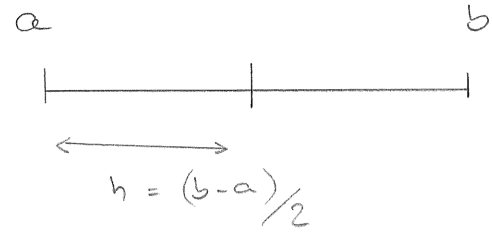
Regra de Simpson - Equação Geral

$$\int_a^b f(x)\,dx = \frac{1}{3}\left(\frac{b-a}{n}\right)\left[f(a) + 4\sum_{K=1}^{(n/2)} f(x_{2K-1}) + 2\sum_{K=1}^{(n/2-1)} f(x_{2K}) + f(b)\right]$$

$n \Rightarrow$ número de subintervalos em que o intervalo $[a, b]$ é subdividido ; n deve ser um nº par

Aplicação para $n = 2$



$$x_1 = a + h = a + \left(\frac{b-a}{2}\right) = \frac{b+a}{2}$$

x_0	x_1	x_2
a	$a+h$	b

$$\Rightarrow \quad 4\sum_{K=1}^{(n/2)} f(x_{2K-1}) = 4\sum_{K=1}^{1} f(x_{2K-1})$$ há um único termo

p/ $K = 1$ $\quad 2K-1 = 1$

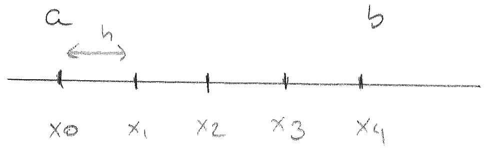
$$= 4\,f(x_1)$$

$$\Rightarrow \quad 2\sum_{K=1}^{(n/2-1)} f(x_{2K}) = 2\sum_{K=1}^{0} f(x_{2K})$$ não há qualquer termo

$$\int_a^b f(x)\,dx = \frac{1}{3}\left(\frac{b-a}{2}\right)\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

$$\Rightarrow \frac{1}{3}\left(\frac{b-a}{2}\right)\left[f(x_0) + 4(f(x_1)) + 2(\emptyset) + f(x_2)\right]$$

Aplicação para $n=4$



$$h = \frac{b-a}{4}$$

$$x_0 = a \qquad x_1 = x_0 + h = a + \frac{b-a}{4} = \frac{3a+b}{4}$$

$$x_2 = x_0 + 2h = a + \frac{2(b-a)}{4} = \frac{b+a}{2}$$

$$x_3 = x_0 + 3h = a + \frac{3(b-a)}{4} = \frac{a+3b}{4}$$

$$\Rightarrow \quad 4 \sum_{k=1}^{n/2} f(x_{2k-1}) = 4 \sum_{k=1}^{2} f(x_{2k-1}) \qquad \text{há } 2 \text{ termos}$$

$$k=1 \qquad 2k-1 = 1 \qquad x_{2k-1} = x_1$$

$$k=2 \qquad 2k-1 = 3 \qquad x_{2k-1} = x_3$$

$$\Rightarrow \quad 4\left(f(x_1) + f(x_3) \right)$$

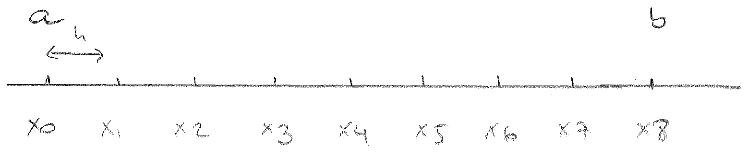
$$\Rightarrow \quad 2 \sum_{k=1}^{n/2 - 1} f(x_{2k}) = 2 \sum_{k=1}^{1} f(x_{2k}) \qquad \text{há } 1 \text{ termo}$$

$$k=1 \qquad 2k = 2 \qquad x_{2k} = x_2$$

$$\Rightarrow \quad 2\left(f(x_2) \right)$$

$$\Rightarrow \quad \frac{1}{3}\left(\frac{b-a}{4}\right)\left[f(x_0) + 4\left(f(x_1) + f(x_3) \right) + 2\left(f(x_2) \right) + f(x_4) \right]$$

Aplicação para $n = 8$



$$h = \frac{b-a}{8}$$

$x_0 = a$

$x_1 = x_0 + h = a + \frac{b-a}{8} = (7a+b)/8$ $\qquad$ $x_2 = x_0 + 2h = a + \frac{2(b-a)}{8} = (6a+2b)/8$

