

Gramática Categorical

Segunda lista de exercícios

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1 “toda menina chutou uma bola”

1.1 Quantificador universal com escopo sobre existencial

toda	menina	chutou	uma	bola
$\frac{(N \uparrow S)/N_c :}{\lambda P_0. \lambda P_1. \forall x. ((P_0 x) \rightarrow (P_1 x))} L$	$\frac{N_c :}{M'_1} L$	$\frac{chutou}{(N \setminus S)/N : C'_5} L$	$\frac{(N \uparrow S)/N_c :}{\lambda Q_0. \lambda Q_1. \exists y. ((Q_0 y) \wedge (Q_1 y))} L$	$\frac{N_c :}{B'} L$
$\frac{N \uparrow S : (\lambda P_0. \lambda P_1. \forall x. ((P_0 x) \rightarrow (P_1 x)) M'_1)}{=_{red.\beta} \lambda P_1. \forall x. ((M'_1 x) \rightarrow (P_1 x))} E/$		$\frac{N \uparrow S : (\lambda Q_0. \lambda Q_1. \exists y. ((Q_0 y) \wedge (Q_1 y)) B')}{=_{red.\beta} \lambda Q_1. \exists y. ((B' y) \wedge (Q_1 y))} E/$		
$\frac{}{N : x_0} 0$		$\frac{}{N : x_1} 1$		
		$\frac{}{N \setminus S : (C'_5 x_1)} E/$		
		$\frac{}{S : ((C'_5 x_1) x_0)} E \setminus$		
		$\frac{S : (\lambda Q_1. \exists y. ((B' y) \wedge (Q_1 y)) \lambda x_1. ((C'_5 x_1) x_0))}{=_{red.\beta} \exists y. ((B' y) \wedge (\lambda x_1. ((C'_5 x_1) x_0) y))}{=_{red.\beta} \exists y. ((B' y) \wedge ((C'_5 y) x_0))} E \uparrow^1$		
		$\frac{S : (\lambda P_1. \forall x. ((M'_1 x) \rightarrow (P_1 x)) \lambda x_0. \exists y. ((B' y) \wedge ((C'_5 y) x_0)))}{=_{red.\beta} \forall x. ((M'_1 x) \rightarrow (\lambda x_0. \exists y. ((B' y) \wedge ((C'_5 y) x_0) x)))}{=_{red.\beta} \forall x. ((M'_1 x) \rightarrow \exists y. ((B' y) \wedge ((C'_5 y) x)))} E \uparrow^0$		

1.2 Quantificador existencial com escopo sobre universal

toda	menina	chutou	uma	bola
$\frac{(N \uparrow S)/N_c :}{\lambda P_0. \lambda P_1. \forall x. ((P_0 x) \rightarrow (P_1 x))} L$	$\frac{N_c :}{M'_1} L$	$\frac{chutou}{(N \setminus S)/N : C'_5} L$	$\frac{(N \uparrow S)/N_c :}{\lambda Q_0. \lambda Q_1. \exists y. ((Q_0 y) \wedge (Q_1 y))} L$	$\frac{N_c :}{B'} L$
$\frac{N \uparrow S : (\lambda P_0. \lambda P_1. \forall x. ((P_0 x) \rightarrow (P_1 x)) M'_1)}{=_{red.\beta} \lambda P_1. \forall x. ((M'_1 x) \rightarrow (P_1 x))} E/$		$\frac{N \uparrow S : (\lambda Q_0. \lambda Q_1. \exists y. ((Q_0 y) \wedge (Q_1 y)) B')}{=_{red.\beta} \lambda Q_1. \exists y. ((B' y) \wedge (Q_1 y))} E/$		
$\frac{}{N : x_0} 0$		$\frac{}{N : x_1} 1$		
		$\frac{}{N \setminus S : (C'_5 x_1)} E/$		
		$\frac{}{S : ((C'_5 x_1) x_0)} E \setminus$		
		$\frac{S : (\lambda P_1. \forall x. ((M'_1 x) \rightarrow (P_1 x)) \lambda x_0. ((C'_5 x_1) x_0))}{=_{red.\beta} \forall x. ((M'_1 x) \rightarrow (\lambda x_0. ((C'_5 x_1) x_0) x))}{=_{red.\beta} \forall x. ((M'_1 x) \rightarrow ((C'_5 x_1) x))} E \uparrow^0$		
		$\frac{S : (\lambda Q_1. \exists y. ((B' y) \wedge (Q_1 y)) \lambda x_1. \forall x. ((M'_1 x) \rightarrow ((C'_5 x_1) x)))}{=_{red.\beta} \exists y. ((B' y) \wedge (\lambda x_1. \forall x. ((M'_1 x) \rightarrow ((C'_5 x_1) x) y))}{=_{red.\beta} \exists y. ((B' y) \wedge \forall x. ((M'_1 x) \rightarrow ((C'_5 y) x)))} E \uparrow^1$		

