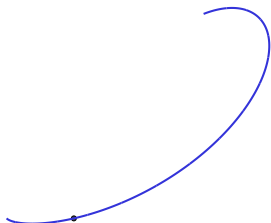
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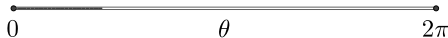
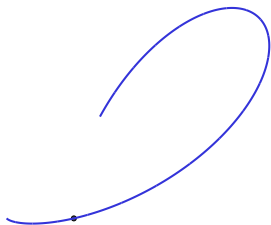
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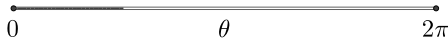
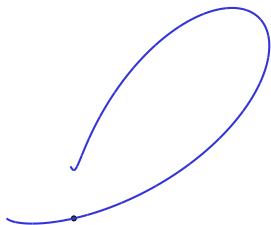
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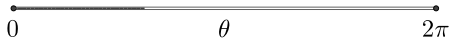
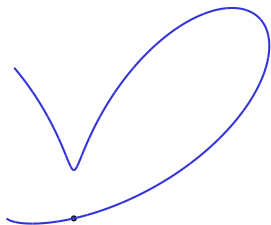
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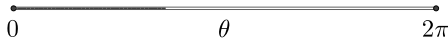
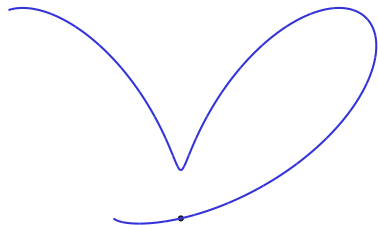
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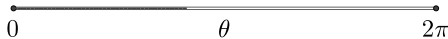
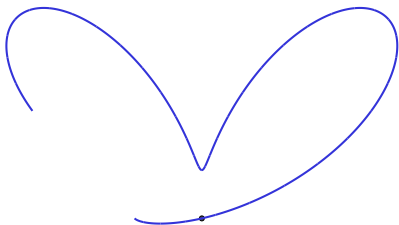
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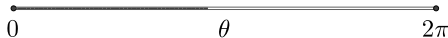
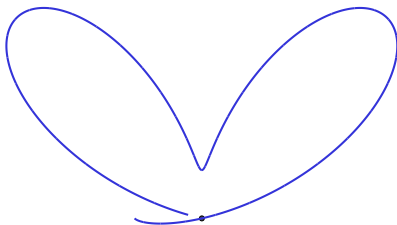
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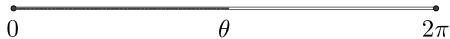
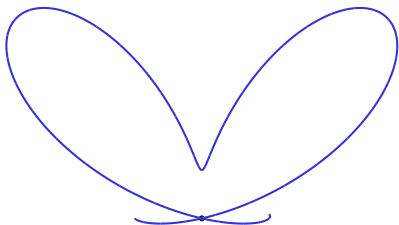
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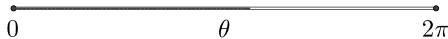
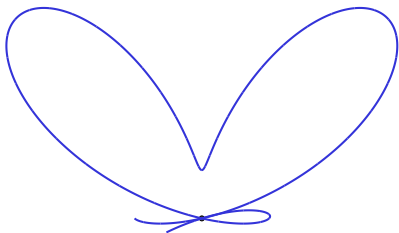
$$r = e^{\sin\theta} - 2\cos(4\theta) + \sin^5\left(\frac{2\theta - \pi}{24}\right).$$



Equações no sistema polar

Equação de uma borboleta

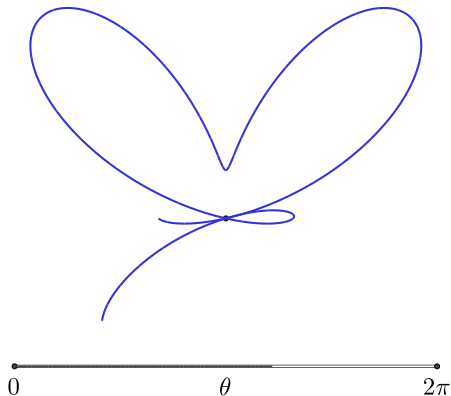
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Equações no sistema polar

Equação de uma borboleta

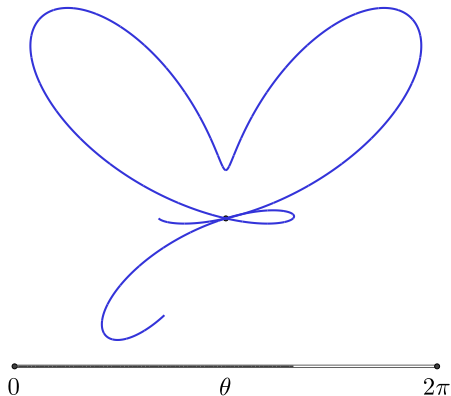
$$r = e^{\sin \theta} - 2 \cos(4\theta) + \sin^5 \left(\frac{2\theta - \pi}{24} \right).$$



Equações no sistema polar

Equação de uma borboleta

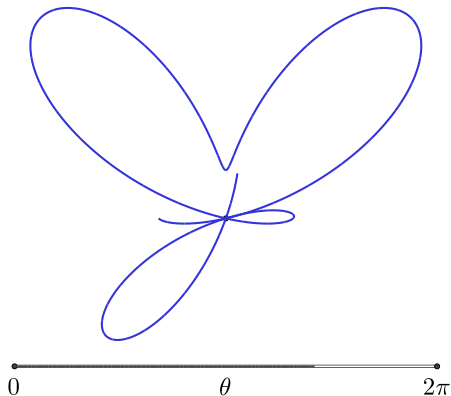
$$r = e^{\sin\theta} - 2\cos(4\theta) + \sin^5\left(\frac{2\theta - \pi}{24}\right).$$



Equações no sistema polar

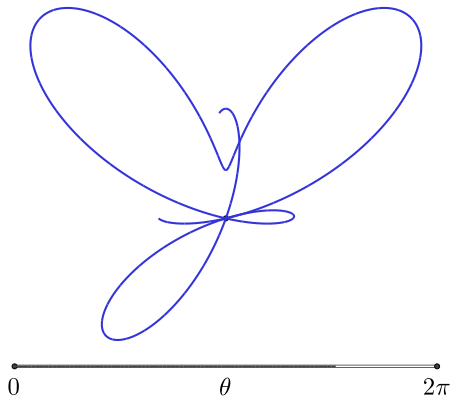
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Equação de uma borboleta

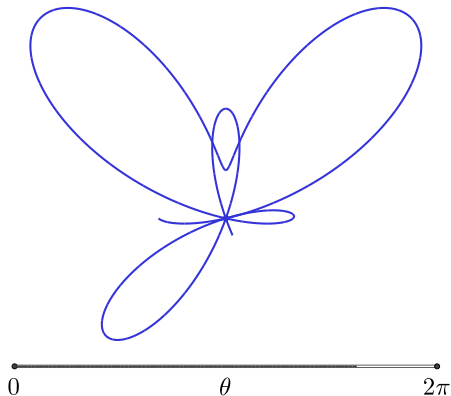
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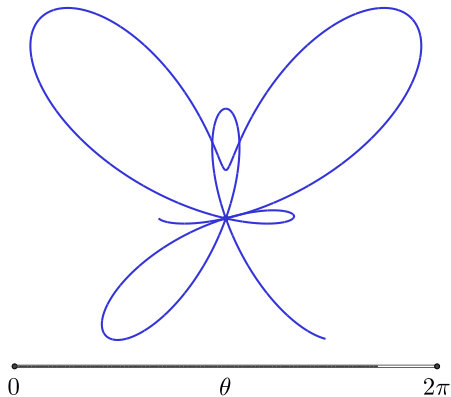
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Equações no sistema polar

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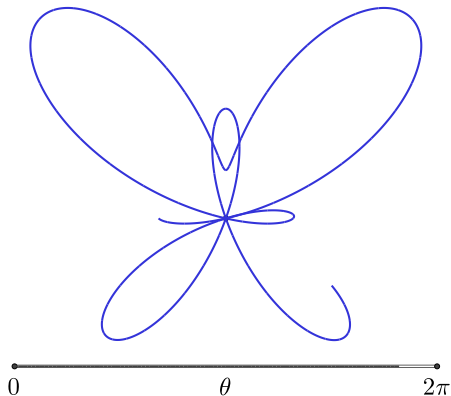
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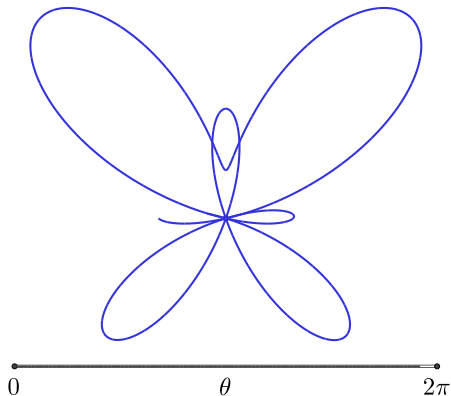
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Equações no sistema polar