$x_3 = x_0 + 3h = a + 3(b-a)/8 = (5a+3b)/8$ $\qquad$ $x_4 = x_0 + 4h = a + 4(b-a)/8 = (4a+4b)/8$

$x_5 = x_0 + 5h = a + 5(b-a)/8 = (3a+5b)/8$ $\qquad$ $x_6 = x_0 + 6h = a + 6(b-a)/8 = (2a+6b)/8$

$x_7 = x_0 + 7h = a + 7(b-a)/8 = (a+7b)/8$ $\qquad$ $x_8 = b$

$$\Rightarrow 4 \sum_{K=1}^{(n/2)} f(x_{2K-1}) = 4 \sum_{K=1}^{4} f(x_{2K-1}) \qquad \text{há 4 termos}$$

$K = 1$	$2K-1 = 1$	$x_{2K-1} = x_1$
$K = 2$	$2K-1 = 3$	$x_{2K-1} = x_3$
$K = 3$	$2K-1 = 5$	$x_{2K-1} = x_5$
$K = 4$	$2K-1 = 7$	$x_{2K-1} = x_7$

$$\Rightarrow 4\left(f(x_1) + f(x_3) + f(x_5) + f(x_7)\right)$$

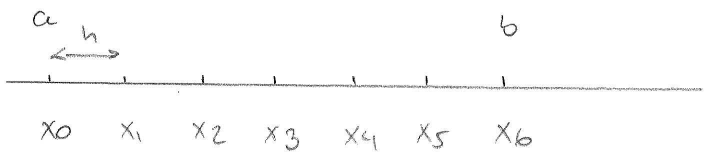
$$\Rightarrow 2 \sum_{K=1}^{(n/2-1)} = 2 \sum_{K=1}^{3} f(x_{2K}) \qquad \text{há 3 termos}$$

$K = 1$	$2K = 2$	$x_{2K} = x_2$
$K = 2$	$2K = 4$	$x_{2K} = x_4$
$K = 3$	$2K = 6$	$x_{2K} = x_6$

$$\Rightarrow 2\left(f(x_2) + f(x_4) + f(x_6)\right)$$

$$\Rightarrow \frac{1}{3}\left(\frac{b-a}{8}\right)\Big[f(x_0) + 4\left(f(x_1) + f(x_3) + f(x_5) + f(x_7)\right) + $$
$$+ 2\left(f(x_2) + f(x_4) + f(x_6)\right) + f(x_8) \Big]$$

Aplicação para $n = 6$



$$h = \frac{b-a}{6}$$

$$x_0 = a \qquad x_1 = x_0 + h = a + \frac{b-a}{6} = \frac{5a+b}{6}$$

$$x_2 = x_0 + 2h = a + \frac{2(b-a)}{6} = \frac{2a+3b}{3}$$

$$x_3 = x_0 + 3h = a + \frac{3(b-a)}{6} = \frac{3a+3b}{6} = \frac{a+b}{2}$$

$$x_4 = x_0 + 4h = a + \frac{4(b-a)}{6} = \frac{2a+4b}{6} = \frac{a+2b}{3}$$

$$x_5 = x_0 + 5h = a + \frac{5(b-a)}{6} = \frac{a+5b}{6}$$

$$\Rightarrow 4\sum_{K=1}^{(n/2)} f(x_{2K-1}) = 4\sum_{K=1}^{3} f(x_{2K-1}) \qquad \text{há 3 termos}$$

$K=1 \quad 2K-1 = 1 \quad x_{2K-1} = x_1$

$K=2 \quad 2K-1 = 3 \quad x_{2K-1} = x_3$

$K=3 \quad 2K-1 = 5 \quad x_{2K-1} = x_5$

$$\Rightarrow 4\left(f(x_1) + f(x_3) + f(x_5)\right)$$

$$\Rightarrow 2\sum_{K=1}^{(n/2-1)} f(x_{2K}) = 2\sum_{K=1}^{2} f(x_{2K}) \qquad \text{há 2 termos}$$

$K=1 \quad 2K = 2 \quad x_{2K} = x_2$

$K=2 \quad 2K = 4 \quad x_{2K} = x_4$

$$\Rightarrow 2\left(f(x_2) + f(x_4)\right)$$

$$\Rightarrow \frac{1}{3}\left(\frac{b-a}{6}\right)\left[f(x_0) + 4\left(f(x_1) + f(x_3) + f(x_5)\right) + 2\left(f(x_2) + f(x_4)\right) + f(x_6)\right]$$