Gramática Categorial: Introdução de \ e de /

Luiz Arthur Pagani (UFPR)

1 Introdução de \

- regra:

$$\cfrac{\cfrac{\begin{matrix}[Y : \upsilon]^n & \vdots \\ \vdots & \vdots\end{matrix}}{X : \phi}}{Y\backslash X : \lambda\upsilon.\phi}\, I\backslash^n$$

- condições:
  - $\upsilon \in Var_{T_Y}$
  - υ nova na derivação

[Baseado em [1, p. 156]]

2 Introdução de /

- regra:

$$\dfrac{\dfrac{\begin{array}{cc} \vdots & [Y : v]^n \\ \vdots & \vdots \end{array}}{X : \phi}}{X/Y : \lambda v.\phi}\, I/^n$$

- condições:
  - $v \in Var_{T_Y}$
  - v nova na derivação

[Baseado em [1, p. 156]]

3 Exemplos

3.1 "Pedro ama e Maria odeia Grosbilda"

3.1.1 "Pedro ama"

- inserção lexical:

$$\dfrac{\text{Pedro}}{N : p}\,L \qquad \dfrac{\text{ama}}{(N\backslash S)/N : A}\,L$$

- suposição inicial:

$$\dfrac{\text{Pedro}}{N : p}\,L \qquad \dfrac{\text{ama}}{(N\backslash S)/N : A}\,L \qquad [N : x]$$

- eliminação de /:

$$
\begin{array}{ccccccccccccccccc}
\underbrace{(N\backslash S)} & / & N & : & A & & N & : & x & & & & & & & & \\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow & & & & & & & & \\
\hline
\multicolumn{1}{|c}{X} & / & Y & : & \phi & + & Y & : & \alpha & \Rightarrow & X & : & (& \phi & \alpha & \multicolumn{1}{c|}{)} \\
\hline
 & & & & & & & & & & \downarrow & | & | & \downarrow & \downarrow & | \\
 & & & & & & & & & & \overbrace{N\backslash S} & : & (& A & x &)
\end{array}
$$

- permite desenhar:

$$
\boxed{
\begin{array}{ccc}
\dfrac{\text{Pedro}}{N : p}\,L & &
\dfrac{\dfrac{\text{ama}}{(N\backslash S)/N : A}\,L \qquad [N : x]}{N\backslash S : (A\;x)}\,E/
\end{array}
}
$$

- eliminação de \:

$$
\begin{array}{ccccccccccccccc}
N & : & p & & N & \backslash & S & : & \underbrace{(A\,x)} & & & & & & \\
\downarrow & | & \downarrow & & \downarrow & | & \downarrow & | & \downarrow & & & & & & \\
\hline
\multicolumn{1}{|c}{Y} & : & \alpha & + & Y & \backslash & X & : & \phi & \Rightarrow & X & : & (& \phi\quad\alpha & \multicolumn{1}{c|}{)} \\
\hline
 & & & & & & & & & & \downarrow & | & | & \downarrow\quad\downarrow & | \\
 & & & & & & & & & & S & : & (& \overbrace{(A\,x)}\quad p &)
\end{array}
$$

- permite agora desenhar:

$$
\boxed{
\dfrac{\dfrac{\text{Pedro}}{N : p}\,L \qquad \dfrac{\dfrac{\text{ama}}{(N\backslash S)/N : A}\,L \qquad [N : x]^1}{N\backslash S : (A\,x)}\,E/}{S : ((A\,x)\,p)}\,E\backslash
}
$$

- introdução de /:

$$\dfrac{\dfrac{\dfrac{\text{Pedro}}{N : p}\,L \qquad \dfrac{\dfrac{\text{ama}}{(N\backslash S)/N : A}\,L \qquad [N : x]^1}{N\backslash S : (A\ x)}\,E/}{S : ((A\ x)\ p)}\,E\backslash}{S/N : \lambda x.((A\ x)\ p)}\,I/^1$$

3.1.2 "Maria odeia"

- inserção lexical:

$$\dfrac{\text{Maria}}{N : m}L \qquad \dfrac{\text{odeia}}{(N\backslash S)/N : O}L$$

- suposição inicial:

$$\dfrac{\text{Maria}}{N : m}L \qquad \dfrac{\text{odeia}}{(N\backslash S)/N : O}L \qquad [N : y]$$

- eliminação de /:

$$
\begin{array}{ccccccccccccccccc}
\multicolumn{1}{c}{\underbrace{(N\backslash S)}} & / & N & : & O & & N & : & y & & & & & & & & \\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow & & & & & & & & \\
\hline
X & / & Y & : & \phi & + & Y & : & \alpha & \Rightarrow & X & : & (& \phi & \alpha &) \\
\hline
 & & & & & & & & & & \downarrow & | & | & \downarrow & \downarrow & | \\
 & & & & & & & & & & \overbrace{N\backslash S} & : & (& O & y &)
\end{array}
$$

- permite desenhar:

$$
\boxed{\dfrac{\dfrac{\text{Maria}}{N : m}\,L \qquad \dfrac{\dfrac{\text{odeia}}{(N\backslash S)/N : O}\,L \qquad [N : y]}{N\backslash S : (O\ y)}\,E/}{}}
$$

- eliminação de \:

$$
\begin{array}{cccccccccccccccc}
N & : & m & & N & \backslash & S & : & \underbrace{(O\ y)} & & & & & & & \\
\downarrow & | & \downarrow & & \downarrow & | & \downarrow & | & \downarrow & & & & & & & \\
\hline
\multicolumn{1}{|c}{Y} & : & \alpha & + & Y & \backslash & X & : & \phi & \Rightarrow & X & : & (& \phi & \alpha & \multicolumn{1}{c|}{)} \\
\hline
& & & & & & & & & & \downarrow & | & | & \downarrow & \downarrow & | \\
& & & & & & & & & & S & : & (& \overbrace{(O\ y)} & m &)
\end{array}
$$

- permite agora desenhar:

$$
\boxed{
\dfrac{
\dfrac{\text{Maria}}{N : m}\,L
\qquad
\dfrac{
\dfrac{\text{odeia}}{(N\backslash S)/N : O}\,L
\qquad
[N : y]
}{N\backslash S : (O\ y)}\,E/
}{S : ((O\ y)\ m)}\,E\backslash
}
$$

- introdução de /:

$$\dfrac{\dfrac{\dfrac{\text{Maria}}{N : m}\,L \qquad \dfrac{\dfrac{\text{odeia}}{(N\backslash S)/N : O}\,L \qquad [N : y]^2}{N\backslash S : (O\ y)}\,E/}{S : ((O\ y)\ m)}\,E\backslash}{S/N : \lambda y.((O\ y)\ m)}\,I/^2$$