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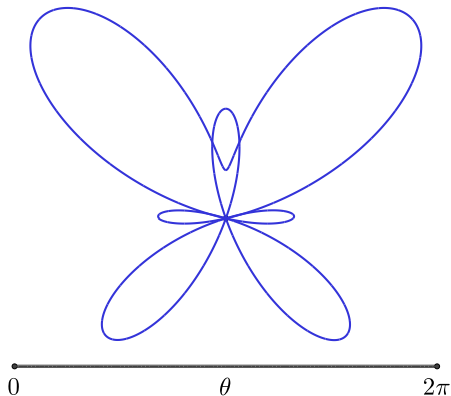
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Equações no sistema polar

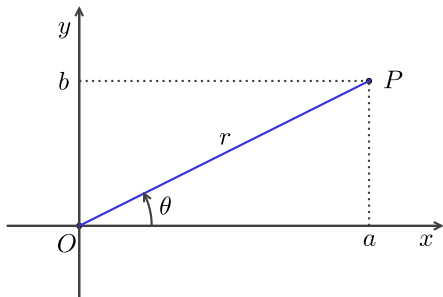
Equação de uma borboleta

$$r = e^{\sin\theta} - 2\cos(4\theta) + \sin^5\left(\frac{2\theta - \pi}{24}\right).$$



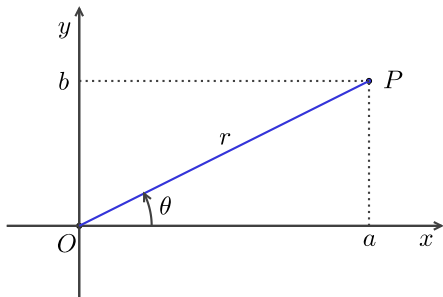
Relação entre as coordenadas nos dois sistemas

- $P = (a, b)$ - coordenadas no sistema cartesiano;
- $P = (r, \theta)$ - coordenadas no sistema polar;



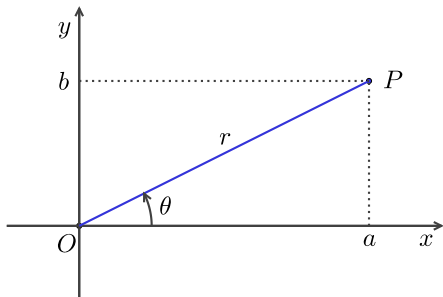
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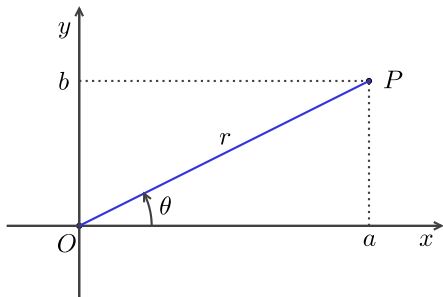
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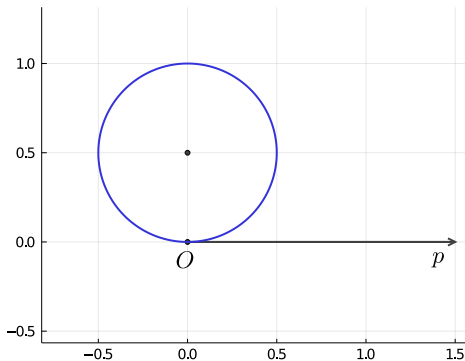
- $P = (a, b)$ - coordenadas no sistema cartesiano;
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- $\cos \theta = \frac{a}{r}$;
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Revisitando o Exercício 2

Exercício 2

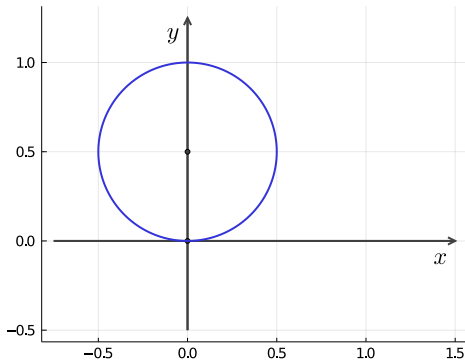
Considere um sistema polar (O, p) e a circunferência de raio $1/2$, centrada no ponto de coordenadas polares $(1/2, \pi/2)$. Prove que esta circunferência é dada por $\mathcal{C} = \{(r, \theta) \mid \theta \in [0, \pi), r = \sin \theta\}$.



Revisitando o Exercício 2

Exercício 2

Considere um sistema polar (O, p) e a circunferência de raio $1/2$, centrada no ponto de coordenadas polares $(1/2, \pi/2)$. Prove que esta circunferência é dada por $C = \{(r, \theta) \mid \theta \in [0, \pi), r = \sin \theta\}$.

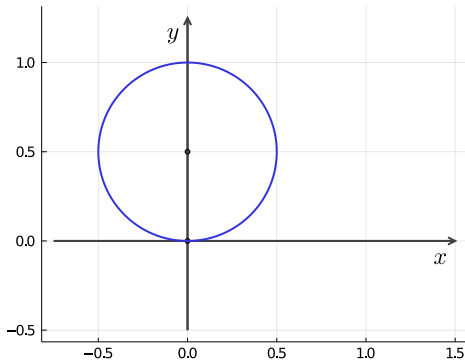


Revisitando o Exercício 2

Exercício 2

Considere um sistema polar (O, p) e a circunferência de raio $1/2$, centrada no ponto de coordenadas polares $(1/2, \pi/2)$. Prove que esta circunferência é dada por $\mathcal{C} = \{(r, \theta) \mid \theta \in [0, \pi), r = \sin \theta\}$.

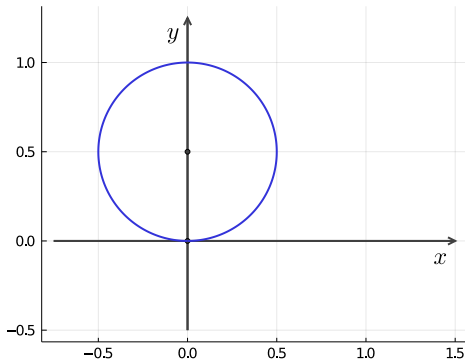
- $x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$;



Exercício 2

Considere um sistema polar (O, p) e a circunferência de raio $1/2$, centrada no ponto de coordenadas polares $(1/2, \pi/2)$. Prove que esta circunferência é dada por $\mathcal{C} = \{(r, \theta) \mid \theta \in [0, \pi), r = \sin \theta\}$.

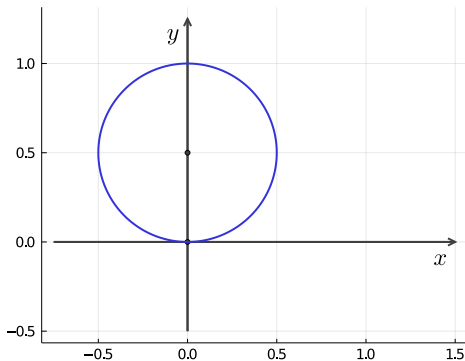
- $x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$;
- $r^2 \cos^2 \theta + (r \sin \theta - \frac{1}{2})^2 = \frac{1}{4}$;



Exercício 2

Considere um sistema polar (O, p) e a circunferência de raio $1/2$, centrada no ponto de coordenadas polares $(1/2, \pi/2)$. Prove que esta circunferência é dada por $\mathcal{C} = \{(r, \theta) \mid \theta \in [0, \pi), r = \sin \theta\}$.

- $x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$;
- $r^2 \cos^2 \theta + (r \sin \theta - \frac{1}{2})^2 = \frac{1}{4}$;
- $r = \sin \theta$.

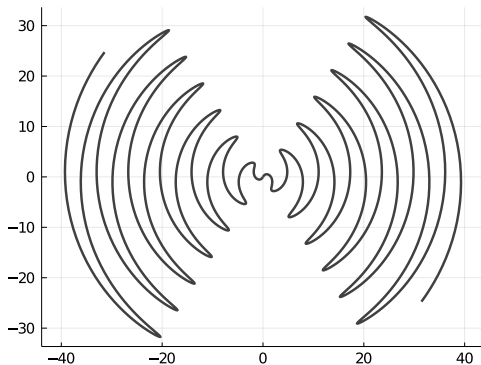


Exemplo 1

$$(\theta - \pi - \cos r)(\theta - \cos r) = 0.$$

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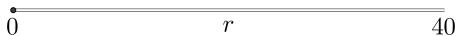
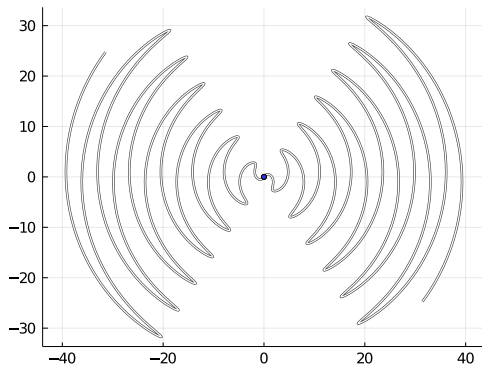
0

r

40

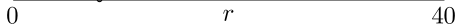
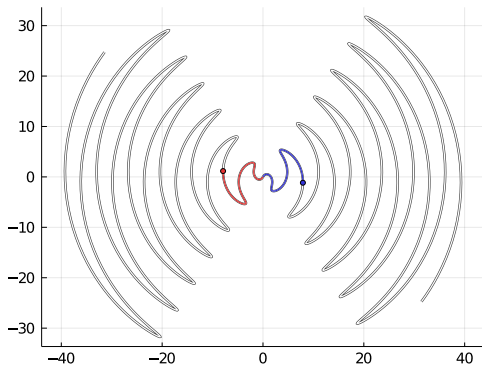
Exemplo 1