1.3 Sugestão para reduzir trabalho duplicado

1.3.1 Derivação comum às duas análises

toda	menina	chutou	uma	bola
$\frac{(N \uparrow S)/N_c :}{\lambda P_0. \lambda P_1. \forall x. ((P_0 x) \rightarrow (P_1 x))} L$	$\frac{N_c :}{M'_1} L$	$\frac{chutou}{(N \setminus S)/N : C'_5} L$	$\frac{(N \uparrow S)/N_c :}{\lambda Q_0. \lambda Q_1. \exists y. ((Q_0 y) \wedge (Q_1 y))} L$	$\frac{N_c :}{B'} L$
$\frac{N \uparrow S : (\lambda P_0. \lambda P_1. \forall x. ((P_0 x) \rightarrow (P_1 x)) M'_1)}{=_{red.\beta} \lambda P_1. \forall x. ((M'_1 x) \rightarrow (P_1 x))} E/$		$\frac{N \uparrow S : (\lambda Q_0. \lambda Q_1. \exists y. ((Q_0 y) \wedge (Q_1 y)) B')}{=_{red.\beta} \lambda Q_1. \exists y. ((B' y) \wedge (Q_1 y))} E/$		
$\frac{}{N : x_0} 0$		$\frac{}{N : x_1} 1$		
		$\frac{}{N \setminus S : (C'_5 x_1)} E/$		
		$\frac{}{S : ((C'_5 x_1) x_0)} E \setminus$		

1.3.2 Quantificador universal com escopo sobre existencial

toda menina chutou uma bola	D_1
$S : ((C'_5 x_1) x_0)$	$E\uparrow^1$
$S : (\lambda Q_1. \exists y. ((B' y) \wedge (Q_1 y)) \lambda x_1. ((C'_5 x_1) x_0))$ $=_{red,\beta} \exists y. ((B' y) \wedge (\lambda x_1. ((C'_5 x_1) x_0) y))$ $=_{red,\beta} \exists y. ((B' y) \wedge ((C'_5 y) x_0))$	$E\uparrow^0$
$S : (\lambda P_1. \forall x. ((M'_1 x) \rightarrow (P_1 x)) \lambda x_0. \exists y. ((B' y) \wedge ((C'_5 y) x_0)))$ $=_{red,\beta} \forall x. ((M'_1 x) \rightarrow (\lambda x_0. \exists y. ((B' y) \wedge ((C'_5 y) x_0) x)))$ $=_{red,\beta} \forall x. ((M'_1 x) \rightarrow \exists y. ((B' y) \wedge ((C'_5 y) x)))$	$E\uparrow^0$

1.3.3 Quantificador existencial com escopo sobre universal

toda menina chutou uma bola	D_1
$S : ((C'_5 x_1) x_0)$	$E\uparrow^0$
$S : (\lambda P_1. \forall x. ((M'_1 x) \rightarrow (P_1 x)) \lambda x_0. ((C'_5 x_1) x_0))$ $=_{red,\beta} \forall x. ((M'_1 x) \rightarrow (\lambda x_0. ((C'_5 x_1) x_0) x))$ $=_{red,\beta} \forall x. ((M'_1 x) \rightarrow ((C'_5 x_1) x))$	$E\uparrow^1$
$S : (\lambda Q_1. \exists y. ((B' y) \wedge (Q_1 y)) \lambda x_1. \forall x. ((M'_1 x) \rightarrow ((C'_5 x_1) x)))$ $=_{red,\beta} \exists y. ((B' y) \wedge (\lambda x_1. \forall x. ((M'_1 x) \rightarrow ((C'_5 x_1) x) y)))$ $=_{red,\beta} \exists y. ((B' y) \wedge \forall x. ((M'_1 x) \rightarrow ((C'_5 y) x)))$	$E\uparrow^1$