3.1.3 “Pedro ama e Maria odeia Grosbilda”

- $\langle \text{e}, ((S/N)\backslash(S/N))/(S/N), \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\rangle \in Lex$
- início da análise:

$$\dfrac{\text{Pedro ama}}{S/N : \lambda x.((A\ x)\ p)}D \qquad \dfrac{\text{e}}{((S/N)\backslash(S/N))/(S/N) : \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))}L \qquad \dfrac{\text{Maria odeia}}{S/N : \lambda y.((O\ y)\ m)}D \qquad \dfrac{\text{Grosbilda}}{N : g}L$$

- eliminação de /:

$$\begin{array}{ccccccccc}
\underbrace{((S/N)\backslash(S/N))} & / & \underbrace{(S/N)} & : & \underbrace{\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))} & & \underbrace{S/N} & : & \underbrace{\lambda y.((O\ y)\ m)} \\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow \\
X & / & Y & : & \phi & + & Y & : & \alpha
\end{array}$$

$$\Downarrow$$

$$\begin{array}{cccccc}
X & : & (& \phi & \alpha &) \\
\downarrow & | & | & \downarrow & \downarrow & | \\
\overbrace{(S/N)\backslash(S/N)} & : & (& \overbrace{\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))} & \overbrace{\lambda y.((O\ y)\ m)} &)
\end{array}$$

- inclusão na análise:

$$\dfrac{\text{Pedro ama}}{S/N : \lambda x.((A\ x)\ p)}D \qquad \dfrac{\dfrac{\text{e}}{((S/N)\backslash(S/N))/(S/N) : \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))}L \quad \dfrac{\text{Maria odeia}}{S/N : \lambda y.((O\ y)\ m)}D}{(S/N)\backslash(S/N) : (\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\ \lambda y.((O\ y)\ m))}E/ \qquad \dfrac{\text{Grosbilda}}{N : g}L$$

- redução-β:

$$\begin{array}{ccccccc}
(& \lambda & P & . & \underbrace{\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))} & \lambda y.((O\ y)\ m) &) \\
| & | & \downarrow & | & \downarrow & \downarrow & | \\
(& \lambda & \upsilon & . & \varphi & \alpha &)
\end{array}$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{\lambda Q.\lambda z.((Q\ z) \wedge (\lambda y.((O\ y)\ m)\ z))}$$

- inclusão na análise:

$$\dfrac{\dfrac{\text{Pedro ama}}{\begin{array}{c}S/N:\\ \lambda x.((A\ x)\ p)\end{array}}D \qquad \dfrac{\dfrac{\text{e}}{\begin{array}{c}((S/N)\backslash(S/N))/(S/N):\\ \lambda P.\lambda Q.\lambda z.((Q\ z)\wedge(P\ z))\end{array}}L \quad \dfrac{\text{Maria odeia}}{\begin{array}{c}S/N:\\ \lambda y.((O\ y)\ m)\end{array}}D}{\begin{array}{c}(S/N)\backslash(S/N):\\ (\lambda P.\lambda Q.\lambda z.((Q\ z)\wedge(P\ z))\ \lambda y.((O\ y)\ m))\\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z)\wedge(\lambda y.((O\ y)\ m)\ z))\end{array}}E/ \qquad \dfrac{\text{Grosbilda}}{N:g}L}{}$$

- redução-β em subfórmula:

$$\lambda Q.\lambda z.((Q\ z) \quad \wedge \quad \underbrace{(\lambda y.((O\ y)\ m)\ z)}_{\text{aqui}} \quad)$$

- 2a. redução-β:

$$
\begin{array}{ccccccc}
(& \lambda & y & . & \underbrace{((O\ y)\ m)} & z &) \\
| & | & \downarrow & | & \downarrow & \downarrow & | \\
\hline
(& \lambda & \upsilon & . & \varphi & \alpha &)
\end{array}
$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{((O\ z)\ m)}$$

- resultado:

$$\underbrace{\lambda Q.\lambda z.((Q\ z) \quad \wedge \quad \underbrace{(\lambda y.((O\ y)\ m)\ z)}_{((O\ z)\ m)} \quad)}_{\lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))}$$

- inclusão na análise:

$$
\dfrac{\dfrac{\text{Pedro ama}}{S/N : \lambda x.((A\ x)\ p)}D
\qquad
\dfrac{\dfrac{\text{e}}{((S/N)\backslash(S/N))/(S/N) : \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))}L
\quad
\dfrac{\text{Maria odeia}}{S/N : \lambda y.((O\ y)\ m)}D}
{\begin{array}{c}(S/N)\backslash(S/N) : \\ (\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\ \lambda y.((O\ y)\ m)) \\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge (\lambda y.((O\ y)\ m)\ z)) \\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\end{array}}E/}{}
\qquad
\dfrac{\text{Grosbilda}}{N : g}L
$$

- eliminação de \:

$$
\begin{array}{ccccccccc}
S/N & : & \underbrace{\lambda x.((A\ x)\ p)} & & \underbrace{(S/N)} & \backslash & \underbrace{(S/N)} & : & \underbrace{\lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))} \\
\downarrow & | & \downarrow & & \downarrow & | & \downarrow & | & \downarrow \\
\hline
Y & : & \alpha & + & Y & \backslash & X & : & \phi \\
\hline
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{cccccc}
\hline
X & : & (& \phi & \alpha &) \\
\hline
\downarrow & | & | & \downarrow & \downarrow & | \\
S/N & : & (& \overbrace{\lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))} & \overbrace{\lambda x.((A\ x)\ p)} &)
\end{array}
$$

- inclusão na análise:

$$
\dfrac{
\dfrac{\text{Pedro ama}}{\begin{array}{c} S/N: \\ \lambda x.((A\ x)\ p) \end{array}} D
\qquad
\dfrac{
\dfrac{\text{e}}{\begin{array}{c} ((S/N)\backslash(S/N))/(S/N): \\ \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z)) \end{array}} L
\quad
\dfrac{\text{Maria odeia}}{\begin{array}{c} S/N: \\ \lambda y.((O\ y)\ m) \end{array}} D
}{
\begin{array}{c}
(S/N)\backslash(S/N): \\
(\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\ \lambda y.((O\ y)\ m)) \\
=_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge (\lambda y.((O\ y)\ m)\ z)) \\
=_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))
\end{array}
} E/
}{
S/N : (\lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\ \lambda x.((A\ x)\ p))
} E\backslash
\qquad
\dfrac{\text{Grosbilda}}{N : g} L
$$