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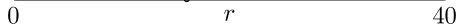
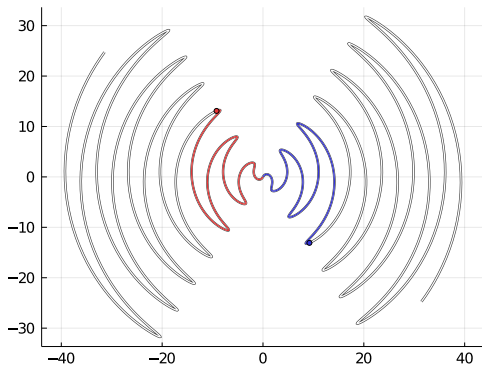
Exemplo 1

$$(\theta - \pi - \cos r)(\theta - \cos r) = 0.$$



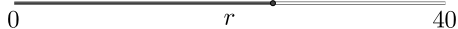
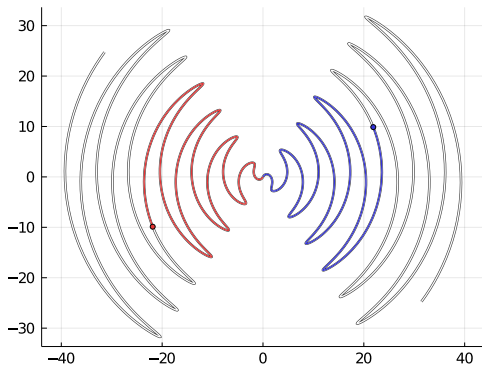
Exemplo 1

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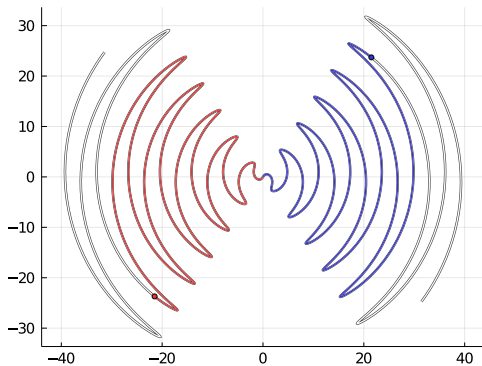
Exemplo 1

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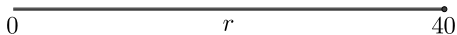
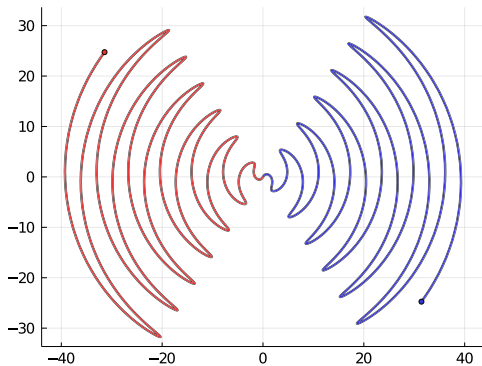
0

r

40

Exemplo 1

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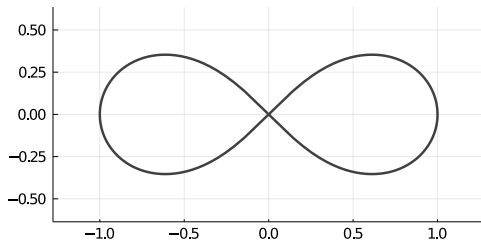


Exemplo 2

$$r = \sqrt{\cos(2\theta)}.$$

Exemplo 2

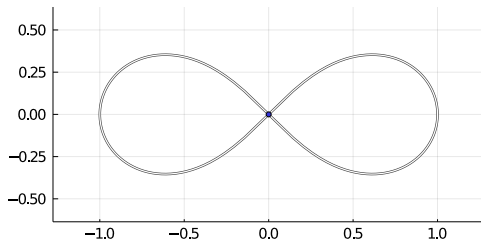
$$r = \sqrt{\cos(2\theta)}.$$



$$\overbrace{-\frac{\pi}{4} \quad \frac{\pi}{4}} \quad \theta \quad \overbrace{\frac{3\pi}{4} \quad \frac{5\pi}{4}}$$

Exemplo 2

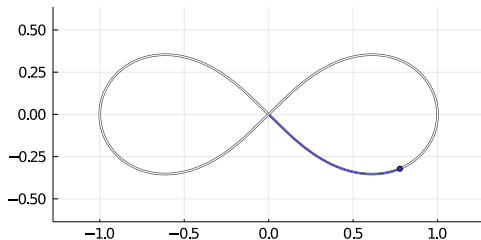
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$$\begin{array}{c} \bullet \\ \hline -\frac{\pi}{4} \qquad \frac{\pi}{4} \end{array} \quad \theta \quad \begin{array}{c} \hline \frac{3\pi}{4} \qquad \frac{5\pi}{4} \end{array}$$

Exemplo 2

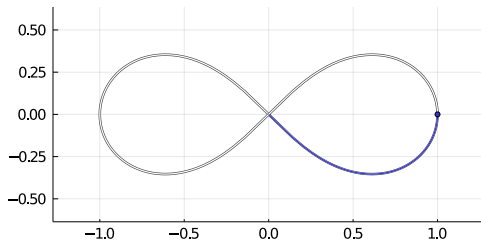
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$$\overline{\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]} \quad \theta \quad \overline{\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]}$$

Exemplo 2

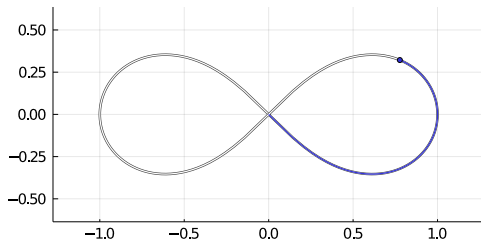
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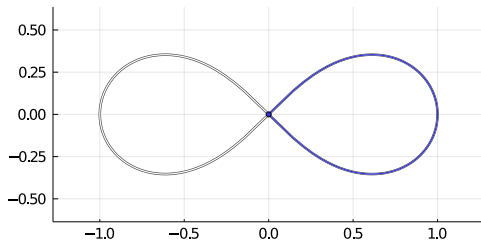
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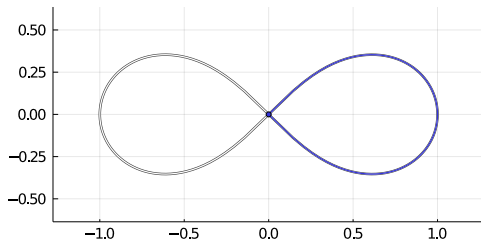


$$\overline{\quad \quad \quad} \quad \theta \quad \overline{\quad \quad \quad}$$

$-\frac{\pi}{4} \quad \frac{\pi}{4} \quad \quad \quad \frac{3\pi}{4} \quad \frac{5\pi}{4}$

Exemplo 2

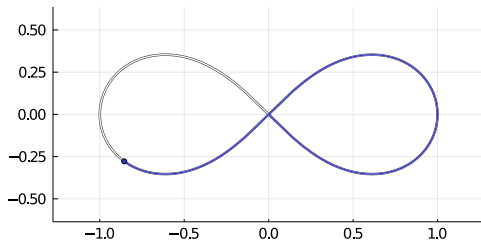
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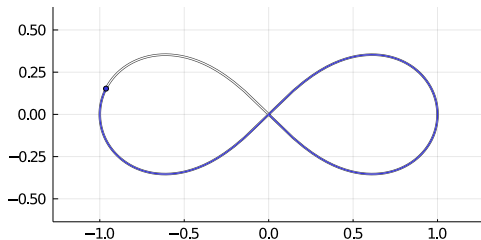
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Exemplo 2

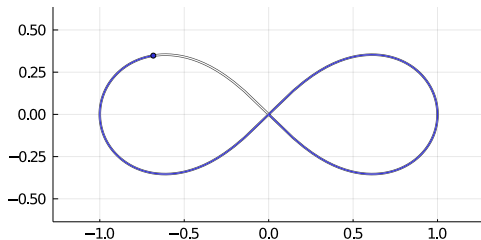
$$r = \sqrt{\cos(2\theta)}.$$



$$\overline{-\frac{\pi}{4} \quad \frac{\pi}{4}} \quad \theta \quad \overline{\frac{3\pi}{4} \quad \frac{5\pi}{4}}$$