2 “Pedro surpreendeu sua mulher e José fez o mesmo”

Aqui, como foi comentado em aula, a interpretação de M' precisaria ser como termo funcional, e não termo predicativo; conforme diferença em [1, p. 17]

2.1 Leitura de identidade estrita

Pedro	surpreendeu	sua	mulher	e	José	fez o mesmo
$[N : p']^0$	$(N \setminus S) / N :$	$(N \setminus N) / N_c :$	$N_c : M'$	$(S \setminus S) / S :$	$N : j'$	$(N \setminus S) \setminus (N \setminus S) :$
S'_1	S'_1	$\lambda P_0. P_0$	$\lambda P_0. P_0$	$\lambda P_1. \lambda P_2. (P_2 \wedge P_1)$	$N : j'$	$\lambda P_3. P_3$
		$N \setminus N : (\lambda P_0. P_0 M')$	M'			$N \setminus S : (\lambda P_3. P_3 (S' (M' p')))$
		$=_{red,\beta} M'$				$=_{red,\beta} (S' (M' p'))$
		$N : (M' p')$				$S : ((S' (M' p')) j')$
		$E/$	$E/$			$E/$
		$[N \setminus S : (S' (M' p'))]^1$		$S \setminus S : (\lambda P_1. \lambda P_2. (P_2 \wedge P_1) ((S' (M' p')) j'))$		$=_{red,\beta} \lambda P_2. (P_2 \wedge ((S' (M' p')) j'))$
		$E\backslash$				$E\backslash$
		$S : ((S' (M' p')) p')$				
		$E\backslash$				
		$S : (\lambda P_2. (P_2 \wedge ((S' (M' p')) j')) ((S' (M' p')) p'))$				
		$=_{red,\beta} ((S' (M' p')) p') \wedge ((S' (M' p')) j')$				

2.2 Leitura de identidade fajuta

$\frac{\text{Pedro}}{N : p} L \quad \frac{}{[N : x_0]^1} 0 \quad \frac{\text{surpreendeu}}{(N \setminus S)/N : S'_1} L \quad \frac{\text{sua}}{(N \setminus N)/N_c : \lambda P_0.P_0} L \quad \frac{\text{mulher}}{N_c : M'} L$	$\frac{\text{e}}{(S \setminus S)/S : \lambda P_1.\lambda P_2.(P_2 \wedge P_1)} L \quad \frac{\text{José}}{N : j} L \quad \frac{\text{fez o mesmo}}{(N \setminus S) (N \setminus S) : \lambda P_3.P_3} L$
$\frac{}{N N : (\lambda P_0.P_0 M')} E/ \quad \frac{}{=_{red.\beta} M'} E/ \quad \frac{}{N : (M' x_0)} E 1$	$\frac{}{N \setminus S : (\lambda P_3.P_3 \lambda x_0.((S' (M' x_0)) x_0))} E 2 \quad \frac{}{=_{red.\beta} \lambda x_0.((S' (M' x_0)) x_0)} E 2$
$\frac{}{N \setminus S : (S' (M' x_0))} E/ \quad \frac{}{S : ((S' (M' x_0)) x_0)} E \setminus$	$\frac{}{S : (\lambda x_0.((S' (M' x_0)) x_0) j)} E \setminus \quad \frac{}{=_{red.\beta} ((S' (M' j)) j)} E \setminus$
$\frac{}{[N \setminus S : \lambda x_0.((S' (M' x_0)) x_0)]^2} I^0 \quad \frac{}{S : (\lambda x_0.((S' (M' x_0)) x_0) p)} E \setminus$	$\frac{}{S \setminus S : (\lambda P_1.\lambda P_2.(P_2 \wedge P_1) ((S' (M' j)) j))} E/ \quad \frac{}{=_{red.\beta} \lambda P_2.(P_2 \wedge ((S' (M' j)) j))} E/$
$\frac{}{=_{red.\beta} ((S' (M' p)) p)} E \setminus$	$\frac{}{S : (\lambda P_2.(P_2 \wedge ((S' (M' j)) j)) ((S' (M' p)) p))} E \setminus$
$\frac{}{=_{red.\beta} ((S' (M' p)) p) \wedge ((S' (M' j)) j)} E \setminus$	

Referências

- [1] Jon Barwise and John Etchemendy. *The Language of First-Order Logic*. CSLI, Stanford, CA, third edition, 1992.