- redução-β:

$$
\begin{array}{ccccccc}
(& \lambda & Q & . & \underbrace{\lambda z.((Q\ z) \wedge ((O\ z)\ m))} & \underbrace{\lambda x.((A\ x)\ p)} &) \\
| & | & \downarrow & | & \downarrow & \downarrow & | \\
(& \lambda & \upsilon & . & \varphi & \alpha &)
\end{array}
$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{\lambda z.((\lambda x.((A\ x)\ p)\ z) \wedge ((O\ z)\ m))}$$

- inclusão na análise:

$$
\dfrac{
 \dfrac{\text{Pedro ama}}{\begin{array}{c}S/N :\\ \lambda x.((A\ x)\ p)\end{array}}D
 \quad
 \dfrac{
   \dfrac{\text{e}}{\begin{array}{c}((S/N)\backslash(S/N))/(S/N) :\\ \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\end{array}}L
   \quad
   \dfrac{\text{Maria odeia}}{\begin{array}{c}S/N :\\ \lambda y.((O\ y)\ m)\end{array}}D
 }{\begin{array}{c}(S/N)\backslash(S/N) :\\ (\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\ \lambda y.((O\ y)\ m))\\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge (\lambda y.((O\ y)\ m)\ z))\\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\end{array}}E/
}{\begin{array}{c}S/N : (\lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\ \lambda x.((A\ x)\ p))\\ =_{red.\beta}\ \lambda z.((\lambda x.((A\ x)\ p)\ z) \wedge ((O\ z)\ m))\end{array}}E\backslash
\qquad
\dfrac{\text{Grosbilda}}{N : g}L
$$

- outra redução-β em subfórmula:

$$\lambda z.(\quad \underbrace{(\lambda x.((A\ x)\ p)\ z)}_{\text{aqui}} \quad \wedge \quad ((O\ z)\ m) \quad)$$

- nova redução-β:

$$\begin{array}{ccccccc} (& \lambda & x & . & \underbrace{((A\ x)\ p)} & z &) \\ | & | & \downarrow & | & \downarrow & \downarrow & | \\ \hline (& \lambda & \upsilon & . & \varphi & \alpha &) \\ \hline \end{array}$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{((A\ z)\ p)}$$

- resultado:

$$\underbrace{\lambda z.(\quad \underbrace{(\lambda x.((A\ x)\ p)\ z)}_{((A\ z)\ p)} \quad \wedge ((O\ z)\ m) \quad)}_{\lambda z.((A\ z)\ p) \wedge ((o\ z)\ m))}$$

- inclusão na análise:

$$
\dfrac{\dfrac{\text{Pedro ama}}{\begin{array}{c}S/N :\\ \lambda x.((A\ x)\ p)\end{array}}D \qquad \dfrac{\dfrac{\text{e}}{\begin{array}{c}((S/N)\backslash(S/N))/(S/N) :\\ \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\end{array}}L \qquad \dfrac{\text{Maria odeia}}{\begin{array}{c}S/N :\\ \lambda y.((O\ y)\ m)\end{array}}D}{\begin{array}{c}(S/N)\backslash(S/N) :\\ (\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\ \lambda y.((O\ y)\ m))\\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge (\lambda y.((O\ y)\ m)\ z))\\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\end{array}}E/}{\begin{array}{l}S/N : (\lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\ \lambda x.((A\ x)\ p))\\ \qquad =_{red.\beta}\ \lambda z.((\lambda x.((A\ x)\ p)\ z) \wedge ((O\ z)\ m))\\ \qquad =_{red.\beta}\ \lambda z.(((A\ z)\ p) \wedge ((O\ z)\ m))\end{array}}E\backslash \qquad \dfrac{\text{Grosbilda}}{N : g}L
$$

- última eliminação de /:

$$
\begin{array}{ccccccccc}
S & / & N & : & \underbrace{\lambda z.(((A\ z)\ p) \wedge ((O\ z)\ m))} & & N & : & g\\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow\\
X & / & Y & : & \phi & + & Y & : & \alpha
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{cccccc}
X & : & (& \phi & \alpha &)\\
\downarrow & | & | & \downarrow & \downarrow & |\\
S & : & (& \overbrace{\lambda z.(((A\ z)\ p) \wedge ((O\ z)\ m))} & g &)
\end{array}
$$

- inclusão na análise:

$$\dfrac{\text{Pedro ama}}{S/N : \lambda x.((A\ x)\ p)}\,D \qquad \dfrac{\text{e}}{((S/N)\backslash(S/N))/(S/N) : \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))}\,L \qquad \dfrac{\text{Maria odeia}}{S/N : \lambda y.((O\ y)\ m)}\,D \qquad \dfrac{\text{Grosbilda}}{N : g}\,L$$

$$\dfrac{((S/N)\backslash(S/N))/(S/N) : \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z)) \qquad S/N : \lambda y.((O\ y)\ m)}{\begin{array}{c}(S/N)\backslash(S/N) : \\ (\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\ \lambda y.((O\ y)\ m)) \\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge (\lambda y.((O\ y)\ m)\ z)) \\ =_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\end{array}}\,E/$$

$$\dfrac{S/N : \lambda x.((A\ x)\ p) \qquad (S/N)\backslash(S/N) : \lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))}{\begin{array}{c}S/N : (\lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\ \lambda x.((A\ x)\ p)) \\ =_{red.\beta}\ \lambda z.((\lambda x.((A\ x)\ p)\ z) \wedge ((O\ z)\ m)) \\ =_{red.\beta}\ \lambda z.(((A\ z)\ p) \wedge ((O\ z)\ m))\end{array}}\,E\backslash$$

$$\dfrac{S/N : \lambda z.(((A\ z)\ p) \wedge ((O\ z)\ m)) \qquad N : g}{S : (\lambda z.(((A\ z)\ p) \wedge ((O\ z)\ m))\ g)}\,E/$$

- última redução-β:

$$\begin{array}{ccccccc}
(& \lambda & z & . & \underbrace{(((A\ z)\ p) \wedge ((O\ z)\ m))} & g &) \\
| & | & \downarrow & | & \downarrow & \downarrow & | \\
\hline
(& \lambda & \upsilon & . & \varphi & \alpha &) \\
\hline
\end{array}$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{(((A\ g)\ p) \wedge ((O\ g)\ m))}$$