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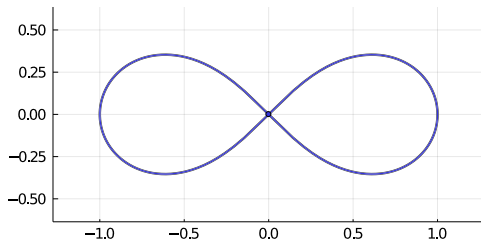
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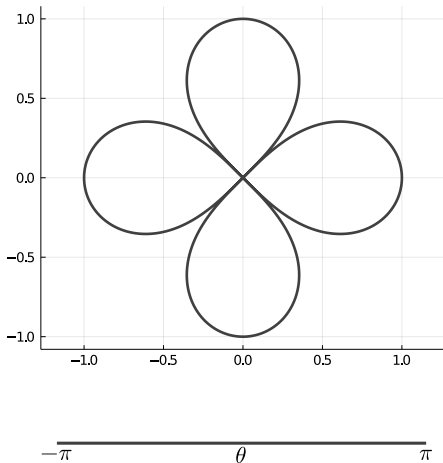
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Exemplo 3

$$r = \sqrt{|\cos(2\theta)|}.$$

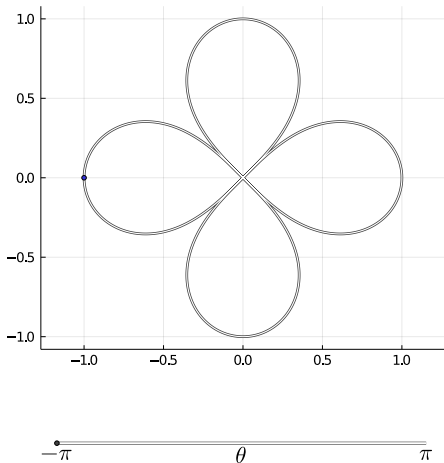
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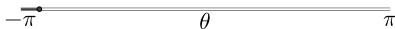
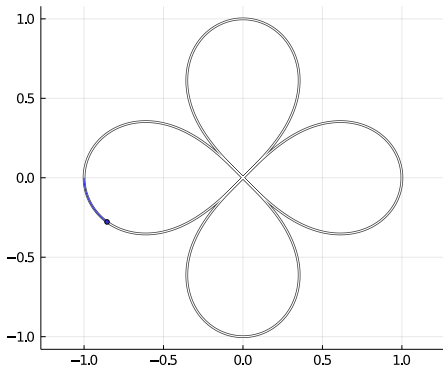
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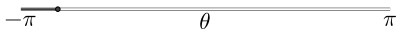
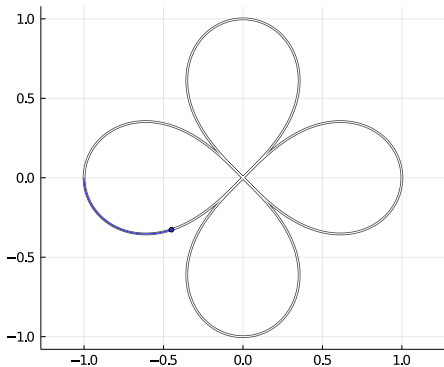
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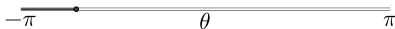
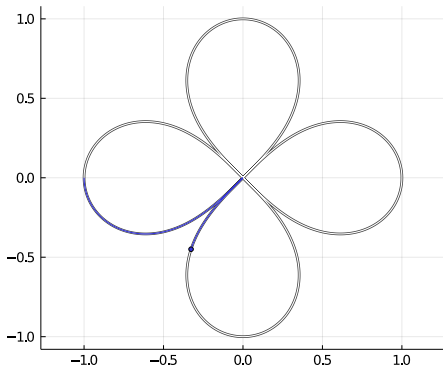
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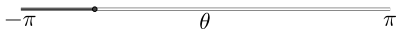
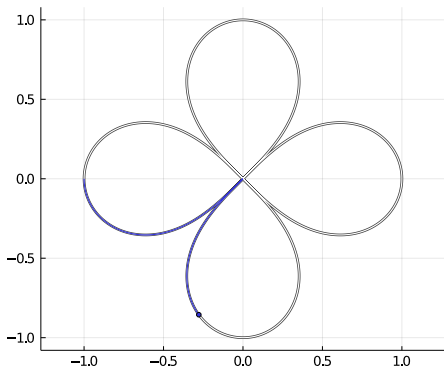
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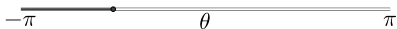
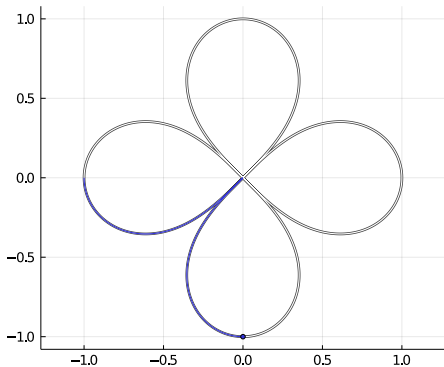
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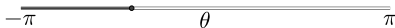
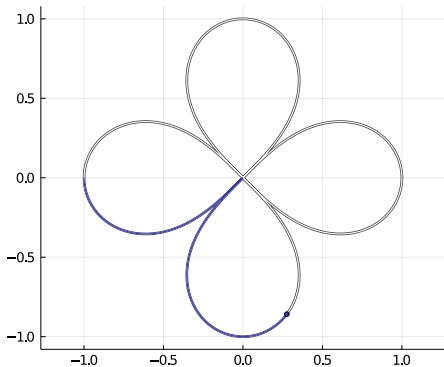
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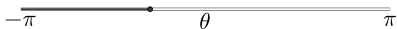
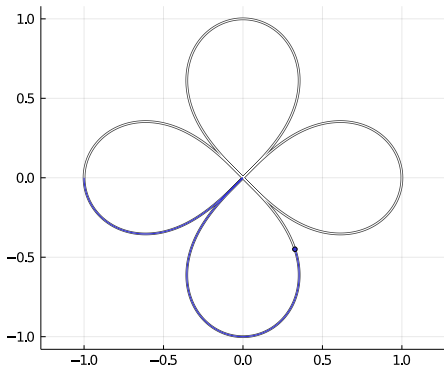
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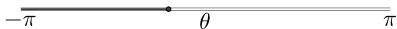
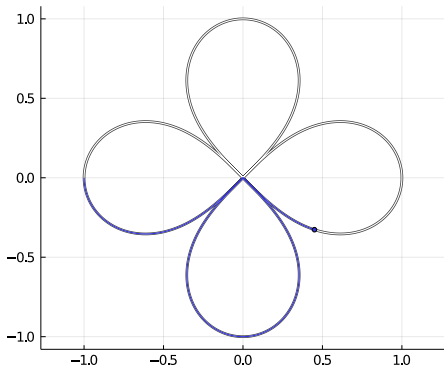
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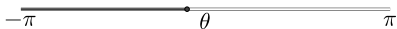
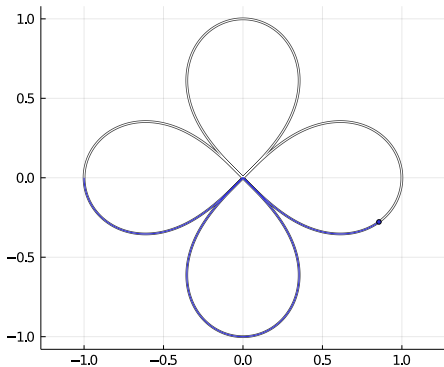
Exemplo 3

