

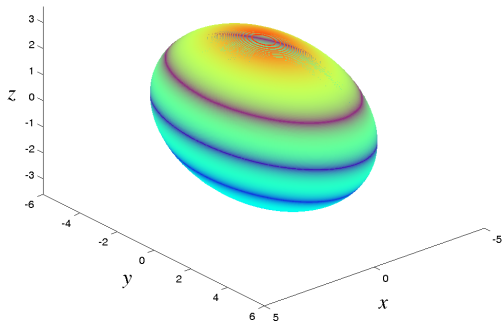
# Geometria Analítica - Quádricas

Elizabeth Wegner Karas  
Ademir Alves Ribeiro

Junho / 2006

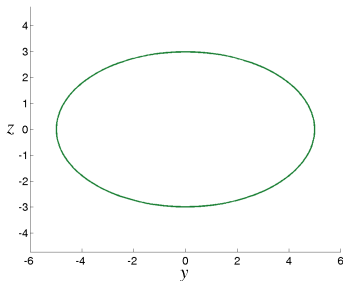
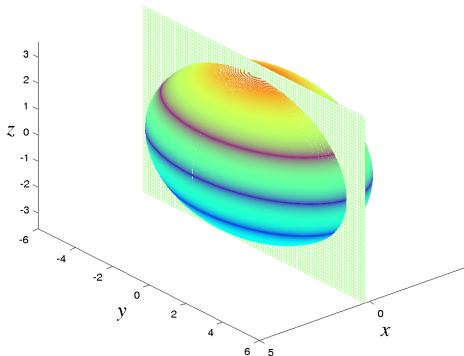
- 1 Elipsóide
- 2 Hiperbolóides
- 3 Parabolóides
- 4 Cilindros

## Elipsóide: gráfico e equação



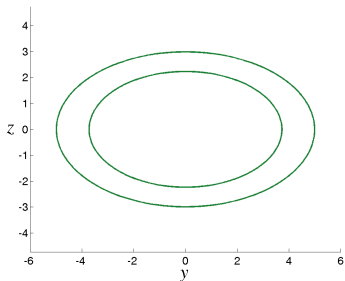
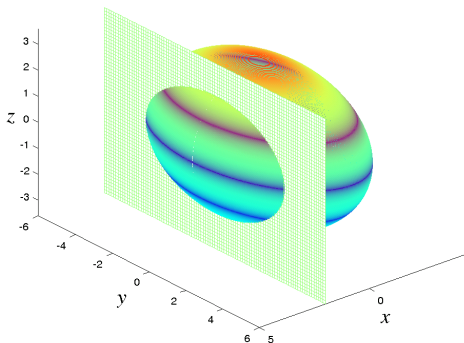
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

## Interseção com planos verticais: $x = k$



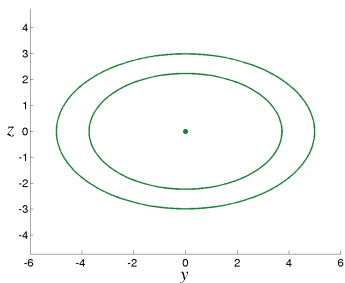
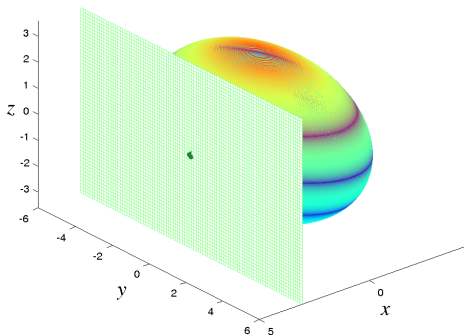
$$\begin{cases} \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 - \frac{k^2}{a^2} \\ x = k \end{cases} : \text{ elipses } (-a < k < a).$$

## Interseção com planos verticais: $x = k$



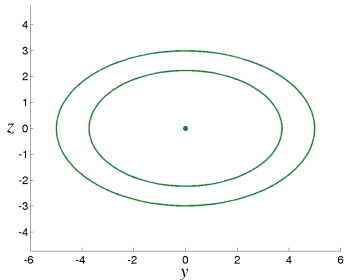
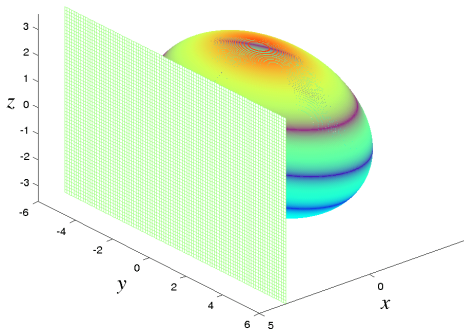
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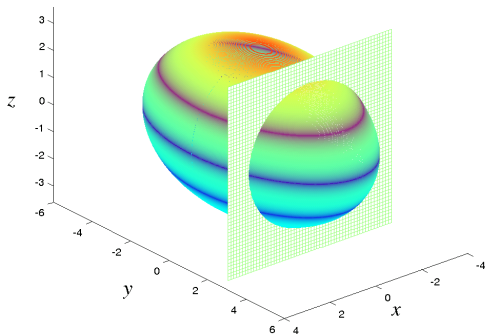
$$\begin{cases} \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0 \\ x = k \end{cases} : \text{um ponto } (k = -a \text{ ou } k = a).$$

## Interseção com planos verticais: $x = k$



- O conjunto vazio quando  $k = < -a$  ou  $k > a$ .

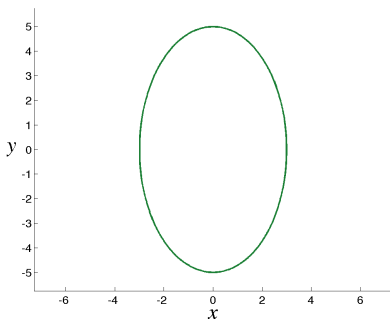
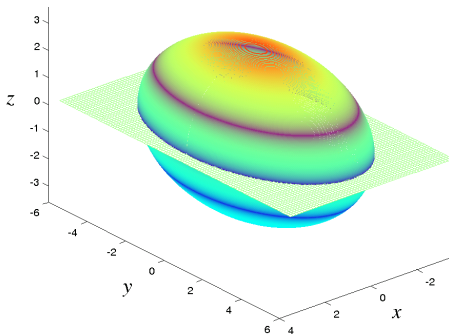
## Interseção com planos verticais: $y = k$



- Análogo ao caso anterior ( $x \leftrightarrow y$ ,  $a \leftrightarrow b$ ).

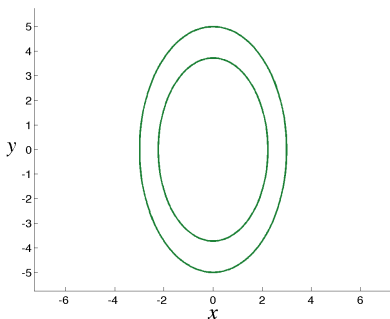
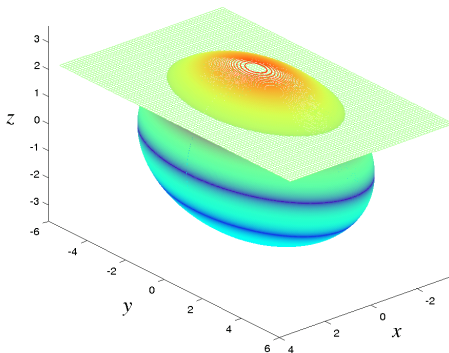


## Interseção com planos horizontais: $z = k$



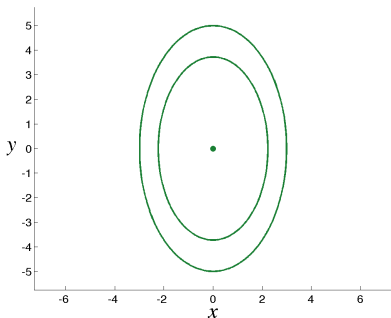
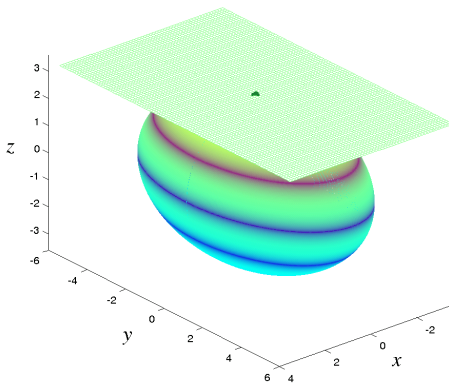
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{k^2}{c^2} \\ z = k \end{cases} : \text{ elipses } (-c < k < c).$$

## Interseção com planos horizontais: $z = k$



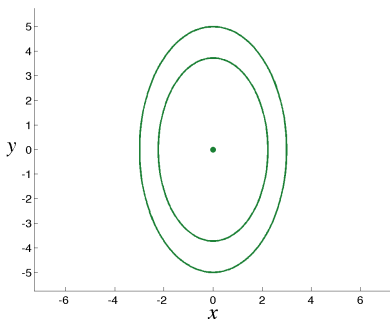
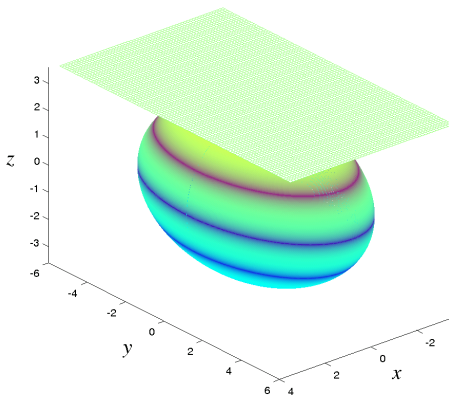
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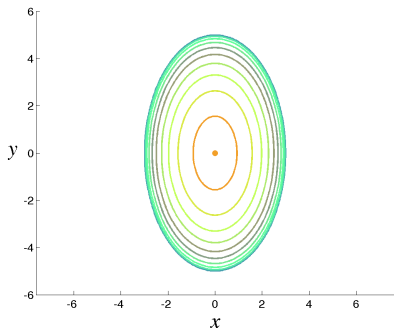
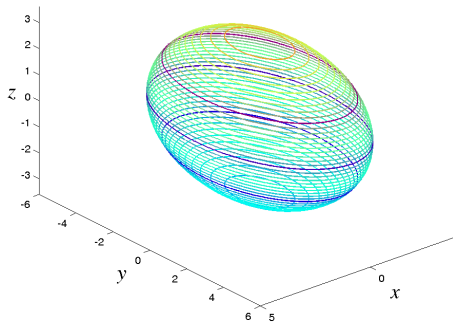
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \\ z = k \end{cases} : \text{um ponto } (k = -c \text{ ou } k = c).$$

## Interseção com planos horizontais: $z = k$



- O conjunto vazio quando  $k \leq -c$  ou  $k > c$ .

## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



- As curvas de nível são elipses para todo  $k \in (-c, c)$ , um ponto para  $k = -c$  ou  $k = c$  e o conjunto vazio para  $k < -c$  ou  $k > c$ .