- finalização da análise:

$$
\dfrac{
\dfrac{
\dfrac{\text{Pedro ama}}{\begin{array}{c} S/N: \\ \lambda x.((A\ x)\ p) \end{array}}\,D
\qquad
\dfrac{
\dfrac{\text{e}}{\begin{array}{c} ((S/N)\backslash(S/N))/(S/N): \\ \lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z)) \end{array}}\,L
\quad
\dfrac{\text{Maria odeia}}{\begin{array}{c} S/N: \\ \lambda y.((O\ y)\ m) \end{array}}\,D
}{
\begin{array}{c}
(S/N)\backslash(S/N): \\
(\lambda P.\lambda Q.\lambda z.((Q\ z) \wedge (P\ z))\ \lambda y.((O\ y)\ m)) \\
=_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge (\lambda y.((O\ y)\ m)\ z)) \\
=_{red.\beta}\ \lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))
\end{array}
}\,E/
}{
\begin{array}{l}
S/N : (\lambda Q.\lambda z.((Q\ z) \wedge ((O\ z)\ m))\ \lambda x.((A\ x)\ p)) \\
\qquad =_{red.\beta}\ \lambda z.((\lambda x.((A\ x)\ p)\ z) \wedge ((O\ z)\ m)) \\
\qquad =_{red.\beta}\ \lambda z.(((A\ z)\ p) \wedge ((O\ z)\ m))
\end{array}
}\,E\backslash
\qquad
\dfrac{\text{Grosbilda}}{N : g}\,L
}{
\begin{array}{l}
S : (\lambda z.(((A\ z)\ p) \wedge ((O\ z)\ m))\ g) \\
\qquad =_{red.\beta}\ (((A\ g)\ p) \wedge ((O\ g)\ m))
\end{array}
}\,E/
$$

3.2 “todo homem ama uma mulher”

- sentença ambígua:
  - para todo homem há uma mulher tal que ele a ama (uma mulher para cada homem)
  - há uma mulher tal que todo homem a ama (uma única mulher para todos os homens)
- itens lexicais que faltam para a análise:
  - $\langle \text{homem}, N_c, H \rangle \in Lex$
  - $\langle \text{mulher}, N_c, M \rangle \in Lex$

3.2.1 "todo homem"

- inserção lexical:

$$\frac{\text{todo}}{(S/(N\backslash S))/N_c : \lambda P.\lambda Q.\forall x.((P\ x) \rightarrow (Q\ x))}L \qquad \frac{\text{homem}}{N_c : H}L$$

- eliminação de /:

$$\begin{array}{ccccccccc}
\underbrace{(S/(N\backslash S))} & / & N_c & : & \underbrace{\lambda P.\lambda Q.\forall x.((P\ x) \rightarrow (Q\ x))} & & N_c & : & H \\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow \\
X & / & Y & : & \phi & + & Y & : & \alpha
\end{array}$$

$$\Downarrow$$

$$\begin{array}{cccccc}
X & : & (& \phi & \alpha &) \\
\downarrow & | & | & \downarrow & \downarrow & | \\
\overbrace{S/(N\backslash S)} & : & (& \overbrace{\lambda P.\lambda Q.\forall x.((P\ x) \rightarrow (Q\ x))} & H &)
\end{array}$$

- incluindo a eliminação no diagrama:

$$\dfrac{\dfrac{\text{todo}}{(S/(N\backslash S))/N_c : \lambda P.\lambda Q.\forall x.((P\ x) \rightarrow (Q\ x))}L \quad \dfrac{\text{homem}}{N_c : H}L}{S/(N\backslash S) : (\lambda P.\lambda Q.\forall x.((P\ x) \rightarrow (Q\ x))\ H)}E/$$

- redução-β:

$$\begin{array}{ccccccc}
(& \lambda & P & . & \underbrace{\lambda Q.\forall x.((P\ x) \rightarrow (Q\ x))} & H &) \\
| & | & \downarrow & | & \downarrow & \downarrow & | \\
\hline
(& \lambda & \upsilon & . & \varphi & \alpha &) \\
\hline
\end{array}$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))}$$

- incluindo a redução-β no diagrama:

$$\cfrac{\cfrac{\text{todo}}{(S/(N\backslash S))/N_c : \lambda P.\lambda Q.\forall x.((P\ x) \rightarrow (Q\ x))}L \quad \cfrac{\text{homem}}{N_c : H}L}{\begin{array}{l} S/(N\backslash S) : (\lambda P.\lambda Q.\forall x.((P\ x) \rightarrow (Q\ x))\ H) \\ \qquad =_{red.\beta} \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x)) \end{array}}E/$$

- finalizado

3.2.2 "uma mulher"

- inserção lexical:

$$\dfrac{\text{uma}}{((S/N)\backslash S)/N_c : \lambda P_1.\lambda Q_1.\exists y.((P_1\ y) \wedge (Q_1\ y))}L \qquad \dfrac{\text{mulher}}{N_c : M}L$$

- eliminação de /:

$$\begin{array}{ccccccccc}
\underbrace{((S/N)\backslash S)} & / & N_c & : & \underbrace{\lambda P_1.\lambda Q_1.\exists y.((P_1\ y) \wedge (Q_1\ y))} & & N_c & : & M \\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow \\
X & / & Y & : & \phi & + & Y & : & \alpha
\end{array}$$

$$\Downarrow$$

$$\begin{array}{cccccc}
X & : & (& \phi & \alpha &) \\
\downarrow & | & | & \downarrow & \downarrow & | \\
\overbrace{(S/N)\backslash S} & : & (& \overbrace{\lambda P_1.\lambda Q_1.\exists y.((P_1\ y) \wedge (Q_1\ y))} & M &)
\end{array}$$

- incluindo a eliminação no diagrama:

$$\boxed{\dfrac{\dfrac{\text{uma}}{((S/N)\backslash S)/N_c : \lambda P_1.\lambda Q_1.\exists y.((P_1\ y) \wedge (Q_1\ y))}L \quad \dfrac{\text{mulher}}{N_c : M}L}{(S/N)\backslash S : (\lambda P_1.\lambda Q_1.\exists y.((P_1\ y) \wedge (Q_1\ y))\ M)}E/}$$

- redução-β:

$$\begin{array}{ccccccc}
(& \lambda & P_1 & . & \underbrace{\lambda Q_1.\exists y.((P_1\ y) \wedge (Q_1\ y))} & M &) \\
| & | & \downarrow & | & \downarrow & \downarrow & | \\
\hline
(& \lambda & \upsilon & . & \varphi & \alpha &) \\
\hline
\end{array}$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))}$$