$$r = \sqrt{|\cos(2\theta)|}.$$



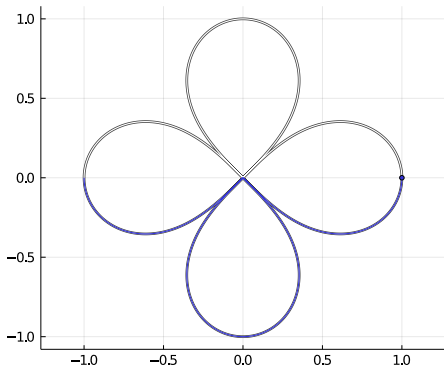
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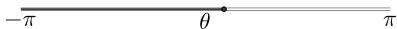
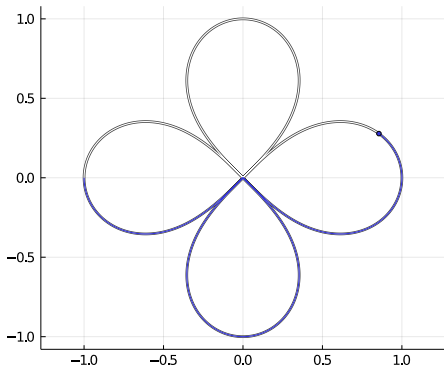
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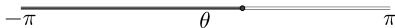
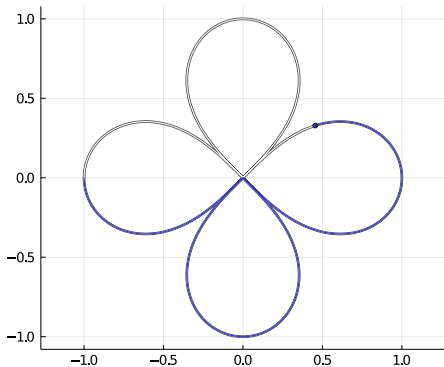
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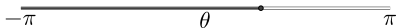
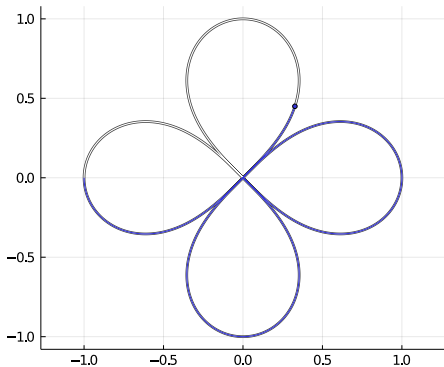
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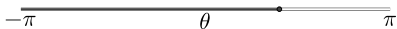
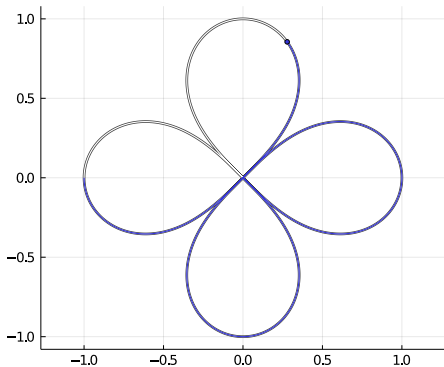
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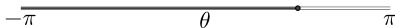
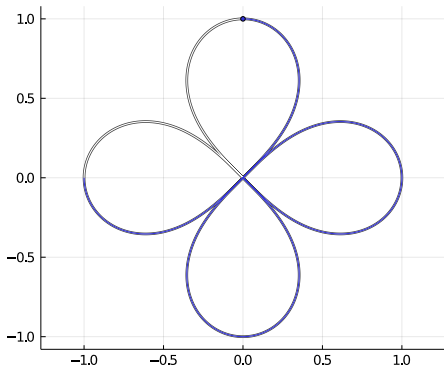
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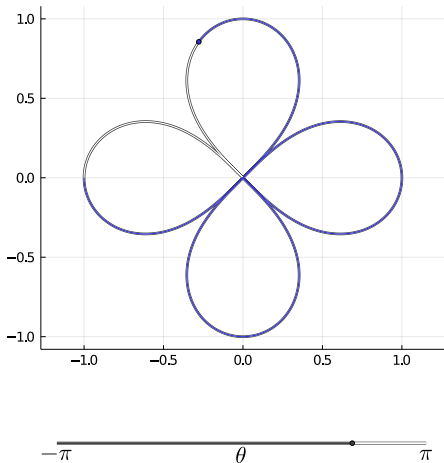
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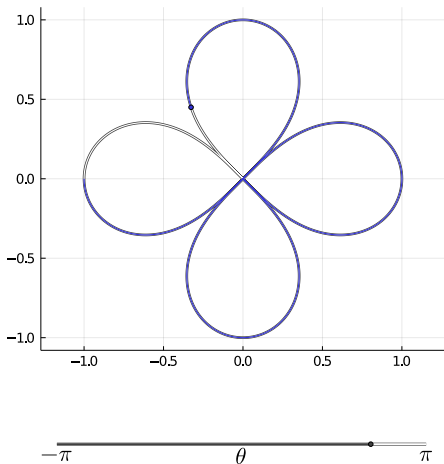
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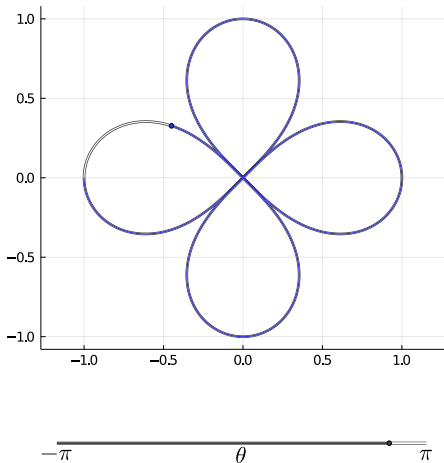
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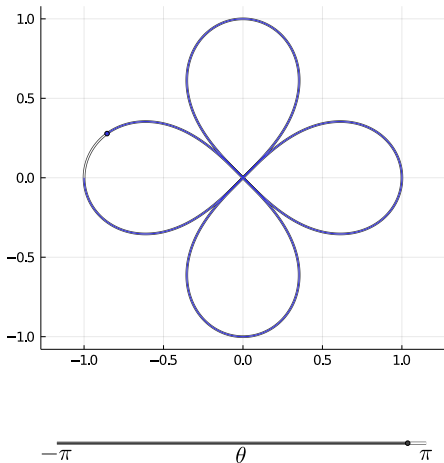
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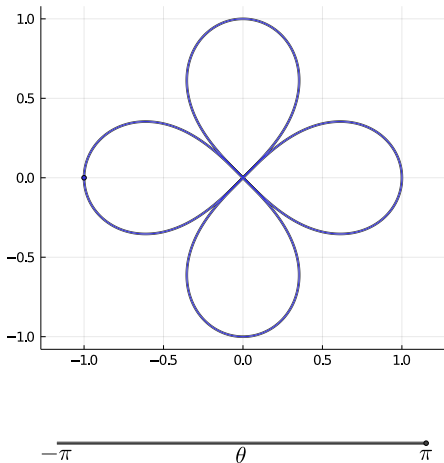
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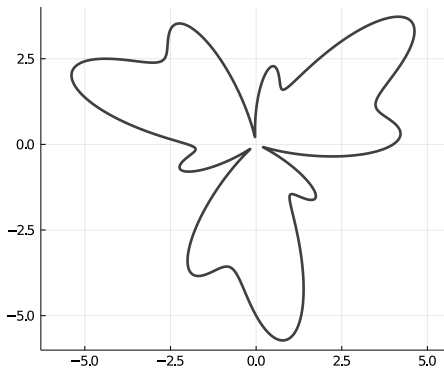