## Equação geral de um hiperbolóide

Equação geral

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = \alpha$$

- $\alpha = 1$ : hiperbolóide de uma folha
- $\alpha = -1$ : hiperbolóide de duas folhas
- $\alpha = 0$ : hiperbolóide degenerado (cone)

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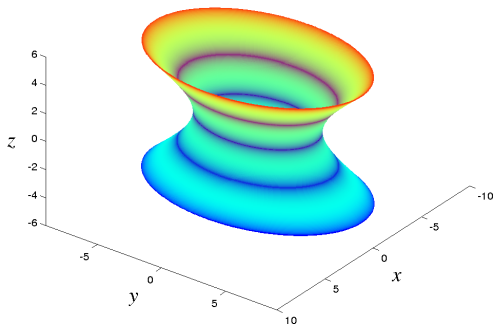
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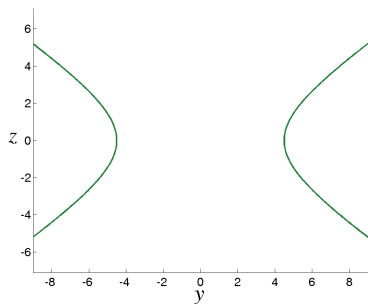
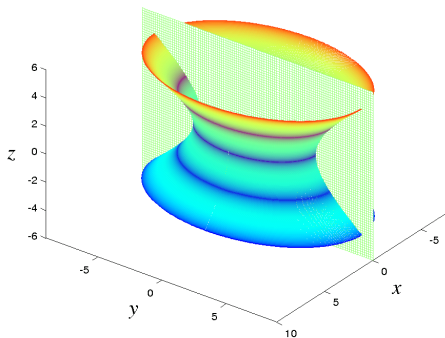
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## Hiperbolóide de uma folha: gráfico e equação



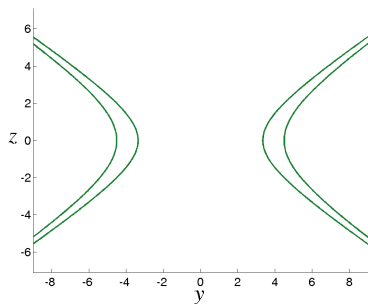
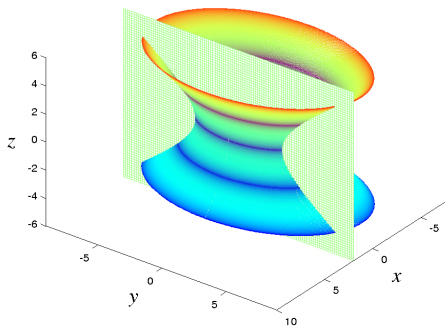
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

## Interseção com planos verticais: $x = k$



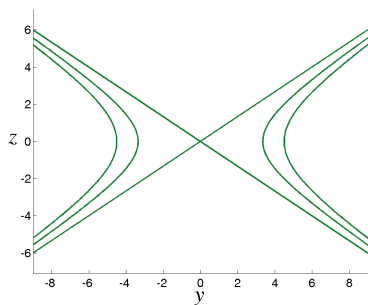
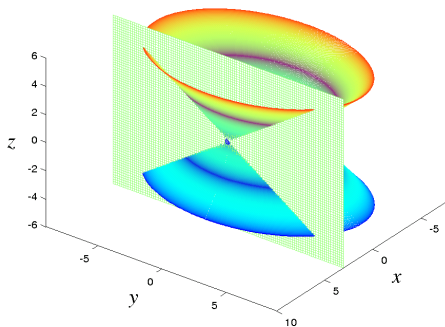
$$\begin{cases} \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 - \frac{k^2}{a^2} \\ x = k \end{cases} : \text{hipérboles } (-a < k < a).$$

## Interseção com planos verticais: $x = k$



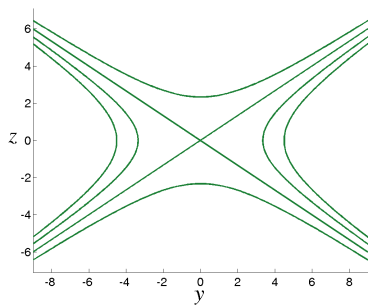
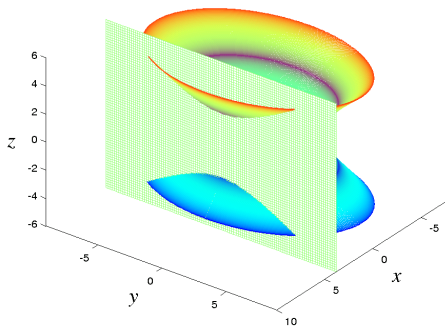
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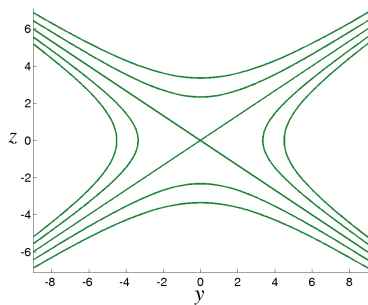
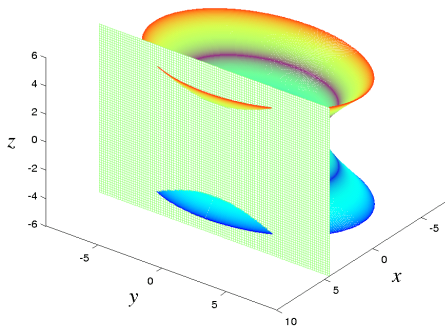
$$\begin{cases} \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \\ x = k \end{cases} : \text{duas retas concorrentes } (k = -a \text{ ou } k = a).$$

## Interseção com planos verticais: $x = k$



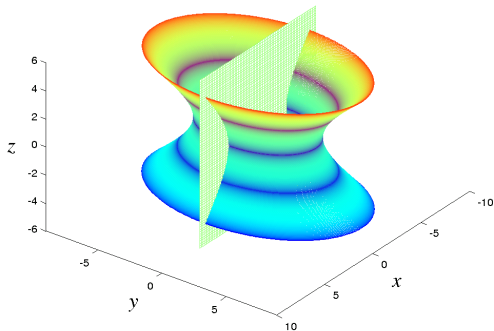
$$\begin{cases} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} - 1 \\ x = k \end{cases} : \text{hipérboles } (k < -a \text{ ou } k > a).$$

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$$\begin{cases} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} - 1 \\ x = k \end{cases} : \text{hipérboles } (k < -a \text{ ou } k > a).$$

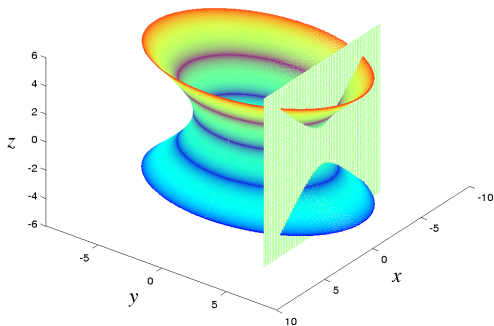
## Interseção com planos verticais: $y = k$



- Análogo ao caso anterior ( $x \leftrightarrow y$ ,  $a \leftrightarrow b$ ).

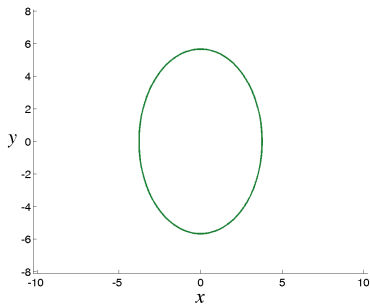
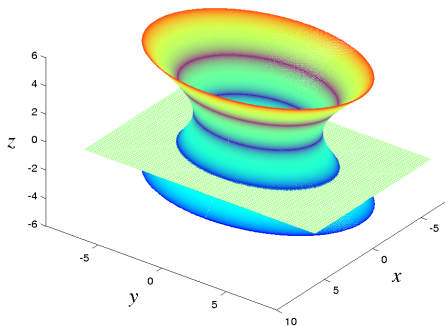


## Interseção com planos verticais: $y = k$



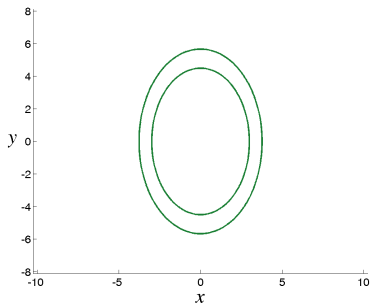
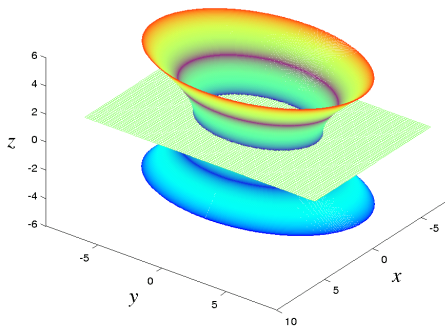
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## Interseção com planos horizontais: $z = k$



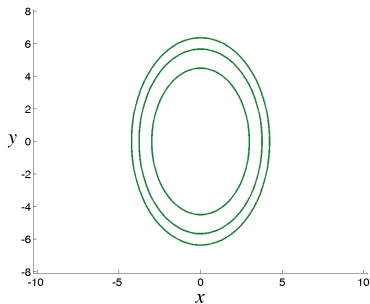
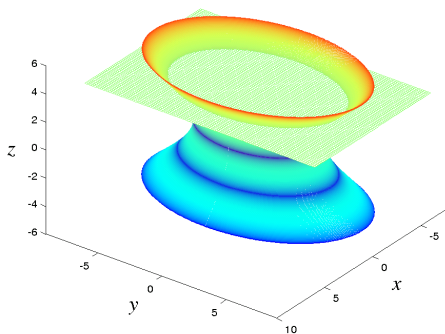
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{k^2}{c^2} \\ z = k \end{cases} : \text{elipses } \forall k \in \mathbb{R}.$$

## Interseção com planos horizontais: $z = k$



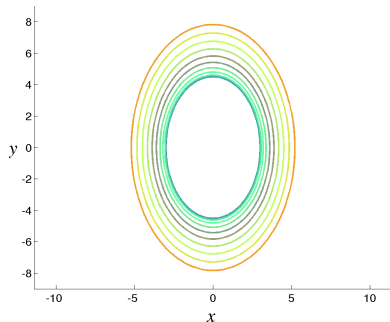
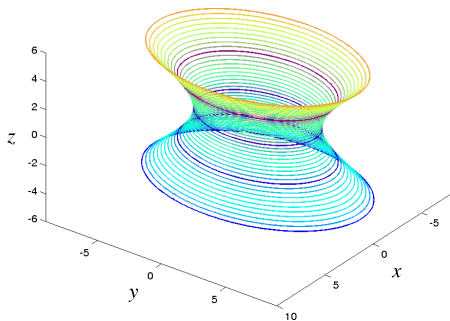
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## Interseção com planos horizontais: $z = k$



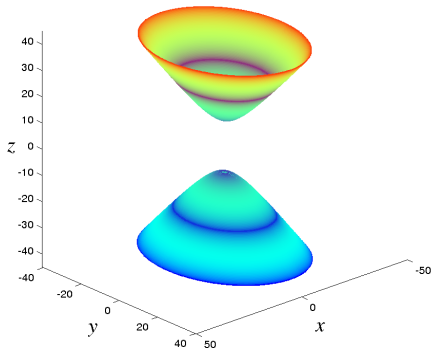
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## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



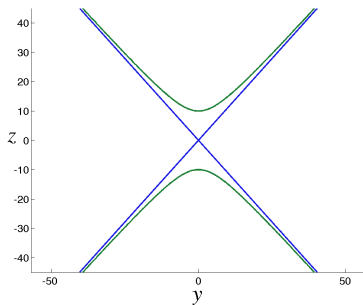
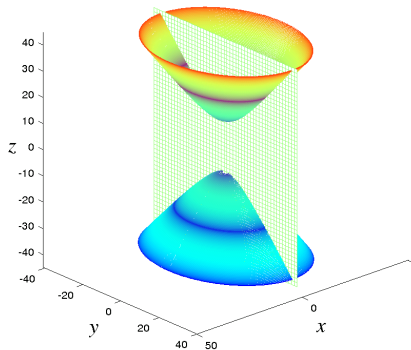
- As curvas de nível são elipses para todo  $k \in \mathbb{R}$ .