- incluindo a redução-β no diagrama:

$$\cfrac{\cfrac{\text{uma}}{((S/N)\backslash S)/N_c : \lambda P_1.\lambda Q_1.\exists y.((P_1\ y) \wedge (Q_1\ y))}L \quad \cfrac{\text{mulher}}{N_c : M}L}{\begin{array}{c}(S/N)\backslash S : (\lambda P_1.\lambda Q_1.\exists y.((P_1\ y) \wedge (Q_1\ y))\ M) \\ =_{red.\beta}\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\end{array}}E/$$

- finalizado

3.2.3 Preparação de “ama”

- postulação de duas suposições:

$$\boxed{[N:z]^1 \quad \dfrac{\text{ama}}{(N\backslash S)/N : A}\,L \quad [N:w]^2}$$

- eliminação de /:

$$\begin{array}{ccccccccccccccccc}
\underbrace{(N\backslash S)} & / & N & : & A & & N & : & w & & & & & & & & \\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow & & & & & & & & \\
\hline
\multicolumn{1}{|c}{X} & / & Y & : & \phi & + & Y & : & \alpha & \Rightarrow & X & : & (& \phi & \alpha & \multicolumn{1}{c|}{)} \\
\hline
 & & & & & & & & & & \downarrow & | & | & \downarrow & \downarrow & | \\
 & & & & & & & & & & \overbrace{N\backslash S} & : & (& A & w &)
\end{array}$$

- incluindo a eliminação de / no diagrama:

$$\boxed{[N:z]^1 \quad \dfrac{\dfrac{\text{ama}}{(N\backslash S)/N : A}\,L \quad [N:w]^2}{N\backslash S : (A\,w)}\,E/}$$

- eliminação de \:

$$\begin{array}{|cccccccccccccccc|}
\multicolumn{1}{c}{N} & : & z & & N & \backslash & S & : & \underbrace{(A\,w)} & & & & & & & \multicolumn{1}{c}{} \\
\multicolumn{1}{c}{\downarrow} & | & \downarrow & & \downarrow & | & \downarrow & | & \downarrow & & & & & & & \multicolumn{1}{c}{} \\
\hline
Y & : & \alpha & + & Y & \backslash & X & : & \phi & \Rightarrow & X & : & (& \phi & \alpha &) \\
\hline
\multicolumn{1}{c}{} & & & & & & & & & & \downarrow & | & | & \downarrow & \downarrow & \multicolumn{1}{c}{|} \\
\multicolumn{1}{c}{} & & & & & & & & & & S & : & (& \overbrace{(A\,w)} & z & \multicolumn{1}{c}{)}
\end{array}$$

- incluindo a eliminação de \ no diagrama:

$$\dfrac{[N:z]^1 \quad \dfrac{\dfrac{\text{ama}}{(N\backslash S)/N : A}L \quad [N:w]^2}{N\backslash S : (A\ w)}E/}{S : ((A\ w)\ z)}E\backslash$$

- finalizado

3.2.4 Primeira interpretação

- descarregando a segunda hipótese — introdução de /:

$$
\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S): \\ \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x)) \end{array}}D
\quad
\dfrac{\dfrac{\text{ama}}{\begin{array}{c} S: \\ ((A\ w)\ z) \end{array}}D}{\begin{array}{c} S/N: \\ \lambda w.((A\ w)\ z) \end{array}}I/^2
\quad
\dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S: \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}D
$$

- eliminação de \:

$$\begin{array}{ccccccccc}
\underbrace{S/N} & : & \underbrace{\lambda w.((A\ w)\ z)} & & \underbrace{(S/N)} & \backslash & S & : & \underbrace{\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))} \\
\downarrow & | & \downarrow & & \downarrow & | & \downarrow & | & \downarrow \\
\hline
Y & : & \alpha & + & Y & \backslash & X & : & \phi \\
\hline
\end{array}$$

$$\Downarrow$$

$$\begin{array}{cccccc}
\hline
X & : & (& \phi & \alpha &) \\
\hline
\downarrow & | & | & \downarrow & \downarrow & | \\
S & : & (& \overbrace{\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))} & \overbrace{\lambda w.((A\ w)\ z)} &)
\end{array}$$

- incluindo no diagrama a eliminação de \:

$$
\dfrac{\text{todo homem}}{\begin{array}{c}S/(N\backslash S):\\ \lambda Q.\forall x.((H\ x)\to(Q\ x))\end{array}}\,D
\qquad
\dfrac{\dfrac{\dfrac{\text{ama}}{\begin{array}{c}S:\\ ((A\ w)\ z)\end{array}}\,D}{\begin{array}{c}S/N:\\ \lambda w.((A\ w)\ z)\end{array}}\,I/^{2}
\qquad
\dfrac{\text{uma mulher}}{\begin{array}{c}(S/N)\backslash S:\\ \lambda Q_1.\exists y.((M\ y)\wedge(Q_1\ y))\end{array}}\,D}{S:(\lambda Q_1.\exists y.((M\ y)\wedge(Q_1\ y))\ \lambda w.((A\ w)\ z))}\,E\backslash
$$

- redução-β:

$$
\begin{array}{ccccccc}
(& \lambda & Q_1 & . & \underbrace{\exists y.((M\ y) \wedge (Q_1\ y))} & \underbrace{\lambda w.((A\ w)\ z)} &) \\
| & | & \downarrow & | & \downarrow & \downarrow & |
\end{array}
$$

$$
\boxed{\begin{array}{ccccccc}
(& \lambda & \upsilon & . & \varphi & \alpha &)
\end{array}}
$$

$$
\Downarrow
$$

$$
\boxed{\varphi[\upsilon \mapsto \alpha]}
$$

$$
\downarrow
$$

$$
\overbrace{\exists y.((M\ y) \wedge (\lambda w.((A\ w)\ z)\ y))}
$$

- incluindo a redução-β no diagrama:

$$
\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S): \\ \lambda Q.\forall x.((H\ x) \to (Q\ x)) \end{array}}D
\quad
\dfrac{\dfrac{\dfrac{\text{ama}}{\begin{array}{c} S: \\ ((A\ w)\ z) \end{array}}D}{\begin{array}{c} S/N: \\ \lambda w.((A\ w)\ z) \end{array}}I/^2
\quad
\dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S: \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}D}{\begin{array}{c} S: (\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\ \lambda w.((A\ w)\ z)) \\ =_{red.\beta} \exists y.((M\ y) \wedge (\lambda w.((A\ w)\ z)\ y)) \end{array}}E\backslash
$$