Exemplo 4

$$r = 3 + \cos(9\theta) + 2 \sin(3\theta).$$

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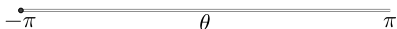
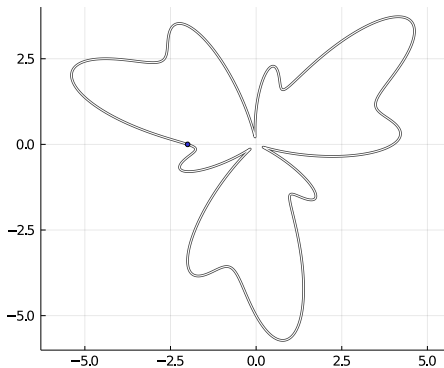
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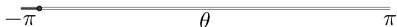
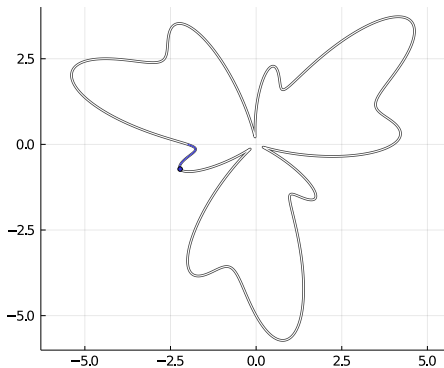
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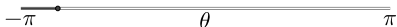
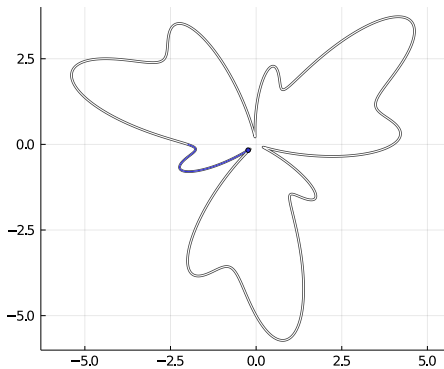
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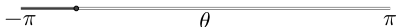
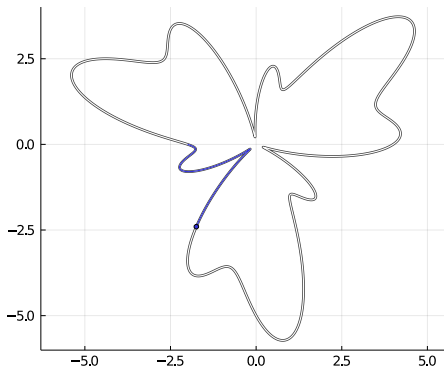
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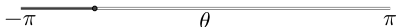
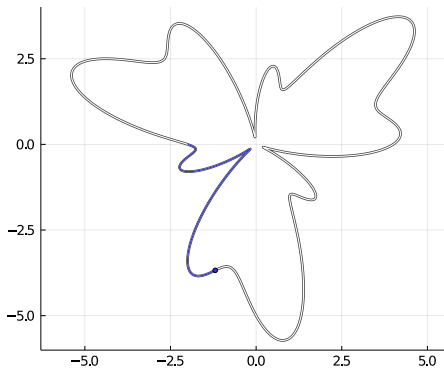
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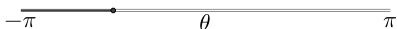
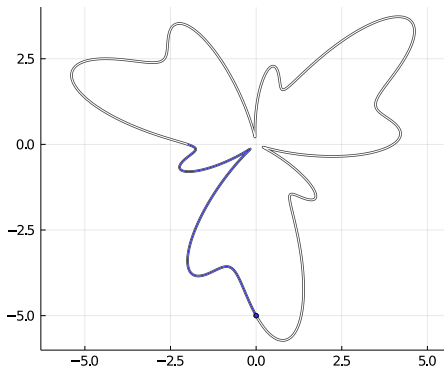
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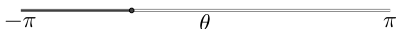
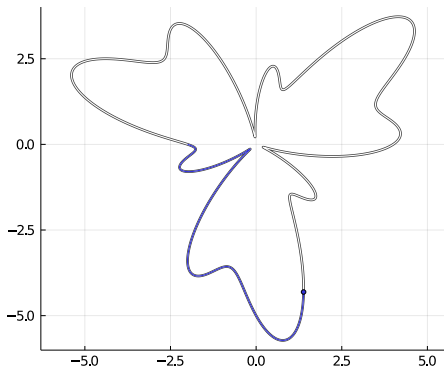
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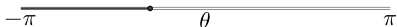
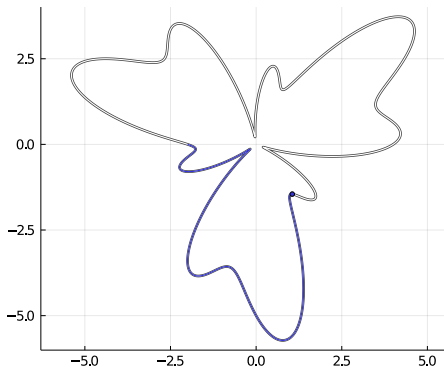
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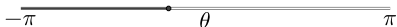
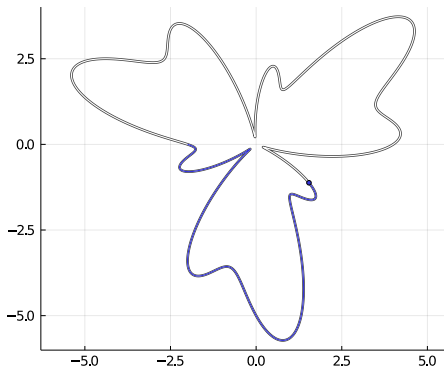
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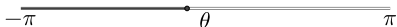
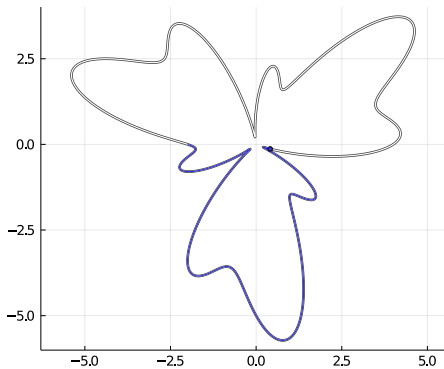
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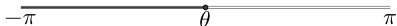
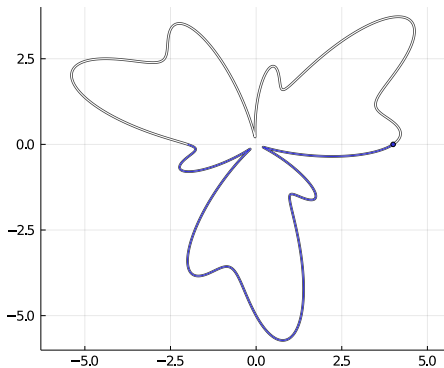
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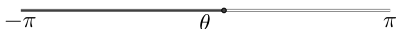
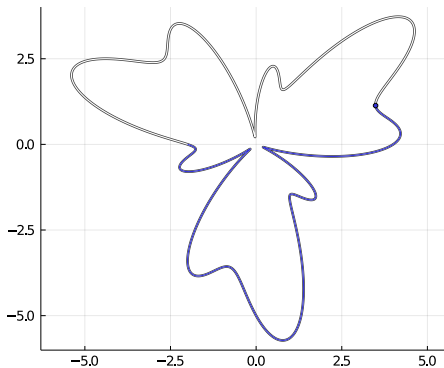
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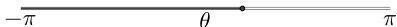
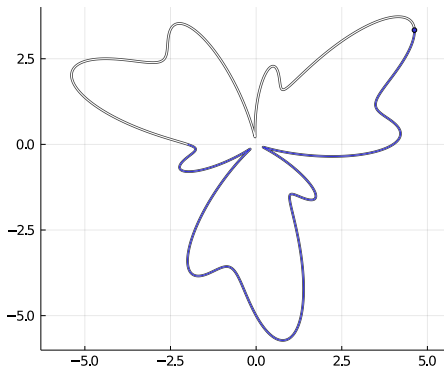
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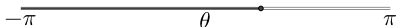
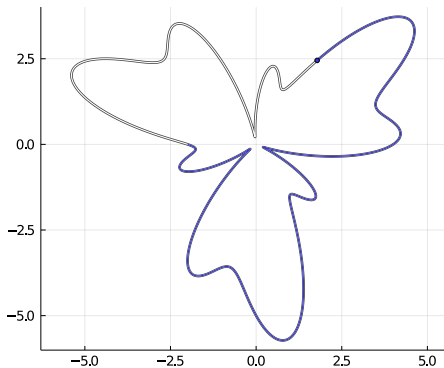
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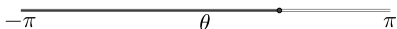
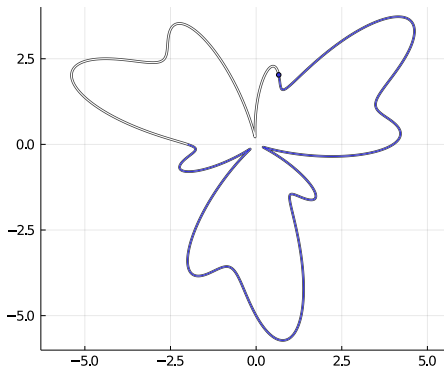
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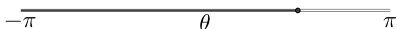
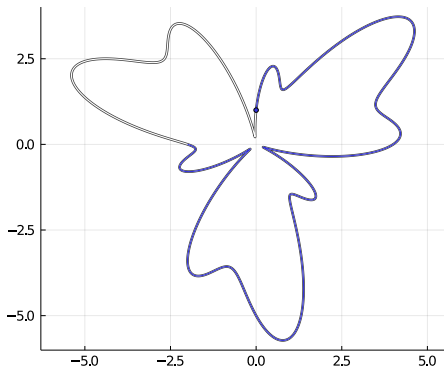
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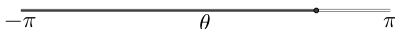
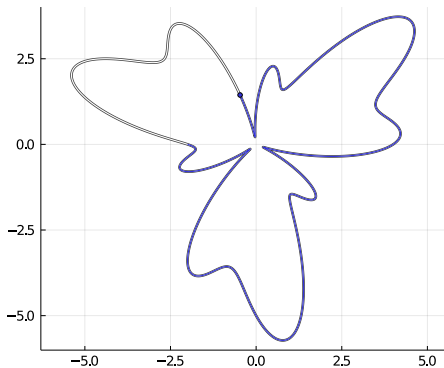
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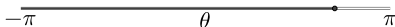
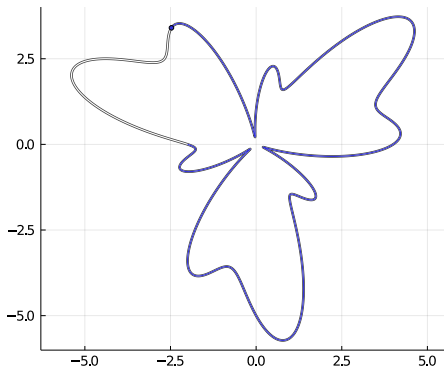
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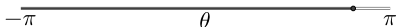
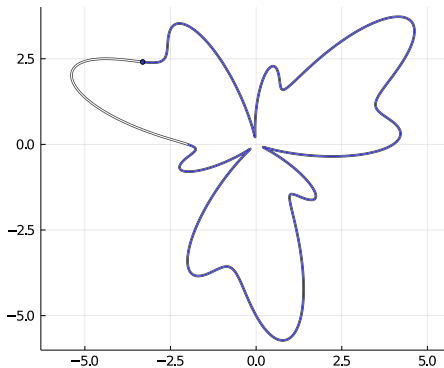
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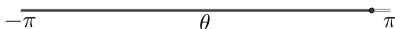
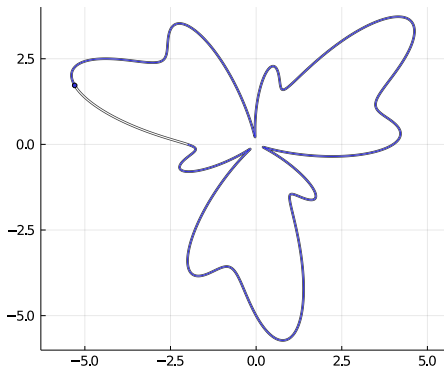
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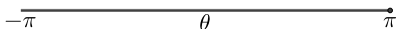
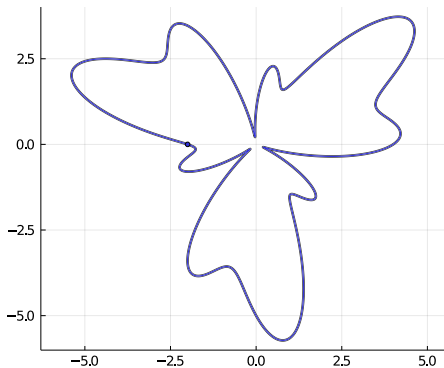
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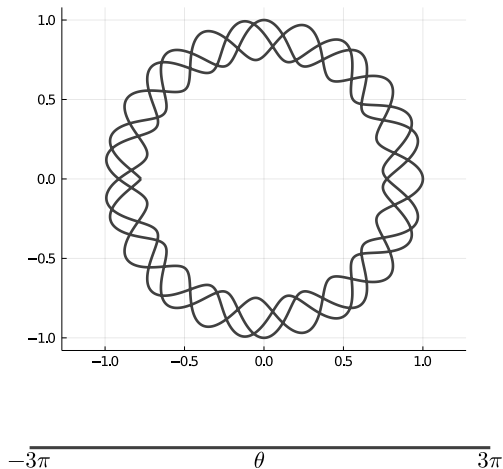