## Hiperbolóide de duas folhas: gráfico e equação



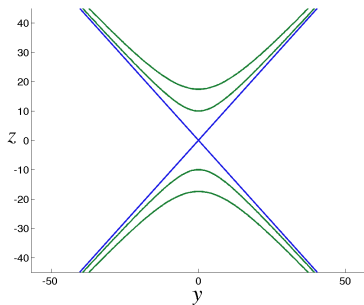
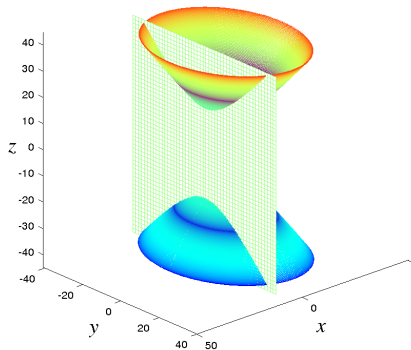
$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

## Interseção com planos verticais: $x = k$



$$\begin{cases} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 + \frac{k^2}{a^2} \\ x = k \end{cases} : \text{hipérboles } \forall k \in \mathbb{R}.$$

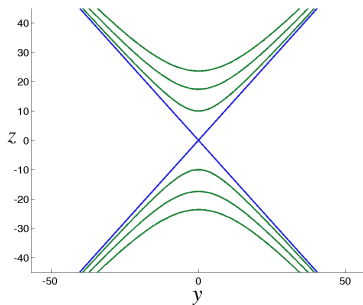
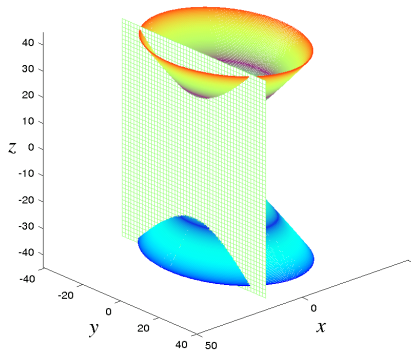
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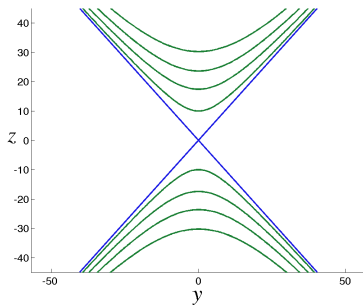
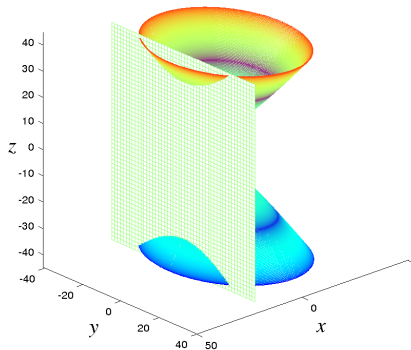


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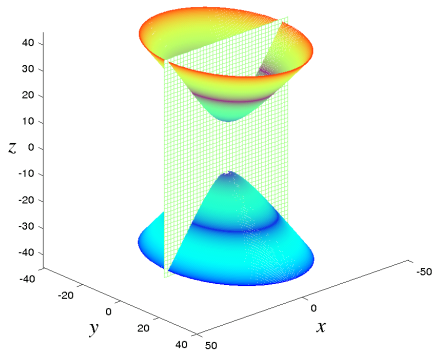
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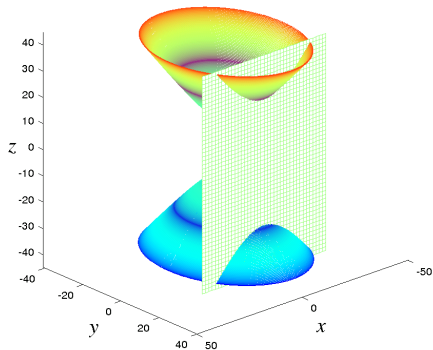
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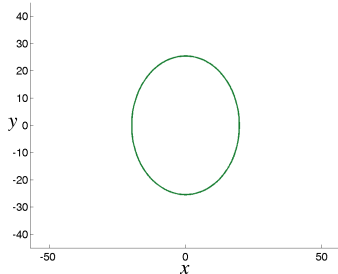
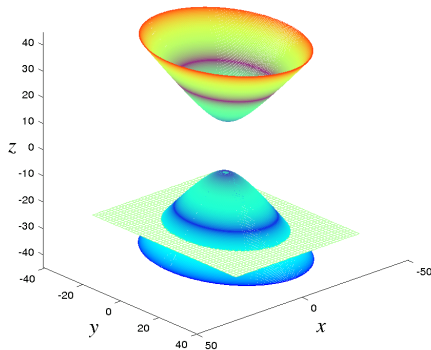
- Análogo ao caso anterior ( $x \leftrightarrow y$ ,  $a \leftrightarrow b$ ).

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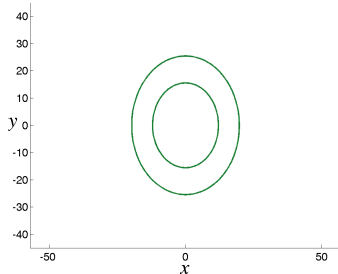
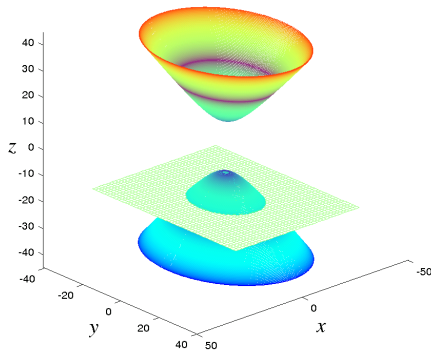
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## Interseção com planos horizontais: $z = k$



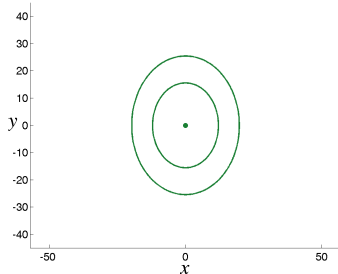
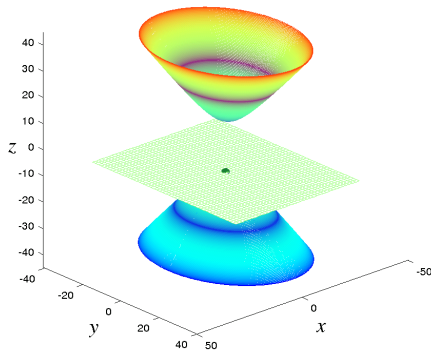
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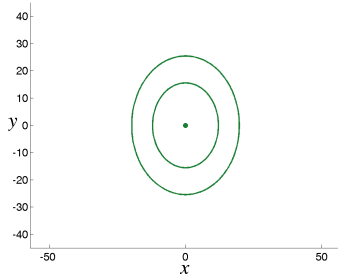
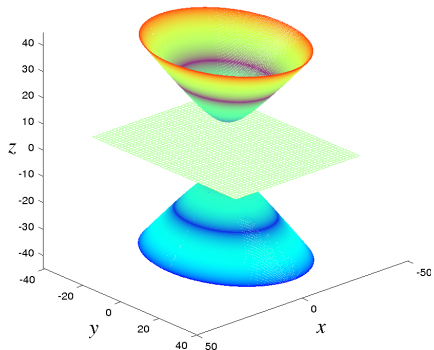
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## Interseção com planos horizontais: $z = k$



$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \\ z = k \end{cases} : \text{um ponto } (k = -c \text{ ou } k = c).$$

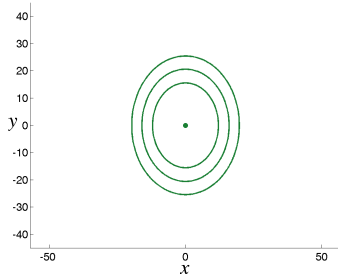
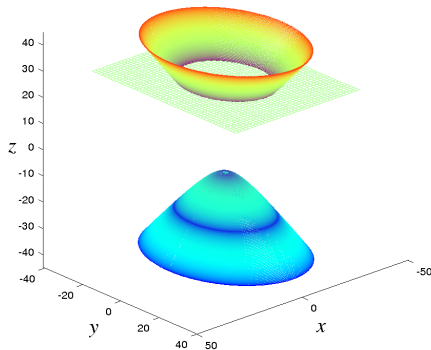
## Interseção com planos horizontais: $z = k$



- O conjunto vazio quando  $-c < k < c$ .

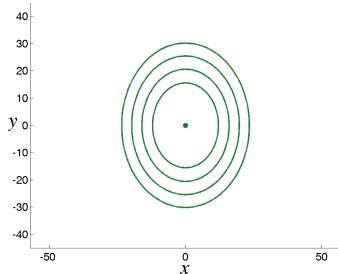
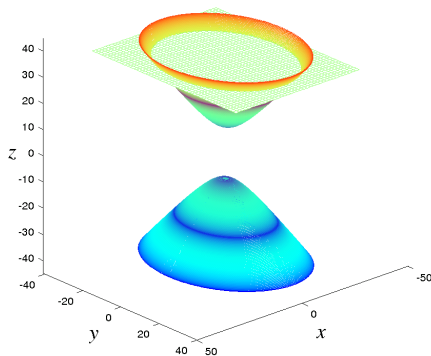


## Interseção com planos horizontais: $z = k$



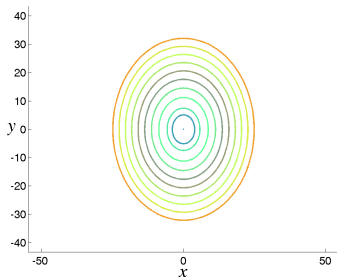
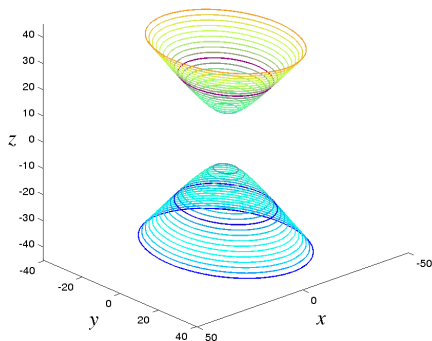
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} - 1 \\ z = k \end{cases} : \text{elipses } (k < -c \text{ ou } k > c).$$

## Interseção com planos horizontais: $z = k$



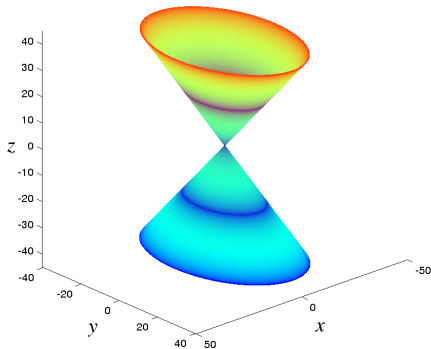
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} - 1 \\ z = k \end{cases} : \text{elipses } (k < -c \text{ ou } k > c).$$

## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



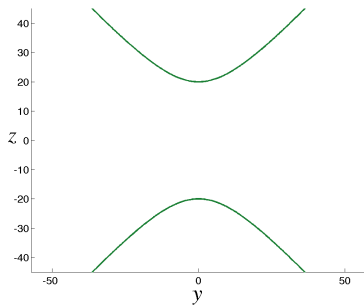
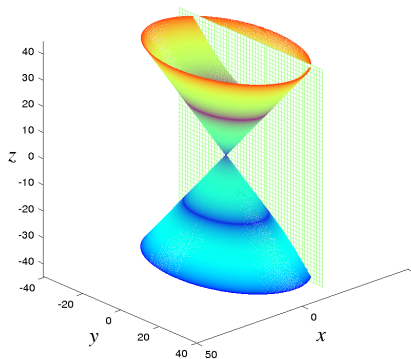
- As curvas de nível são elipses para todo  $k < -c$  ou  $k > c$ , um ponto para  $k = -c$  ou  $k = c$  e o conjunto vazio para  $k \in (-c, c)$ .