- subfórmula a ser reduzida:

$$\exists y.((M\ y) \quad \wedge \quad \underbrace{(\lambda w.((A\ w)\ z)\ y)}_{\text{aqui}} \quad)$$

- redução-β:

$$
\begin{array}{ccccccccc}
(& \lambda & w & . & \underbrace{((A\ w)\ z)} & y &) & & \\
| & | & \downarrow & | & \downarrow & \downarrow & | & & \\
\hline
\multicolumn{1}{|c}{(} & \lambda & \upsilon & . & \varphi & \alpha &) & \Rightarrow & \multicolumn{1}{c|}{\varphi[\upsilon \mapsto \alpha]} \\
\hline
 & & & & & & & & \downarrow \\
 & & & & & & & & \overbrace{((A\ y)\ z))}
\end{array}
$$

- resultado:

$$\underbrace{\exists y.((M\ y) \quad \wedge \quad \underbrace{(\lambda w.((A\ w)\ z)\ y)}_{((A\ y)\ z)} \quad)}_{\exists y.((M\ y) \wedge ((A\ y)\ z))}$$

- incluindo a segunda redução-β no diagrama:

$$
\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S): \\ \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x)) \end{array}}D
\qquad
\dfrac{
\dfrac{\dfrac{\text{ama}}{\begin{array}{c} S: \\ ((A\ w)\ z) \end{array}}D}{\begin{array}{c} S/N: \\ \lambda w.((A\ w)\ z) \end{array}}I/^2
\qquad
\dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S: \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}D
}{
\begin{array}{l} S: (\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\ \lambda w.((A\ w)\ z)) \\ \quad =_{red.\beta} \exists y.((M\ y) \wedge (\lambda w.((A\ w)\ z)\ y)) \\ \quad =_{red.\beta} \exists y.((M\ y) \wedge ((A\ y)\ z)) \end{array}
}E\backslash
$$

- descarregando a primeira hipótese — introdução de \:

$$
\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S) : \\ \lambda Q.\forall x.((H\ x) \to (Q\ x)) \end{array}}\,D
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\text{ama}}{\begin{array}{c} S : \\ ((A\ w)\ z) \end{array}}\,D}{\begin{array}{c} S/N : \\ \lambda w.((A\ w)\ z) \end{array}}\,I/^{2}
\qquad
\dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S : \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}\,D}{\begin{array}{l} S : (\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\ \lambda w.((A\ w)\ z)) \\ \quad =_{red.\beta} \exists y.((M\ y) \wedge (\lambda w.((A\ w)\ z)\ y)) \\ \quad =_{red.\beta} \exists y.((M\ y) \wedge ((A\ y)\ z)) \end{array}}\,E\backslash}{N\backslash S : \lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))}\,I\backslash^{1}
$$

- eliminação de /:

$$
\begin{array}{ccccccccccc}
S & / & \underbrace{(N\backslash S)} & : & \underbrace{\lambda Q.\forall x.((H\ x) \to (Q\ x))} & & N\backslash S & : & \underbrace{\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))} \\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow \\
\hline
\multicolumn{1}{|c}{X} & / & Y & : & \phi & + & Y & : & \multicolumn{1}{c|}{\alpha} \\
\hline
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{cccccc}
\hline
\multicolumn{1}{|c}{X} & : & (& \phi & \alpha & \multicolumn{1}{c|}{)} \\
\hline
\downarrow & | & | & \downarrow & \downarrow & | \\
S & : & (& \overbrace{\lambda Q.\forall x.((H\ x) \to (Q\ x))} & \overbrace{\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))} &)
\end{array}
$$

- incluindo no diagrama a eliminação de /:

$$
\dfrac{
  \dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S) : \\ \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x)) \end{array}}\,D
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{\dfrac{\text{ama}}{\begin{array}{c} S : \\ ((A\ w)\ z) \end{array}}\,D}{\begin{array}{c} S/N : \\ \lambda w.((A\ w)\ z) \end{array}}\,I/^{2}
      \qquad
      \dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S : \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}\,D
    }{\begin{array}{l} S : (\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\ \lambda w.((A\ w)\ z)) \\ \quad =_{red.\beta} \exists y.((M\ y) \wedge (\lambda w.((A\ w)\ z)\ y)) \\ \quad =_{red.\beta} \exists y.((M\ y) \wedge ((A\ y)\ z)) \end{array}}\,E\backslash
  }{N\backslash S : \lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))}\,I\backslash^{1}
}{S : (\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))\ \lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z)))}\,E\backslash
$$

- redução-β:

$$
\begin{array}{cccccccc}
(& \lambda & Q & . & \underbrace{\forall x.((H\ x) \rightarrow (Q\ x))} & \underbrace{\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))} &) \\
| & | & \downarrow & | & \downarrow & \downarrow & | \\
\hline
\multicolumn{1}{|c}{(} & \lambda & \upsilon & . & \varphi & \alpha & \multicolumn{1}{c|}{)} \\
\hline
\end{array}
$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{\forall x.((H\ x) \rightarrow (\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))\ x))}$$

- incluindo no diagrama a redução-β:

$$
\dfrac{
\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S) : \\ \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x)) \end{array}}\,D
\quad
\dfrac{
\dfrac{
\dfrac{\dfrac{\text{ama}}{\begin{array}{c} S : \\ ((A\ w)\ z) \end{array}}\,D}{\begin{array}{c} S/N : \\ \lambda w.((A\ w)\ z) \end{array}}\,I/^2
\quad
\dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S : \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}\,D
}{\begin{array}{c} S : (\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\ \lambda w.((A\ w)\ z)) \\ =_{red.\beta}\ \exists y.((M\ y) \wedge (\lambda w.((A\ w)\ z)\ y)) \\ =_{red.\beta}\ \exists y.((M\ y) \wedge ((A\ y)\ z)) \end{array}}\,E\backslash
}{N\backslash S : \lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))}\,I\backslash^1
}{\begin{array}{c} S : (\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))\ \lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))) \\ =_{red.\beta}\ \forall x.((H\ x) \rightarrow (\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))\ x)) \end{array}}\,E\backslash
$$

- subfórmula a ser reduzida:

$$\forall x.((H\ x) \quad \rightarrow \quad \underbrace{(\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))\ x)}_{\text{aqui}} \quad)$$