Exemplo 5

$$r = \sin^2\left(\frac{12}{5}\theta\right) + \cos^4\left(\frac{12}{5}\theta\right).$$

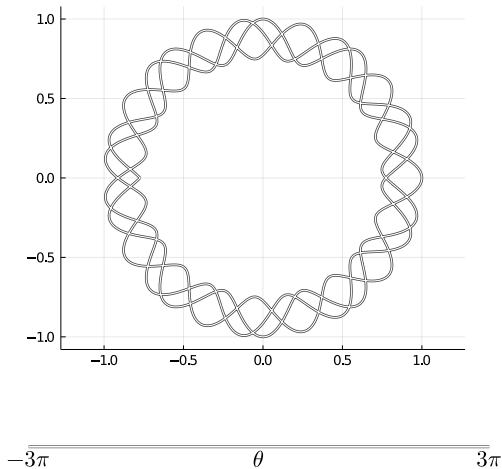
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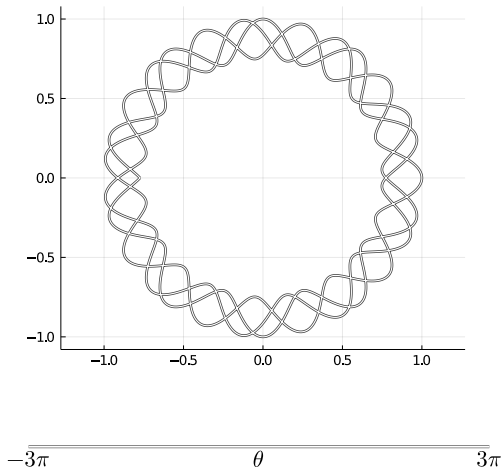
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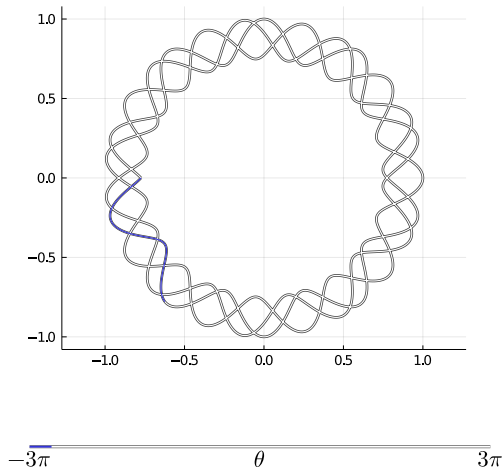
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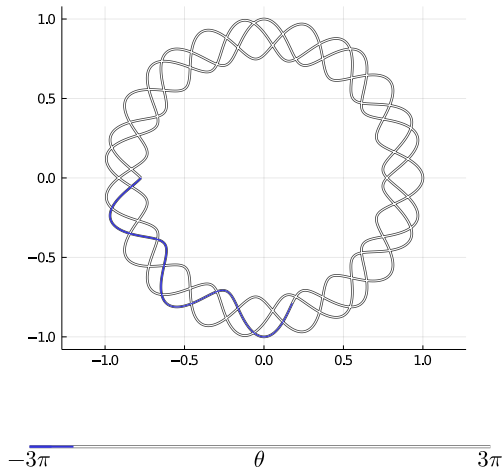
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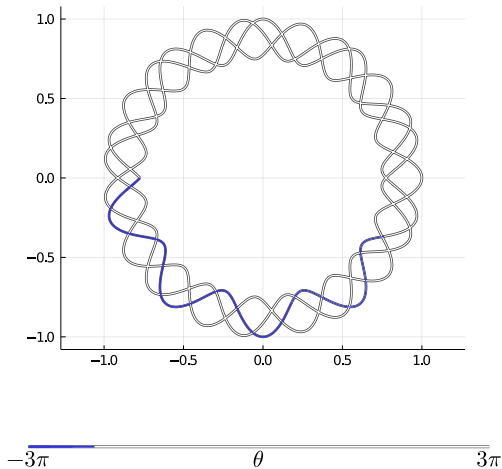
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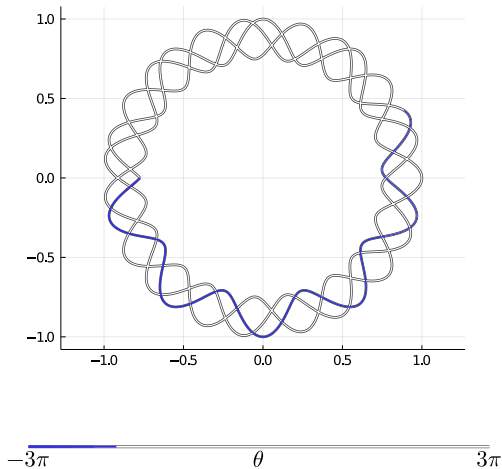
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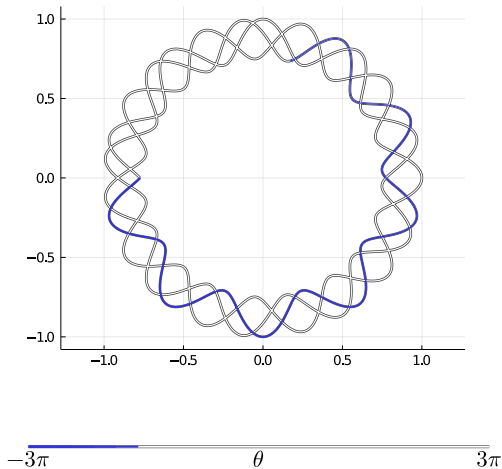
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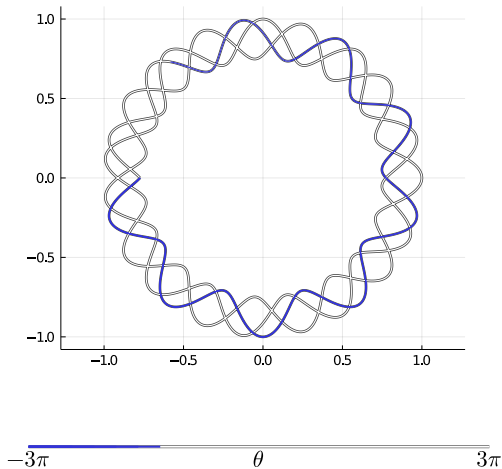
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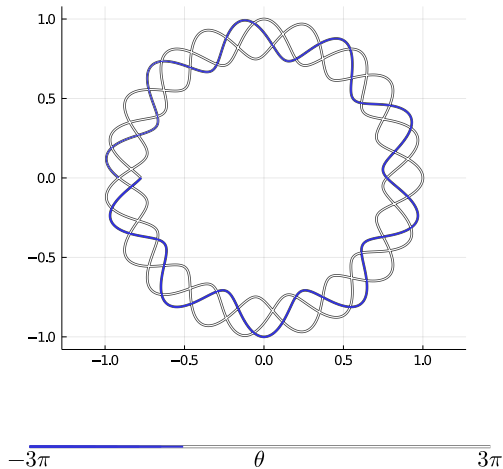
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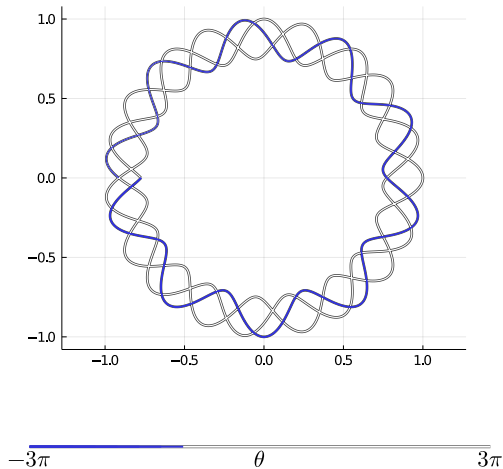
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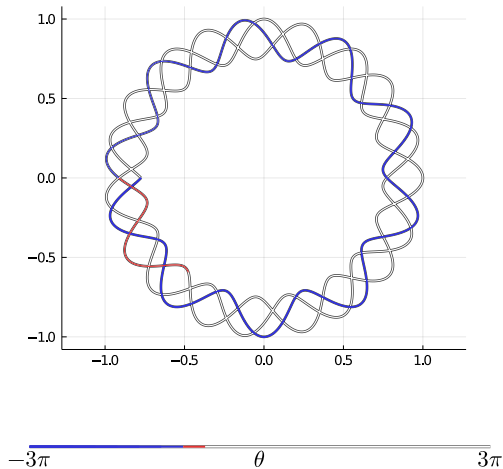
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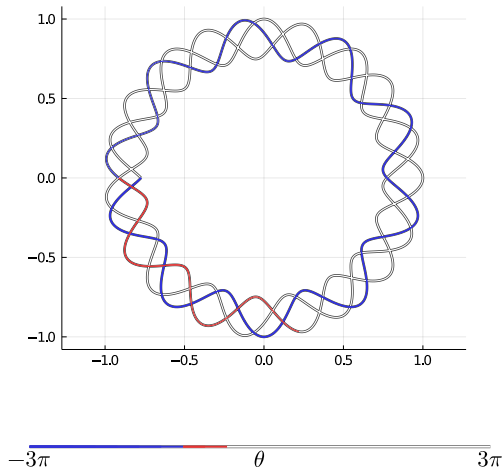