## Cone: gráfico e equação



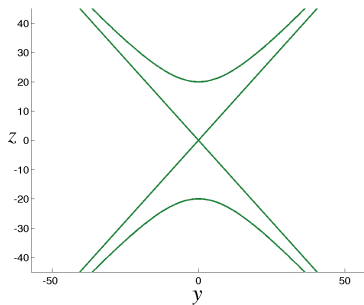
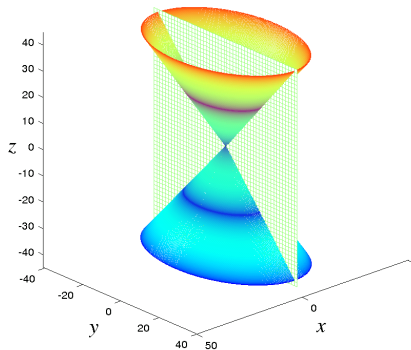
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

## Interseção com planos verticais: $x = k$



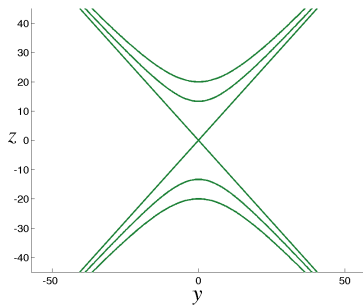
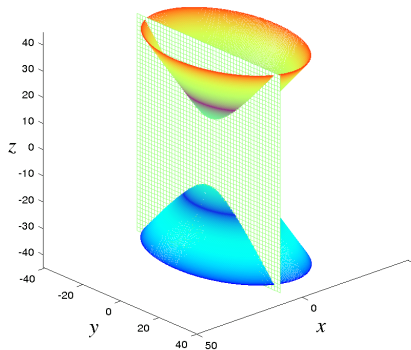
$$\begin{cases} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} \\ x = k \end{cases} : \text{hipérboles } \forall k \neq 0.$$

## Interseção com planos verticais: $x = k$



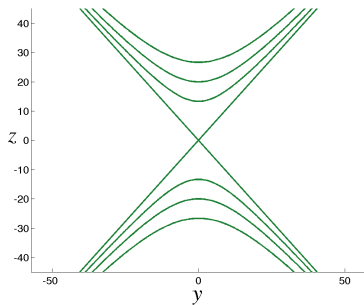
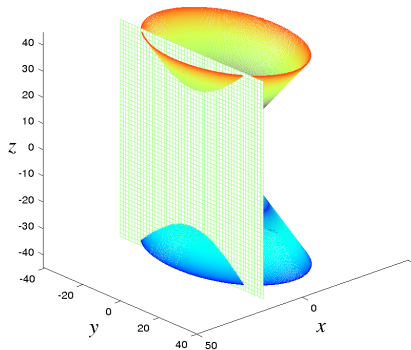
$$\begin{cases} \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \\ x = 0 \end{cases} : \text{duas retas concorrentes } (k = 0).$$

# Interseção com planos verticais: $x = k$



$$\begin{cases} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} \\ x = k \end{cases} : \text{hipérboles } \forall k \neq 0.$$

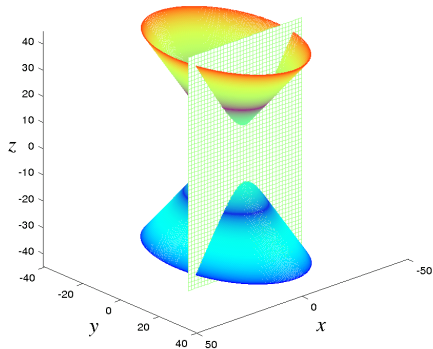
## Interseção com planos verticais: $x = k$



$$\begin{cases} -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{k^2}{a^2} \\ x = k \end{cases} : \text{hipérboles } \forall k \neq 0.$$

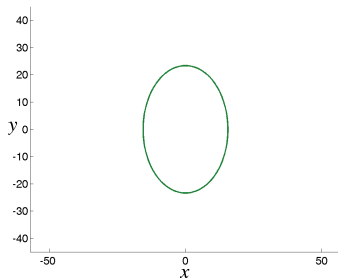
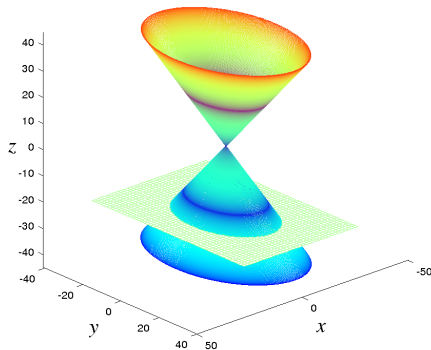


## Interseção com planos verticais: $y = k$



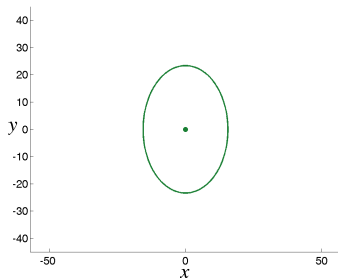
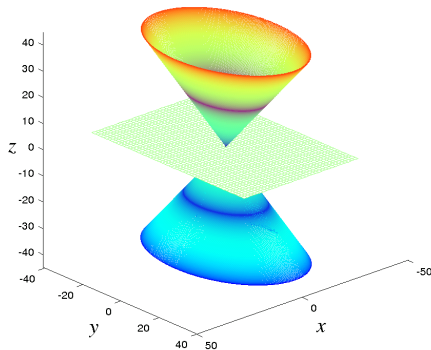
- Análogo ao caso anterior ( $x \leftrightarrow y$ ,  $a \leftrightarrow b$ ).

## Interseção com planos horizontais: $z = k$



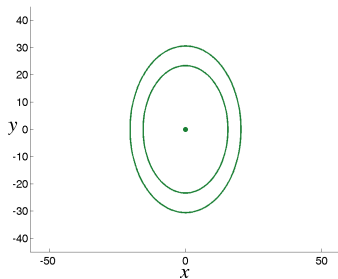
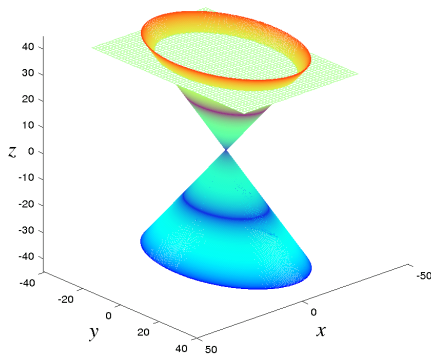
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} \\ z = k \end{cases} : \text{elipses } \forall k \neq 0.$$

## Interseção com planos horizontais: $z = k$



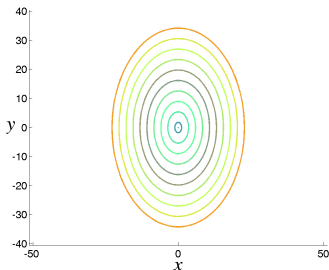
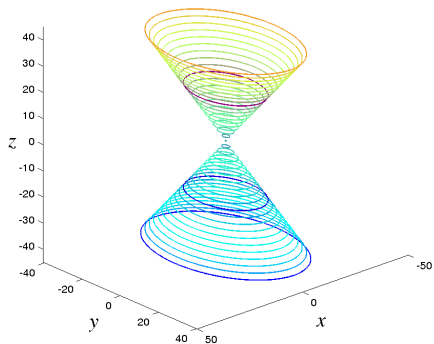
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \\ z = 0 \end{cases} : \text{um ponto } (k = 0).$$

## Interseção com planos horizontais: $z = k$



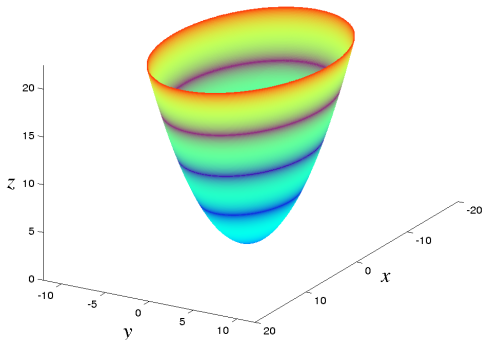
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{k^2}{c^2} \\ z = k \end{cases} : \text{elipses } \forall k \neq 0.$$

## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



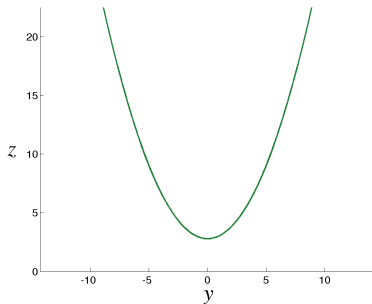
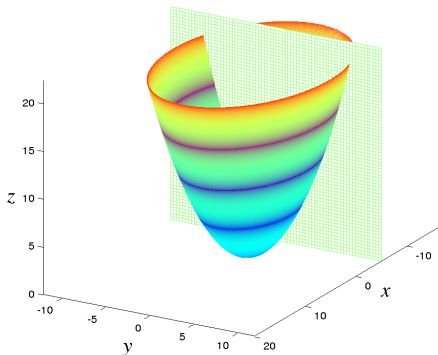
- As curvas de nível são elipses para todo  $k \neq 0$  e um ponto para  $k = 0$ .

## Parabolóide elíptico: gráfico e equação



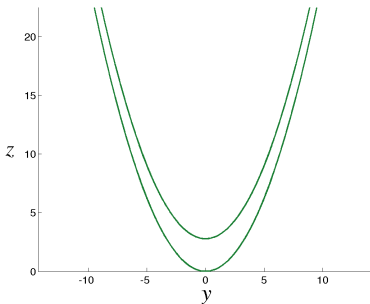
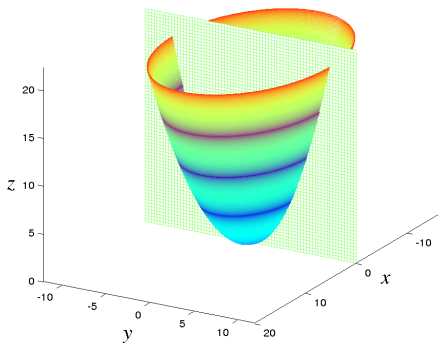
$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

## Interseção com planos verticais: $x = k$



$$\begin{cases} z = \frac{k^2}{a^2} + \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas } \forall k \in \mathbb{R}.$$

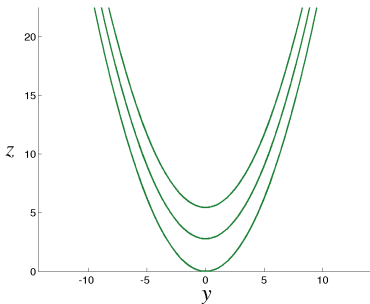
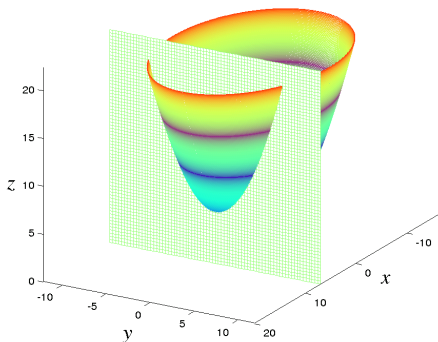
## Interseção com planos verticais: $x = k$



$$\begin{cases} z = \frac{k^2}{a^2} + \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas } \forall k \in \mathbb{R}.$$

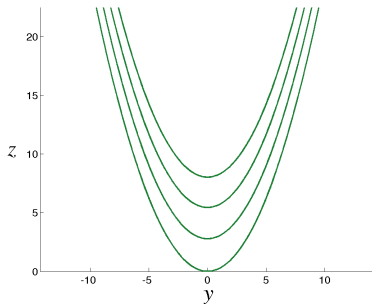
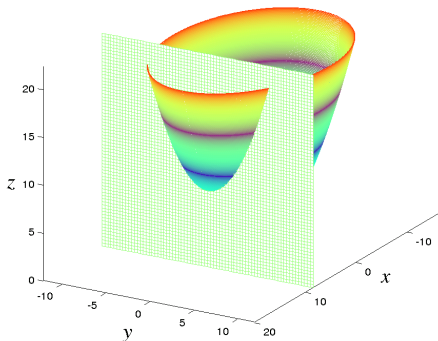


## Interseção com planos verticais: $x = k$



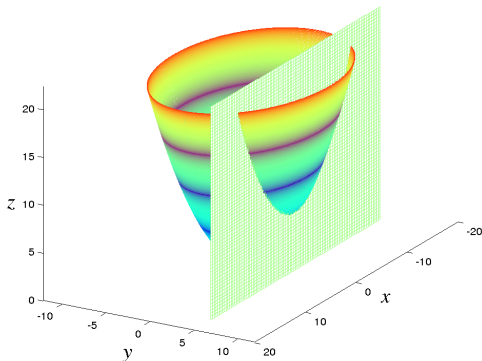
$$\begin{cases} z = \frac{k^2}{a^2} + \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas } \forall k \in \mathbb{R}.$$

## Interseção com planos verticais: $x = k$



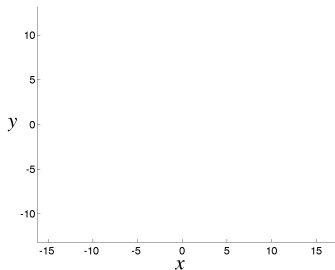
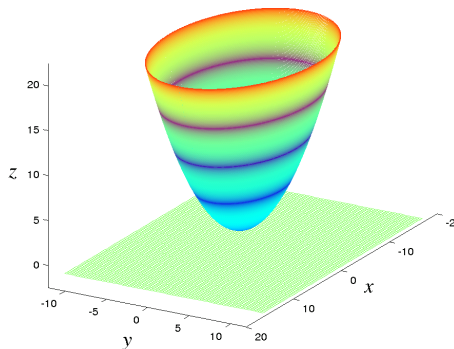
$$\begin{cases} z = \frac{k^2}{a^2} + \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas } \forall k \in \mathbb{R}.$$

## Interseção com planos verticais: $y = k$



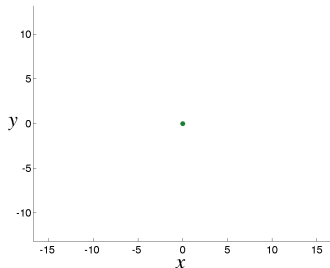
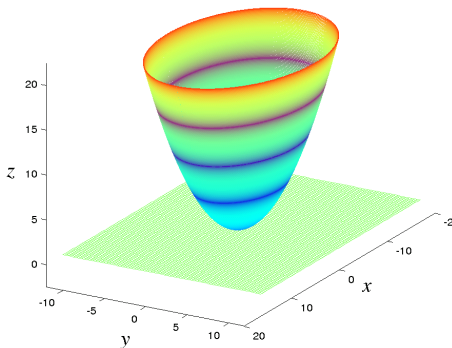
- Análogo ao caso anterior ( $x \leftrightarrow y$ ,  $a \leftrightarrow b$ ).