- redução-β:

$$\begin{array}{ccccccccc}
(& \lambda & z & . & \underbrace{\exists y.((M\ y) \wedge ((A\ y)\ z))} & x &) & & \\
| & | & \downarrow & | & \downarrow & \downarrow & | & & \\
(& \lambda & \upsilon & . & \varphi & \alpha &) & \Rightarrow & \varphi[\upsilon \mapsto \alpha] \\
 & & & & & & & & \downarrow \\
 & & & & & & & & \overbrace{\exists y.((M\ y) \wedge ((A\ y)\ x))}
\end{array}$$

- resultado:

$$\underbrace{\forall x.((H\ x) \quad \rightarrow \quad \underbrace{(\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))\ x)}_{\exists y.((M\ y) \wedge ((A\ y)\ x))} \quad)}_{\forall x.((H\ x)\ \rightarrow \exists y.((M\ y) \wedge ((A\ y)\ x)))}$$

- incluindo no diagrama a segunda redução-β:

$$
\dfrac{
\dfrac{\text{todo homem}}{\begin{array}{c}S/(N\backslash S):\\ \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))\end{array}}D
\qquad
\dfrac{
\dfrac{\dfrac{\text{ama}}{\begin{array}{c}S:\\ ((A\ w)\ z)\end{array}}D}{\begin{array}{c}S/N:\\ \lambda w.((A\ w)\ z)\end{array}}I/^2
\qquad
\dfrac{\text{uma mulher}}{\begin{array}{c}(S/N)\backslash S:\\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\end{array}}D
}{\dfrac{\begin{array}{c}S:(\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\ \lambda w.((A\ w)\ z))\\ =_{red.\beta}\ \exists y.((M\ y) \wedge (\lambda w.((A\ w)\ z)\ y))\\ =_{red.\beta}\ \exists y.((M\ y) \wedge ((A\ y)\ z))\end{array}}{N\backslash S:\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))}I\backslash^1}E\backslash
}{\begin{array}{c}S:(\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))\ \lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z)))\\ =_{red.\beta}\ \forall x.((H\ x) \rightarrow (\lambda z.\exists y.((M\ y) \wedge ((A\ y)\ z))\ x))\\ =_{red.\beta}\ \forall x.((H\ x)\ \rightarrow \exists y.((M\ y) \wedge ((A\ y)\ x)))\end{array}}E\backslash
$$

3.2.5 Segunda interpretação

- descarregando a primeira hipótese — introdução de \:

$$
\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S): \\ \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x)) \end{array}} D
\quad
\dfrac{\dfrac{\text{ama}}{\begin{array}{c} S: \\ ((A\ w)\ z) \end{array}} D}{\begin{array}{c} N\backslash S: \\ \lambda z.((A\ w)\ z) \end{array}} I\backslash^{1}
\quad
\dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S: \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}} D
$$

- eliminação de /:

$$
\begin{array}{cccccccccc}
S & / & \underbrace{(N\backslash S)} & : & \underbrace{\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))} & & \underbrace{N\backslash S} & : & \underbrace{\lambda z.((A\ w)\ z)} \\
\downarrow & | & \downarrow & | & \downarrow & & \downarrow & | & \downarrow \\
\hline
X & / & Y & : & \phi & + & Y & : & \alpha \\
\hline
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{cccccc}
\hline
X & : & (& \phi & \alpha &) \\
\hline
\downarrow & | & | & \downarrow & \downarrow & | \\
S & : & (& \overbrace{\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))} & \overbrace{\lambda z.((A\ w)\ z)} &)
\end{array}
$$

- incluindo no diagrama a eliminação de /:

$$\dfrac{\dfrac{\text{todo homem}}{S/(N\backslash S) : \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))}D \quad \dfrac{\dfrac{\text{ama}}{S : ((A\ w)\ z)}D}{N\backslash S : \lambda z.((A\ w)\ z)}I\backslash^1}{S : (\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))\ \lambda z.((A\ w)\ z))}E/ \qquad \dfrac{\text{uma mulher}}{(S/N)\backslash S : \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))}D$$

- redução-β:

$$\begin{array}{ccccccc} (& \lambda & Q & . & \underbrace{\forall x.((H\ x) \to (Q\ x))} & \underbrace{\lambda z.((A\ w)\ z)} &) \\ | & | & \downarrow & | & \downarrow & \downarrow & | \\ (& \lambda & \upsilon & . & \varphi & \alpha &) \end{array}$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{\forall x.((H\ x) \to (\lambda z.((A\ w)\ z)\ x))}$$

- incluindo a redução-β no diagrama:

$$
\dfrac{\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S) : \\ \lambda Q.\forall x.((H\ x) \to (Q\ x)) \end{array}}D \quad \dfrac{\dfrac{\text{ama}}{\begin{array}{c} S : \\ ((A\ w)\ z) \end{array}}D}{\begin{array}{c} N\backslash S : \\ \lambda z.((A\ w)\ z) \end{array}}I\backslash^1}{\begin{array}{l} S : (\lambda Q.\forall x.((H\ x) \to (Q\ x))\ \lambda z.((A\ w)\ z)) \\ \quad =_{red.\beta} \forall x.((H\ x) \to (\lambda z.((A\ w)\ z)\ x)) \end{array}}E/ \qquad \dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S : \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}D
$$

- subfórmula a ser reduzida:

$$
\forall x.((H\ x) \quad \to \quad \underbrace{(\lambda z.((A\ w)\ z)\ x)}_{\text{aqui}} \quad)
$$

- redução-β:

$$\begin{array}{ccccccccc}
(& \lambda & z & . & \underbrace{((A\,w)\,z)} & x &) & & \\
| & | & \downarrow & | & \downarrow & \downarrow & | & & \\
\hline
(& \lambda & \upsilon & . & \varphi & \alpha &) & \Rightarrow & \varphi[\upsilon \mapsto \alpha] \\
\hline
 & & & & & & & & \downarrow \\
 & & & & & & & & \overbrace{((A\,w)\,x))}
\end{array}$$

- resultado:

$$\underbrace{\forall x.((H\ x) \quad \rightarrow \quad \underbrace{(\lambda z.((A\ w)\ z)\ x)}_{((A\ w)\ x)} \quad)}_{\forall x.((H\ x) \rightarrow ((A\ w)\ x))}$$

- incluindo a segunda redução-β no diagrama:

$$
\cfrac{
\cfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S) : \\ \lambda Q.\forall x.((H\ x) \to (Q\ x)) \end{array}}\,D
\qquad
\cfrac{\cfrac{\text{ama}}{\begin{array}{c} S : \\ ((A\ w)\ z) \end{array}}\,D}{\begin{array}{c} N\backslash S : \\ \lambda z.((A\ w)\ z) \end{array}}\,I\backslash^1
}{
\begin{array}{l} S : (\lambda Q.\forall x.((H\ x) \to (Q\ x))\ \lambda z.((A\ w)\ z)) \\ \quad =_{red.\beta} \forall x.((H\ x) \to (\lambda z.((A\ w)\ z)\ x)) \\ \quad =_{red.\beta} \forall x.((H\ x) \to ((A\ w)\ x)) \end{array}
}\,E/
\qquad
\cfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S : \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}\,D
$$