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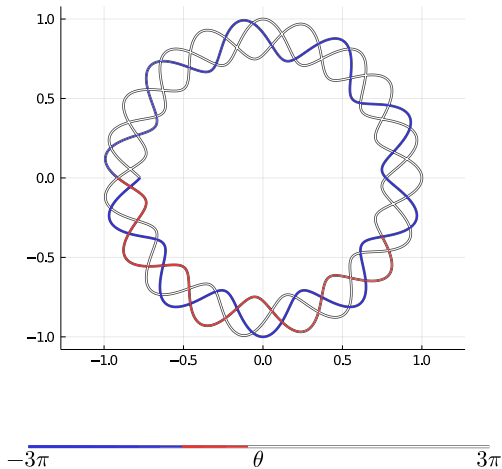
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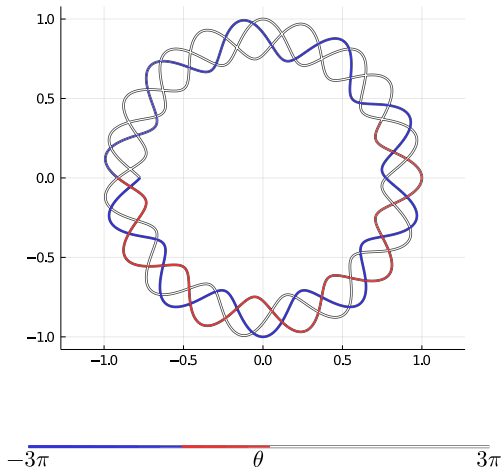
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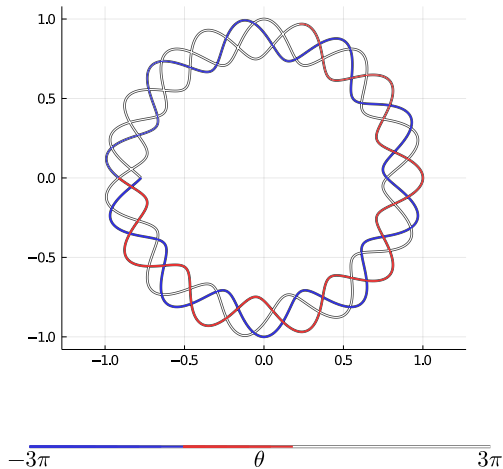
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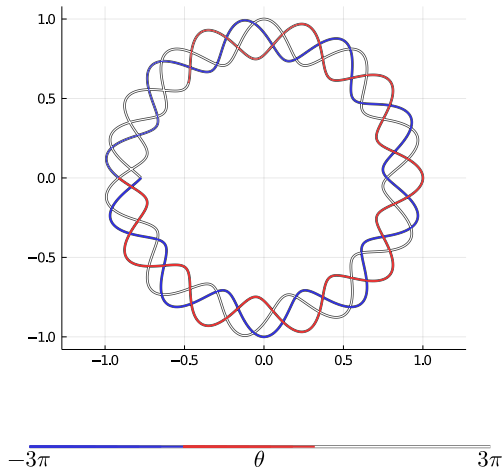
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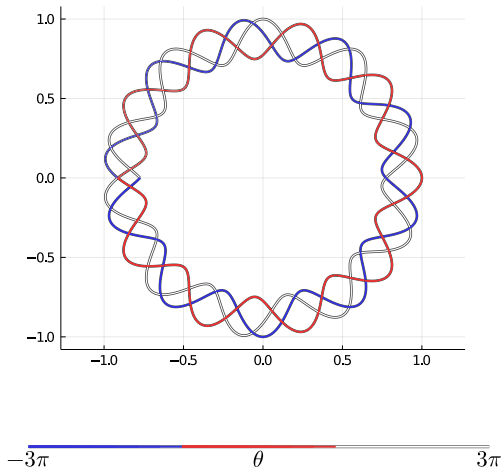
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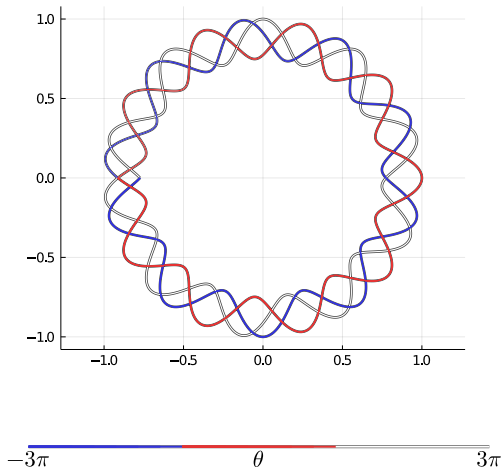
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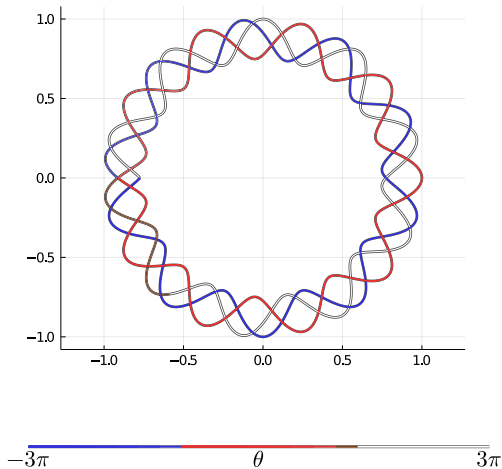
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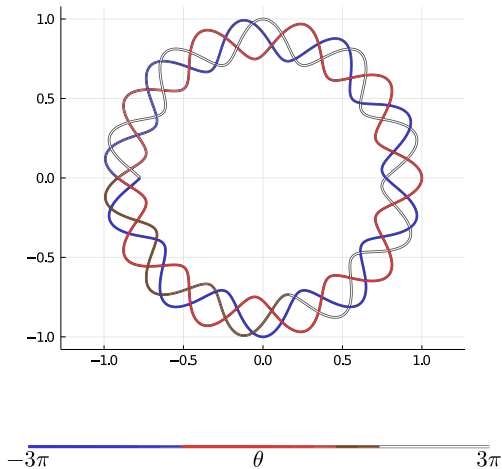
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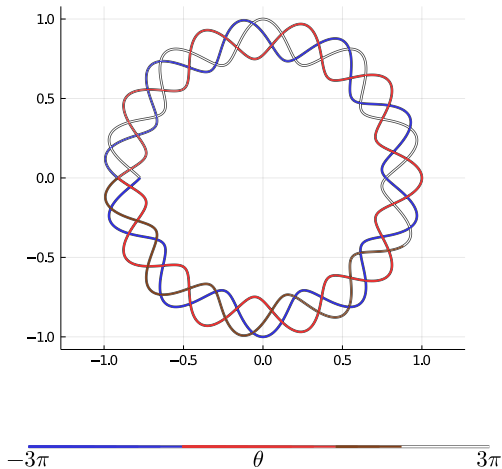
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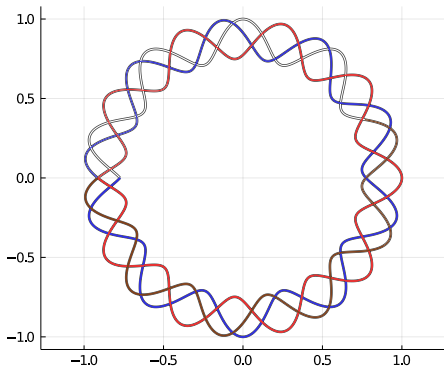
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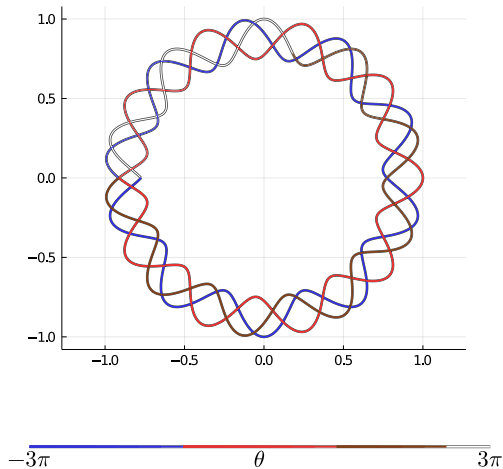
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-3π θ 3π

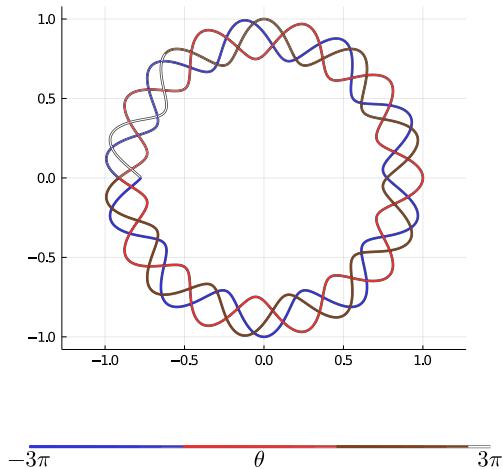
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Exemplo 5

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