## Interseção com planos horizontais: $z = k$



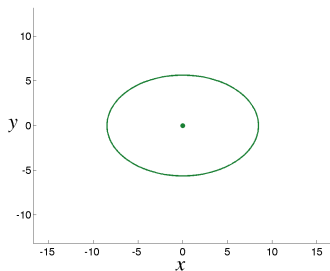
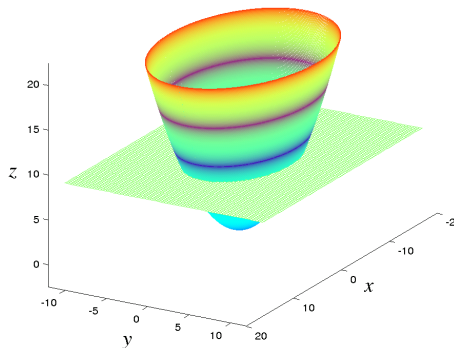
- O conjunto vazio quando  $k < 0$ .

## Interseção com planos horizontais: $z = k$



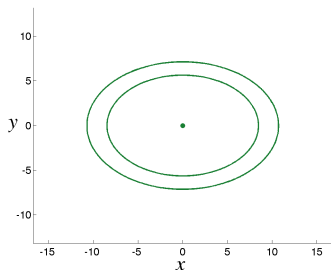
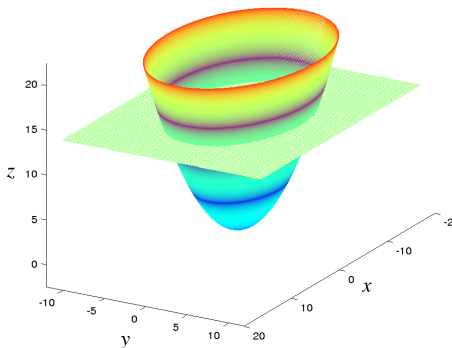
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \\ z = 0 \end{cases} : \text{um ponto } (k = 0).$$

## Interseção com planos horizontais: $z = k$



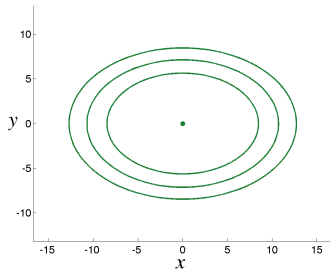
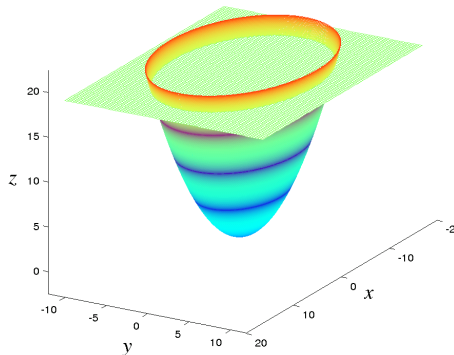
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = k \\ z = k \end{cases} : \text{elipses } (k > 0).$$

## Interseção com planos horizontais: $z = k$



$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = k \\ z = k \end{cases} : \text{elipses } (k > 0).$$

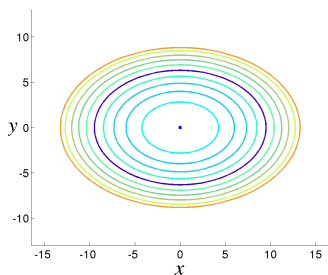
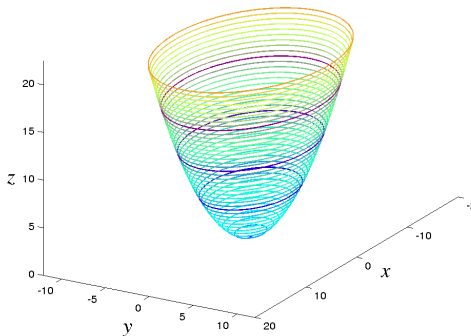
## Interseção com planos horizontais: $z = k$



$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = k \\ z = k \end{cases} : \text{elipses } (k > 0).$$

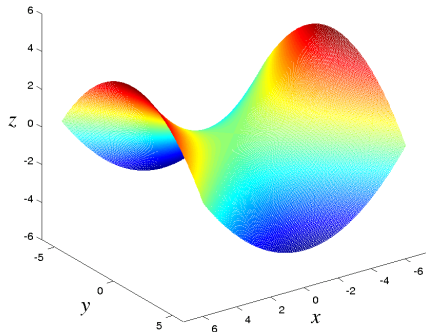


## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



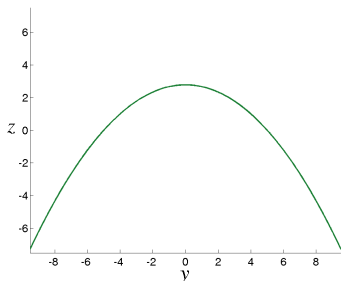
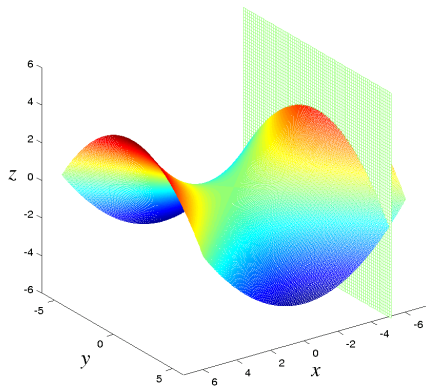
- As curvas de nível são elipses para todo  $k > 0$ , um ponto para  $k = 0$  e o conjunto vazio para  $k < 0$ .

## Parabolóide hiperbólico (sela): gráfico e equação



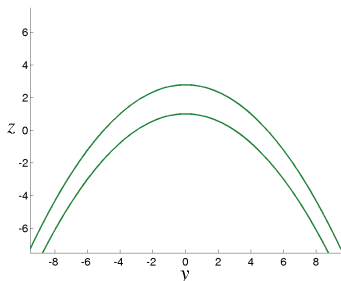
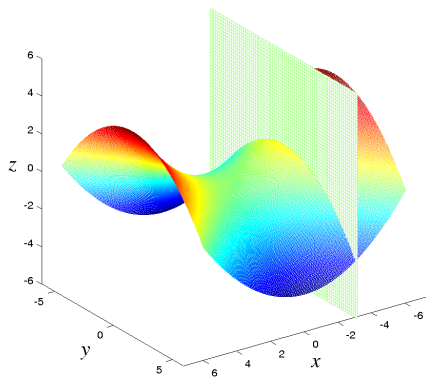
$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

## Interseção com planos verticais: $x = k$



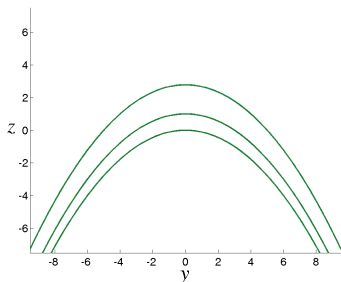
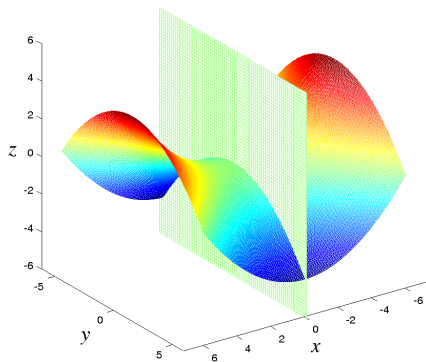
$$\begin{cases} z = \frac{k^2}{a^2} - \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas côncavas } \forall k \in \mathbb{R}.$$

## Interseção com planos verticais: $x = k$



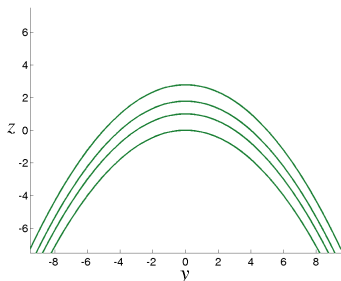
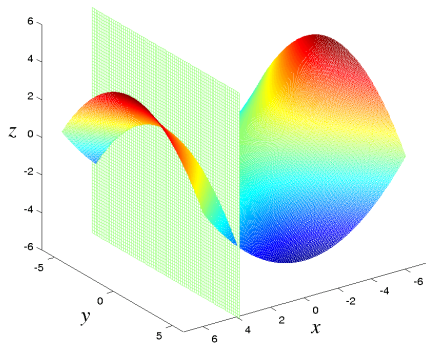
$$\begin{cases} z = \frac{k^2}{a^2} - \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas côncavas } \forall k \in \mathbb{R}.$$

## Interseção com planos verticais: $x = k$



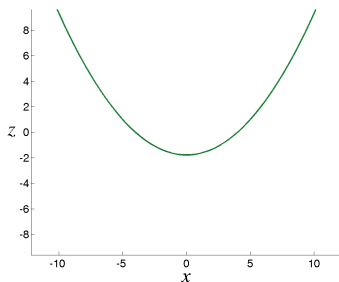
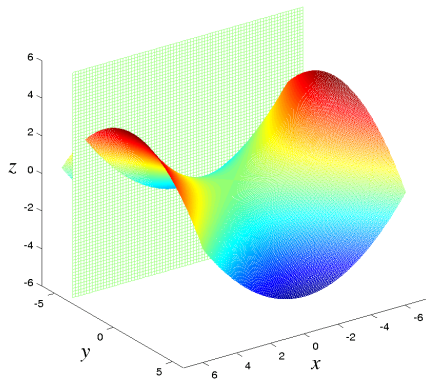
$$\begin{cases} z = \frac{k^2}{a^2} - \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas côncavas } \forall k \in \mathbb{R}.$$

## Interseção com planos verticais: $x = k$



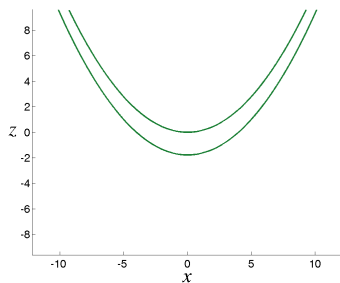
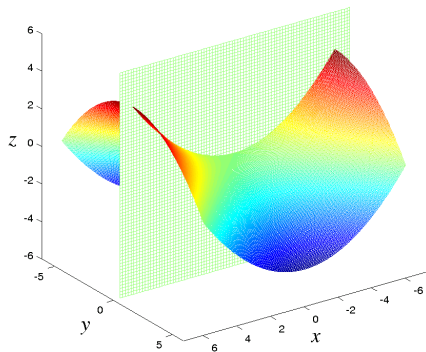
$$\begin{cases} z = \frac{k^2}{a^2} - \frac{y^2}{b^2} \\ x = k \end{cases} : \text{parábolas côncavas } \forall k \in \mathbb{R}.$$

## Interseção com planos verticais: $y = k$



$$\begin{cases} z = \frac{x^2}{a^2} - \frac{k^2}{b^2} \\ y = k \end{cases} : \text{parábolas convexas } \forall k \in \mathbb{R}.$$

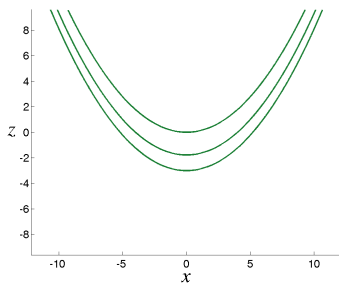
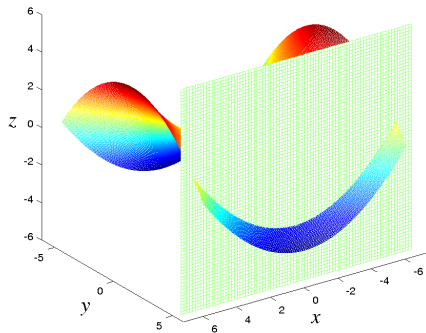
## Interseção com planos verticais: $y = k$



$$\begin{cases} z = \frac{x^2}{a^2} - \frac{k^2}{b^2} \\ y = k \end{cases} : \text{parábolas convexas } \forall k \in \mathbb{R}.$$

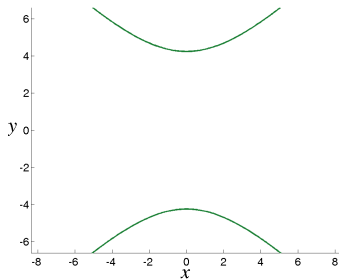
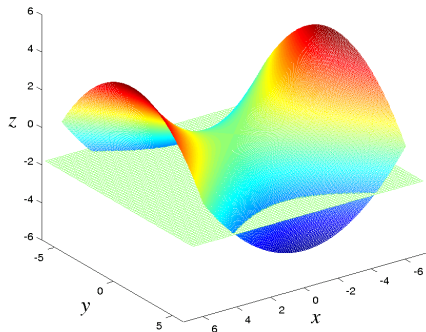


## Interseção com planos verticais: $y = k$



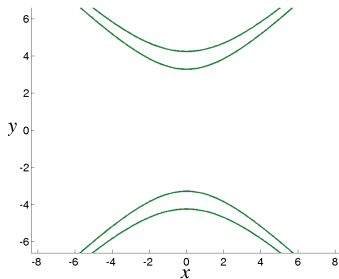
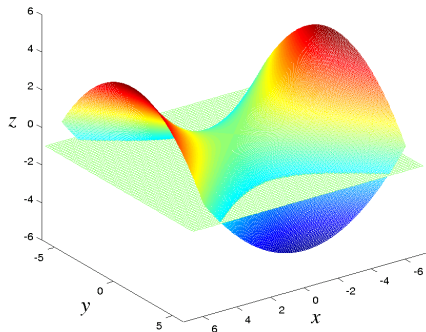
$$\begin{cases} z = \frac{x^2}{a^2} - \frac{k^2}{b^2} \\ y = k \end{cases} : \text{parábolas convexas } \forall k \in \mathbb{R}.$$

## Interseção com planos horizontais: $z = k$



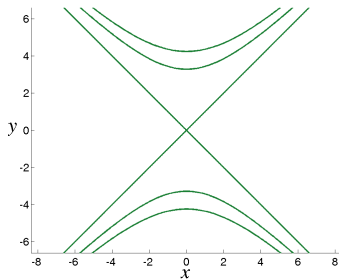
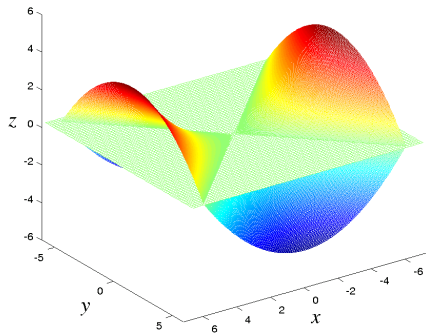
$$\begin{cases} -\frac{x^2}{a^2} + \frac{y^2}{b^2} = -k \\ z = k \end{cases} : \text{hipérboles na vertical } \forall k < 0.$$

## Interseção com planos horizontais: $z = k$



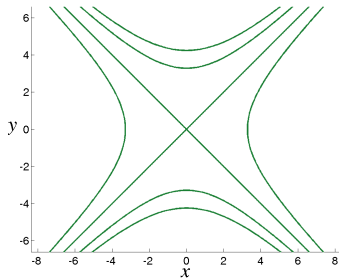
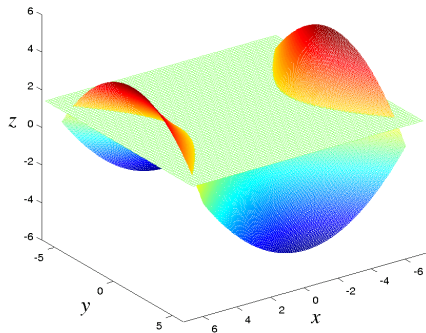
$$\begin{cases} -\frac{x^2}{a^2} + \frac{y^2}{b^2} = -k \\ z = k \end{cases} : \text{hipérboles na vertical } \forall k < 0.$$

## Interseção com planos horizontais: $z = k$



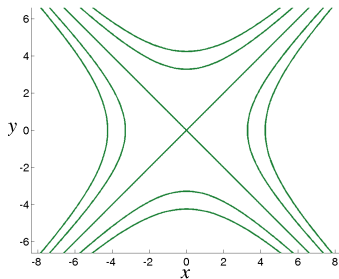
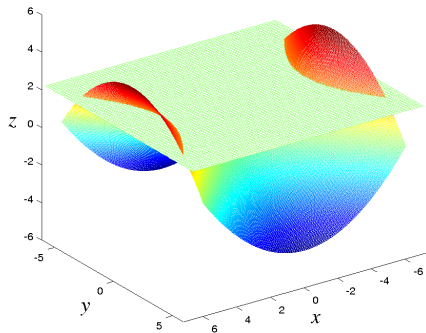
$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \\ z = 0 \end{cases} : \text{duas retas concorrentes } (k = 0).$$

## Interseção com planos horizontais: $z = k$



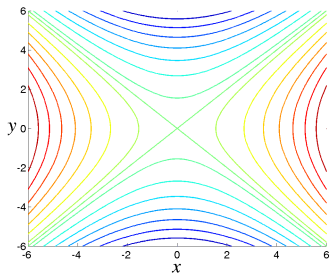
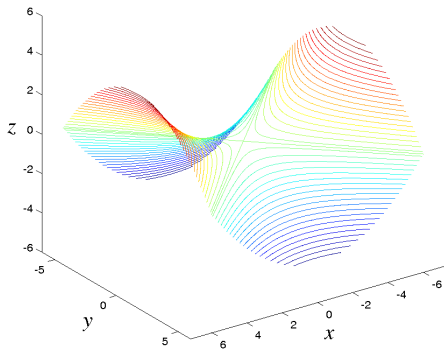
$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = k \\ z = k \end{cases} : \text{hipérboles na horizontal } \forall k > 0.$$

## Interseção com planos horizontais: $z = k$



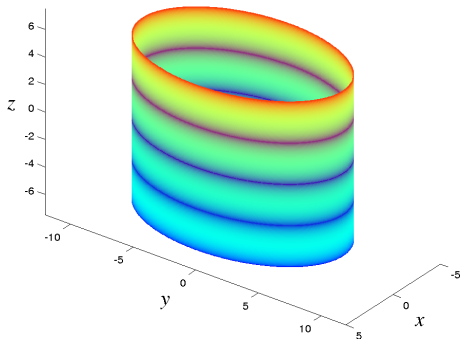
$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = k \\ z = k \end{cases} : \text{hipérboles na horizontal } \forall k > 0.$$

## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



- As curvas de nível são hipérbolas para todo  $k \neq 0$  e duas retas concorrentes para  $k = 0$ .

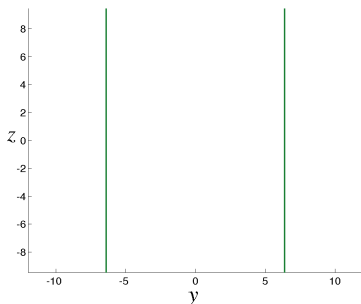
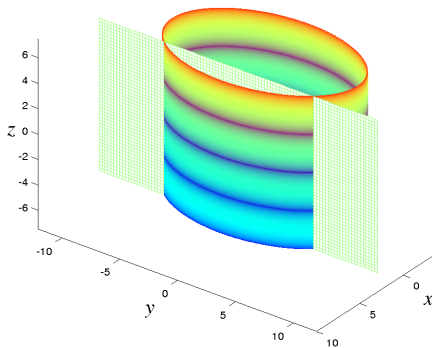
## Cilindro elíptico: gráfico e equação



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

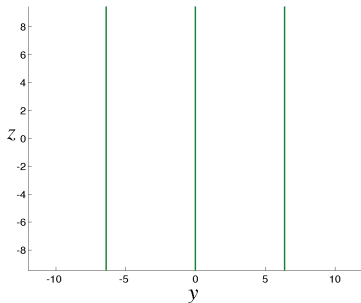
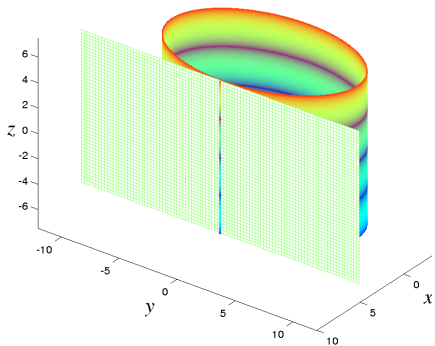


## Interseção com planos verticais: $x = k$



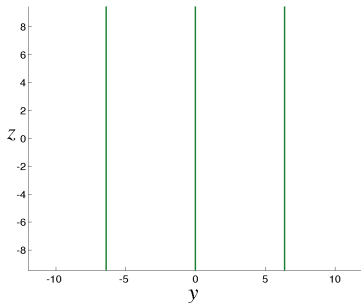
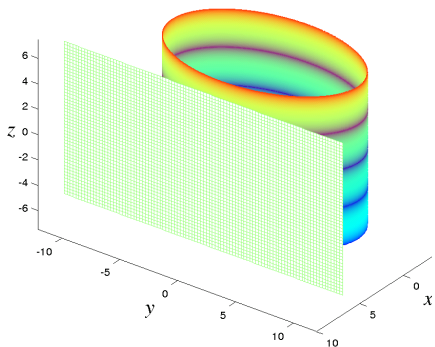
$$\begin{cases} \frac{y^2}{b^2} = 1 - \frac{k^2}{a^2} \\ x = k \end{cases} : \text{duas retas paralelas } (-a < k < a).$$

## Interseção com planos verticais: $x = k$



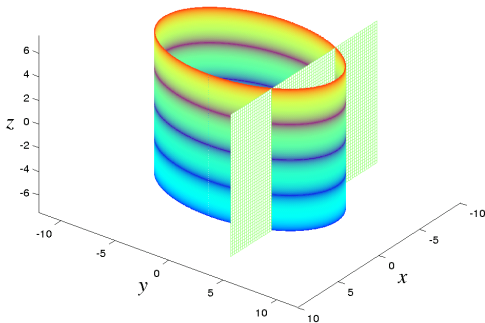
$$\begin{cases} y = 0 \\ x = k \end{cases} : \text{ uma única reta } (k = -a \text{ ou } k = a).$$

## Interseção com planos verticais: $x = k$



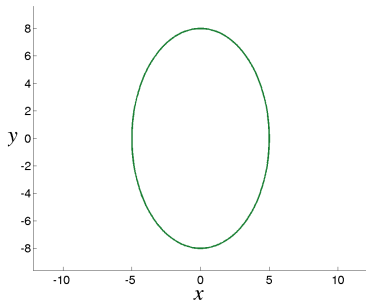
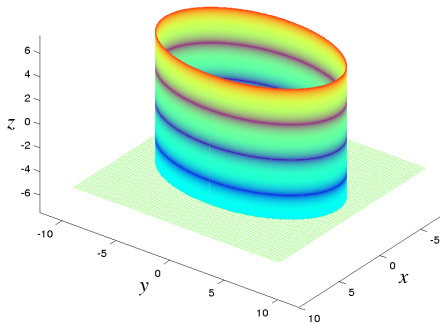
- O conjunto vazio quando  $k < -a$  ou  $k > a$ .

## Interseção com planos verticais: $y = k$



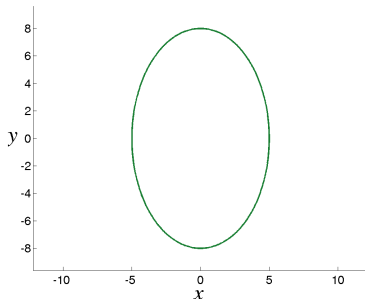
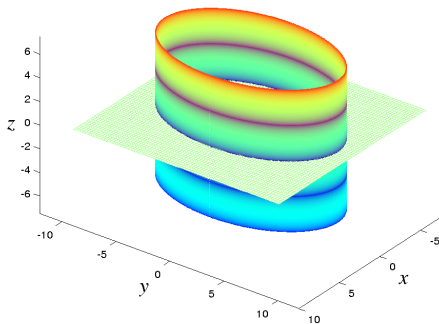
- Análogo ao caso anterior ( $x \leftrightarrow y$ ,  $a \leftrightarrow b$ ).

## Interseção com planos horizontais: $z = k$



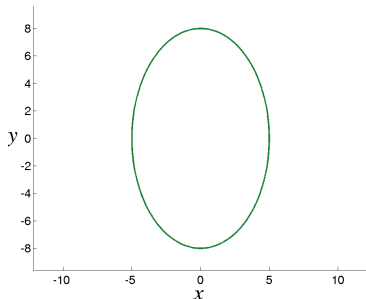
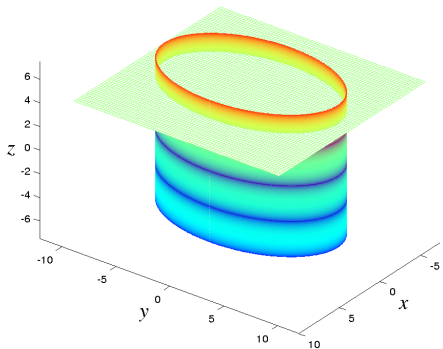
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ z = k \end{cases} : \text{ elipses iguais } \forall k \in \mathbb{R}.$$

## Interseção com planos horizontais: $z = k$



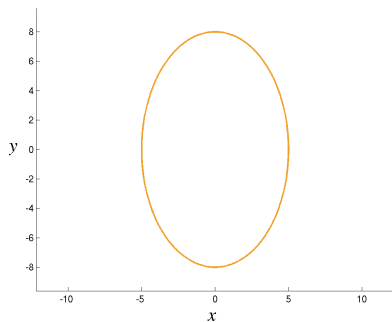
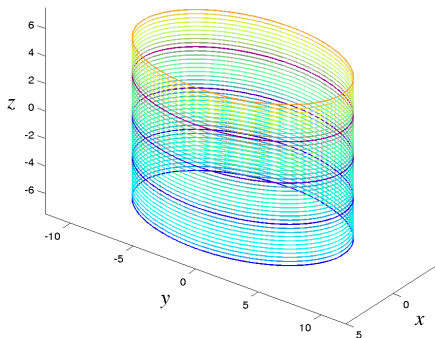
$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ z = k \end{cases} : \text{ elipses iguais } \forall k \in \mathbb{R}.$$

## Interseção com planos horizontais: $z = k$



$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ z = k \end{cases} : \text{ elipses iguais } \forall k \in \mathbb{R}.$$

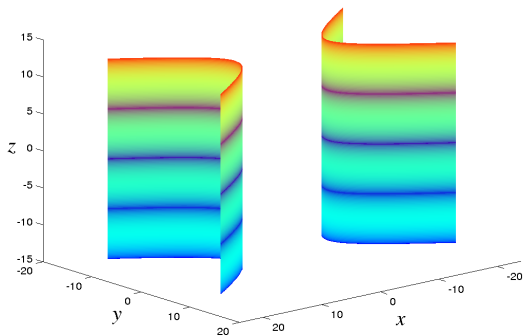
## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



- As curvas de nível são elipses iguais para todo  $k \in \mathbb{R}$ .

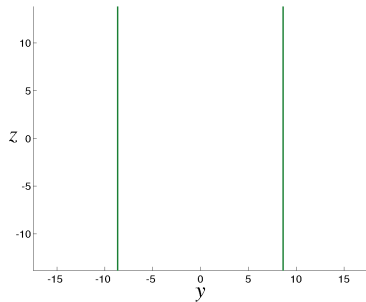
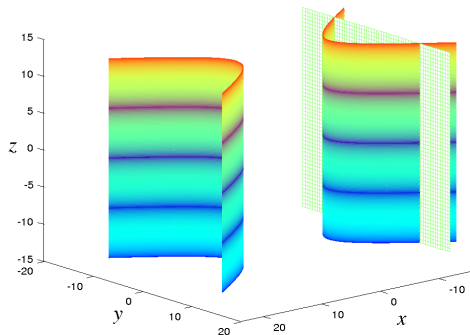


## Cilindro hiperbólico: gráfico e equação



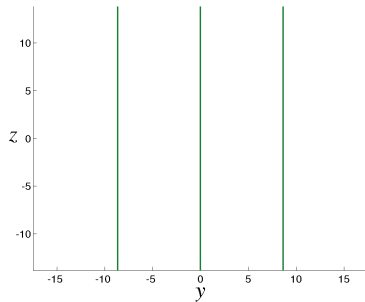
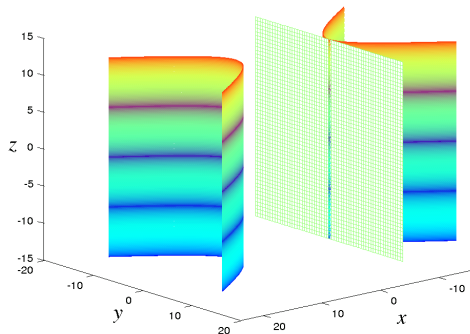
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

## Interseção com planos verticais: $x = k$



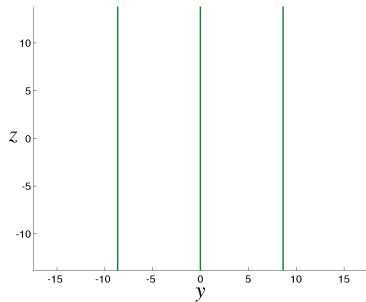
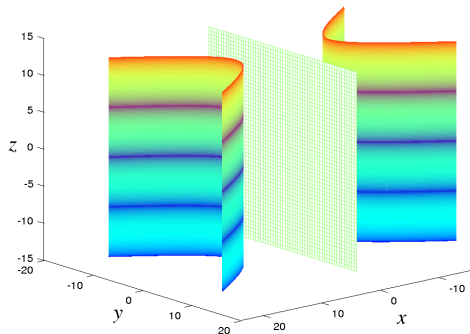
$$\begin{cases} \frac{y^2}{b^2} = \frac{k^2}{a^2} - 1 \\ x = k \end{cases} : \text{duas retas paralelas } (k < -a \text{ ou } k > a).$$

## Interseção com planos verticais: $x = k$



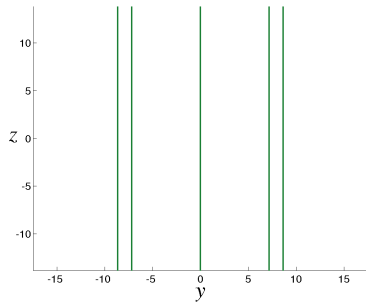
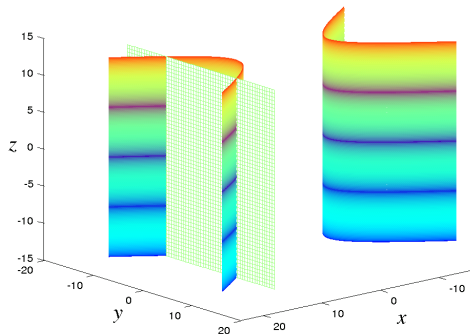
$$\begin{cases} y = 0 \\ x = k \end{cases} : \text{ uma única reta } (k = -a \text{ ou } k = a).$$

## Interseção com planos verticais: $x = k$



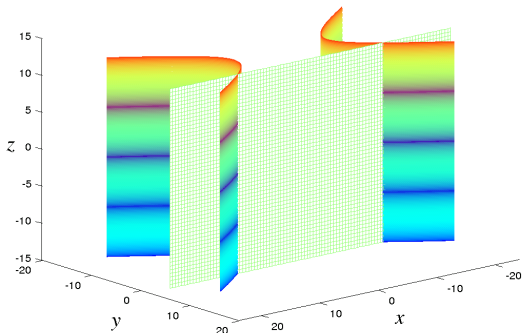
- O conjunto vazio quando  $-a < k < a$ .

## Interseção com planos verticais: $x = k$



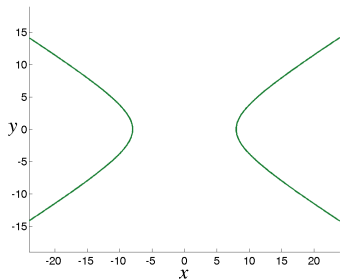
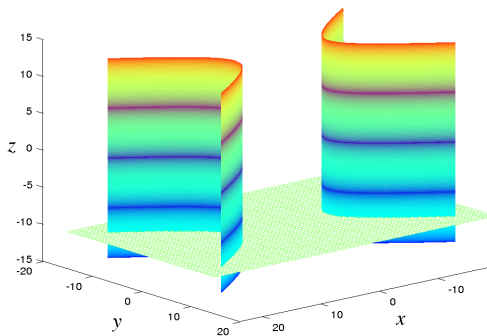
$$\begin{cases} \frac{y^2}{b^2} = \frac{k^2}{a^2} - 1 \\ x = k \end{cases} : \text{duas retas paralelas } (k < -a \text{ ou } k > a).$$

## Interseção com planos verticais: $y = k$



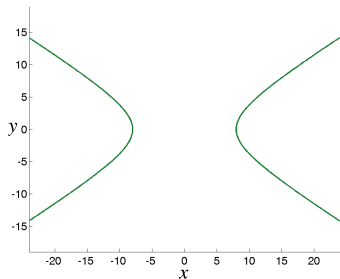
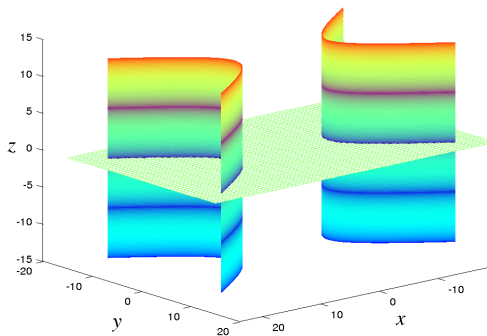
$$\begin{cases} \frac{x^2}{a^2} = 1 + \frac{k^2}{b^2} \\ y = k \end{cases} : \text{duas retas paralelas } \forall k \in \mathbb{R}.$$

## Interseção com planos horizontais: $z = k$



$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ z = k \end{cases} : \text{hipérboles iguais } \forall k \in \mathbb{R}.$$

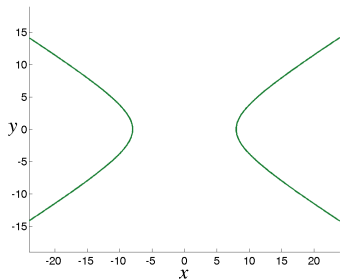
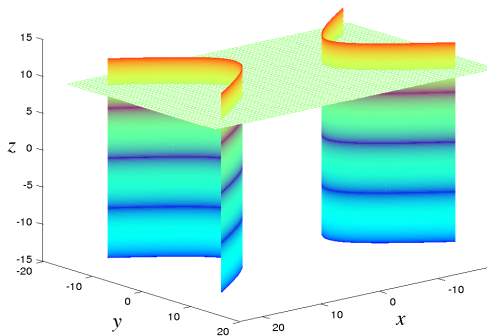
## Interseção com planos horizontais: $z = k$



$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ z = k \end{cases} : \text{hipérboles iguais } \forall k \in \mathbb{R}.$$

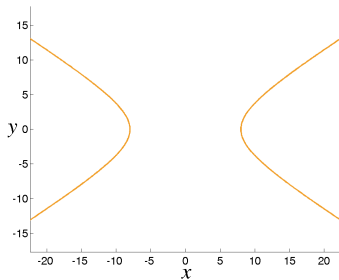
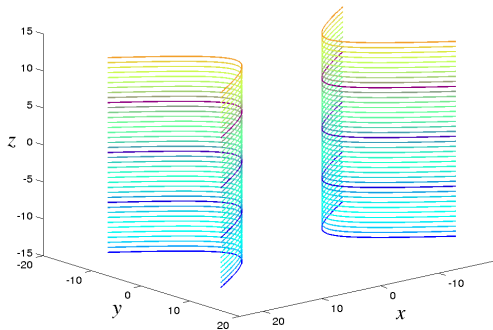


## Interseção com planos horizontais: $z = k$



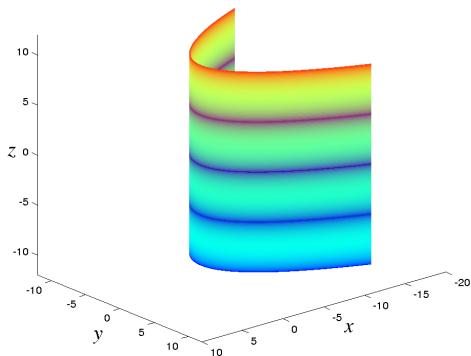
$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ z = k \end{cases} : \text{hipérboles iguais } \forall k \in \mathbb{R}.$$

## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



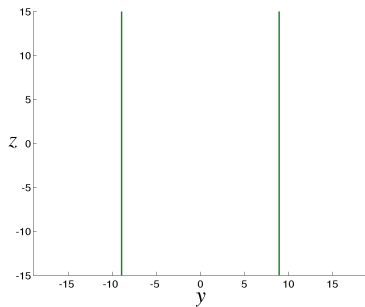
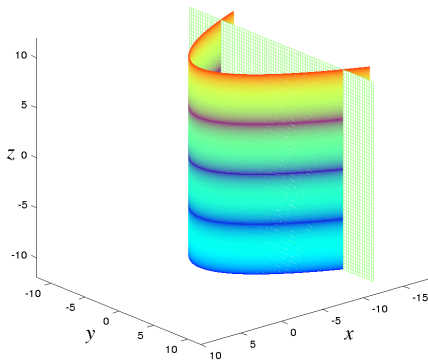
- As curvas de nível são hipérbolas iguais para todo  $k \in \mathbb{R}$ .

## Cilindro parabólico: gráfico e equação



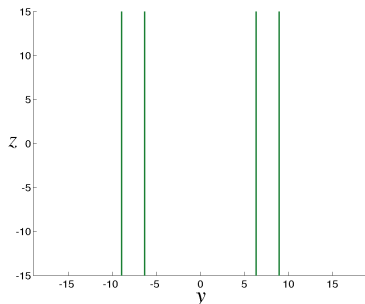
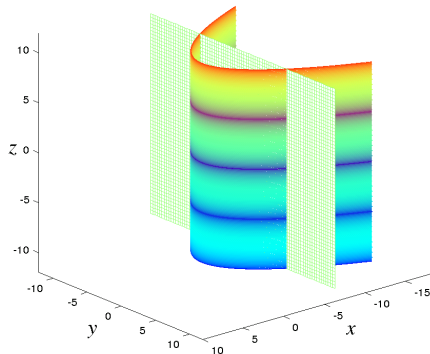
$$y^2 = -2px$$

## Interseção com planos verticais: $x = k$



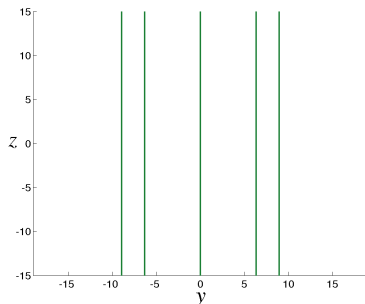
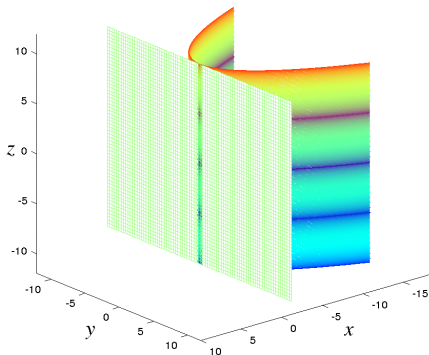
$$\begin{cases} y^2 = -2pk \\ x = k \end{cases} : \text{duas retas paralelas } (k < 0).$$

## Interseção com planos verticais: $x = k$



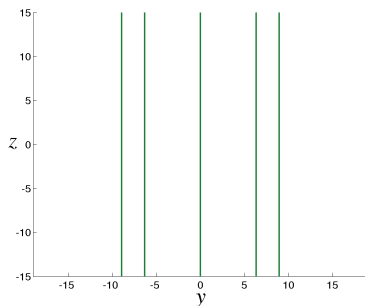
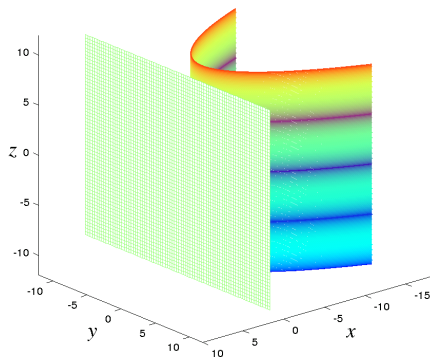
$$\begin{cases} y^2 = -2pk \\ x = k \end{cases} : \text{duas retas paralelas } (k < 0).$$

## Interseção com planos verticais: $x = k$



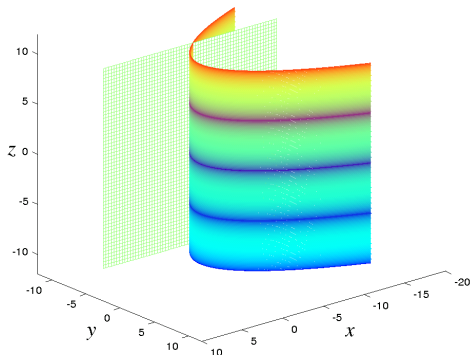
$$\begin{cases} y = 0 \\ x = k \end{cases} : \text{ uma única reta } (k = 0).$$

## Interseção com planos verticais: $x = k$



- O conjunto vazio quando  $k > 0$ .

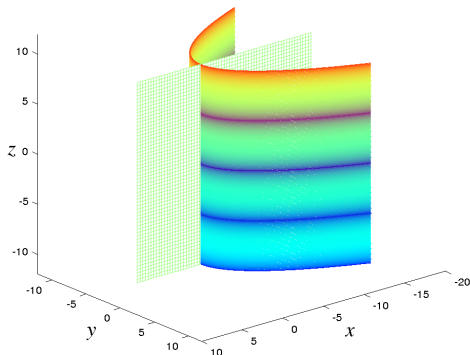
## Interseção com planos verticais: $y = k$



$$\begin{cases} -2px = k^2 \\ y = k \end{cases} : \text{uma única reta } \forall k \in \mathbb{R}.$$

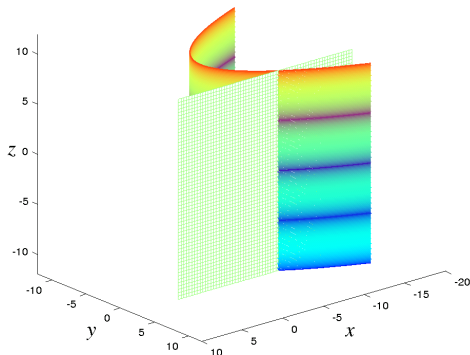


## Interseção com planos verticais: $y = k$



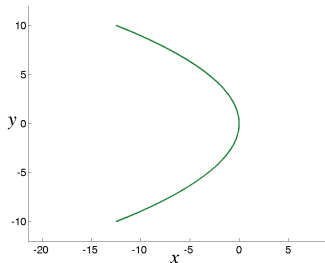
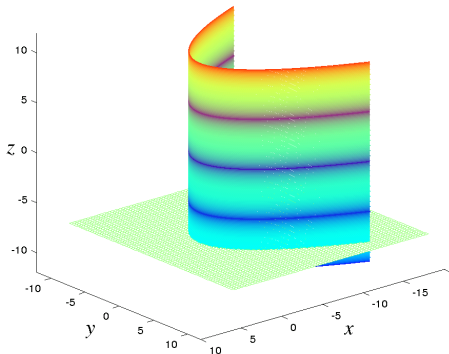
$$\begin{cases} -2px = k^2 \\ y = k \end{cases} : \text{ uma única reta } \forall k \in \mathbb{R}.$$

## Interseção com planos verticais: $y = k$



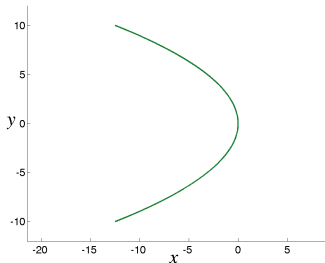
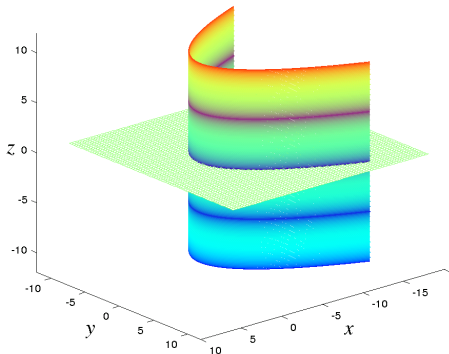
$$\begin{cases} -2px = k^2 \\ y = k \end{cases} : \text{uma única reta } \forall k \in \mathbb{R}.$$

## Interseção com planos horizontais: $z = k$



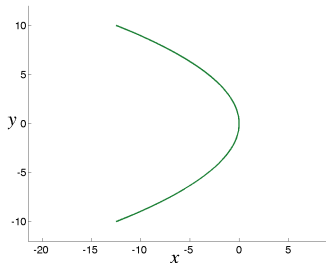
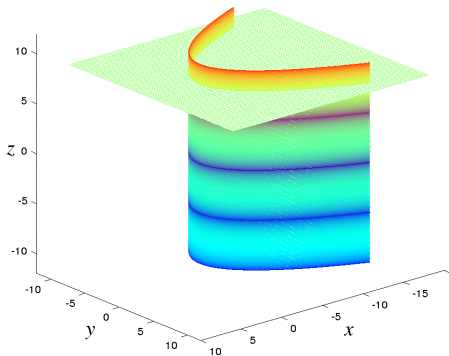
$$\begin{cases} y^2 = -2px \\ z = k \end{cases} : \text{parábolas iguais } \forall k \in \mathbb{R}.$$

## Interseção com planos horizontais: $z = k$



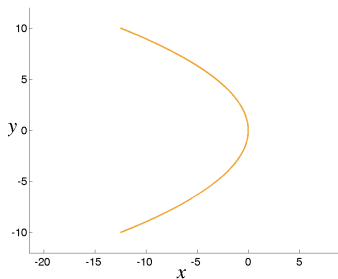
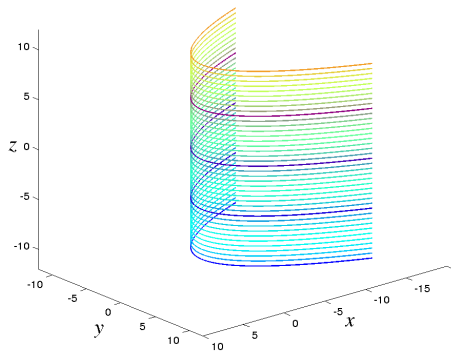
$$\begin{cases} y^2 = -2px \\ z = k \end{cases} : \text{parábolas iguais } \forall k \in \mathbb{R}.$$

## Interseção com planos horizontais: $z = k$



$$\begin{cases} y^2 = -2px \\ z = k \end{cases} : \text{parábolas iguais } \forall k \in \mathbb{R}.$$

## Curvas de nível em $\mathbb{R}^3$ e em $\mathbb{R}^2$



- As curvas de nível são parábolas iguais para todo  $k \in \mathbb{R}$ .