- descarregando a segunda hipótese — introdução de /:

$$
\dfrac{
\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S) : \\ \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x)) \end{array}}\,D
\quad
\dfrac{\dfrac{\text{ama}}{\begin{array}{c} S : \\ ((A\ w)\ z) \end{array}}\,D}{\begin{array}{c} N\backslash S : \\ \lambda z.((A\ w)\ z) \end{array}}\,I\backslash^{1}
}{
\dfrac{\begin{array}{l} S : (\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))\ \lambda z.((A\ w)\ z)) \\ \quad =_{red.\beta} \forall x.((H\ x) \rightarrow (\lambda z.((A\ w)\ z)\ x)) \\ \quad =_{red.\beta} \forall x.((H\ x) \rightarrow ((A\ w)\ x)) \end{array}}{S/N : \lambda w.\forall x.((H\ x) \rightarrow ((A\ w)\ x))}\,I/^{2}
}\,E/
\qquad
\dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S : \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}\,D
$$

- eliminação de \:

$$
\begin{array}{|ccccccccc|}
\multicolumn{1}{c}{\underbrace{S/N}} & : & \underbrace{\lambda w.\forall x.((H\ x) \to ((A\ w)\ x))} & & \underbrace{(S/N)} & \backslash & S & : & \multicolumn{1}{c}{\underbrace{\lambda Q_1.\exists y.((M\ y) \land (Q_1\ y))}} \\
\multicolumn{1}{c}{\downarrow} & | & \downarrow & & \downarrow & | & \downarrow & | & \multicolumn{1}{c}{\downarrow} \\
\hline
Y & : & \alpha & + & Y & \backslash & X & : & \phi \\
\hline
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{|ccccc|}
\hline
X & : & (\qquad \phi & \alpha &) \\
\hline
\multicolumn{1}{c}{\downarrow} & | & | \qquad \downarrow & \downarrow & \multicolumn{1}{c}{|} \\
\multicolumn{1}{c}{S} & : & (\quad \overbrace{\lambda Q_1.\exists y.((M\ y) \land (Q_1\ y))} & \overbrace{\lambda w.\forall x.((H\ x) \to ((A\ w)\ x))} & \multicolumn{1}{c}{)} \\
\end{array}
$$

- incluindo no diagrama a eliminação de \:

$$
\dfrac{
\dfrac{
\dfrac{\text{todo homem}}{\begin{array}{c} S/(N\backslash S) : \\ \lambda Q.\forall x.((H\ x) \rightarrow (Q\ x)) \end{array}}\ D
\qquad
\dfrac{\dfrac{\text{ama}}{\begin{array}{c} S : \\ ((A\ w)\ z) \end{array}}\ D}{\begin{array}{c} N\backslash S : \\ \lambda z.((A\ w)\ z) \end{array}}\ I\backslash^{1}
}{
\begin{array}{l}
S : (\lambda Q.\forall x.((H\ x) \rightarrow (Q\ x))\ \lambda z.((A\ w)\ z)) \\
\quad =_{red.\beta} \forall x.((H\ x) \rightarrow (\lambda z.((A\ w)\ z)\ x)) \\
\quad =_{red.\beta} \forall x.((H\ x) \rightarrow ((A\ w)\ x))
\end{array}
}\ E/
}{
S/N : \lambda w.\forall x.((H\ x) \rightarrow ((A\ w)\ x))
}\ I/^{2}
\qquad
\dfrac{\text{uma mulher}}{\begin{array}{c} (S/N)\backslash S : \\ \lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y)) \end{array}}\ D
$$

$$
\overline{S : (\lambda Q_1.\exists y.((M\ y) \wedge (Q_1\ y))\ \lambda w.\forall x.((H\ x) \rightarrow ((A\ w)\ x)))}\ E\backslash
$$

- redução-β:

$$\begin{array}{ccccccc} (& \lambda & Q_1 & . & \underbrace{\exists y.((M\ y) \wedge (Q_1\ y))} & \underbrace{\lambda w.\forall x.((H\ x) \rightarrow ((A\ w)\ x))} &) \\ | & | & \downarrow & | & \downarrow & \downarrow & | \end{array}$$

$$\boxed{(\quad \lambda \quad \upsilon \quad . \quad \varphi \quad \alpha \quad)}$$

$$\Downarrow$$

$$\boxed{\varphi[\upsilon \mapsto \alpha]}$$

$$\downarrow$$

$$\overbrace{\exists y.((M\ y) \wedge (\lambda w.\forall x.((H\ x) \rightarrow ((A\ w)\ x))\ y))}$$

- incluindo no diagrama a redução-β:

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{\text{todo homem}}{\begin{array}{c}S/(N\backslash S):\\ \lambda Q.\forall x.((H\ x)\to(Q\ x))\end{array}}\,D
\qquad
\dfrac{\dfrac{\text{ama}}{\begin{array}{c}S:\\ ((A\ w)\ z)\end{array}}\,D}{\begin{array}{c}N\backslash S:\\ \lambda z.((A\ w)\ z)\end{array}}\,I\backslash^{1}
}{\begin{array}{l}S:(\lambda Q.\forall x.((H\ x)\to(Q\ x))\ \lambda z.((A\ w)\ z))\\ \quad =_{red.\beta}\forall x.((H\ x)\to(\lambda z.((A\ w)\ z)\ x))\\ \quad =_{red.\beta}\forall x.((H\ x)\to((A\ w)\ x))\end{array}}\,E/
}{S/N:\lambda w.\forall x.((H\ x)\to((A\ w)\ x))}\,I/^{2}
\qquad
\dfrac{\text{uma mulher}}{\begin{array}{c}(S/N)\backslash S:\\ \lambda Q_1.\exists y.((M\ y)\wedge(Q_1\ y))\end{array}}\,D
}{\begin{array}{l}S:(\lambda Q_1.\exists y.((M\ y)\wedge(Q_1\ y))\ \lambda w.\forall x.((H\ x)\to((A\ w)\ x)))\\ \quad =_{red.\beta}\exists y.((M\ y)\wedge(\lambda w.\forall x.((H\ x)\to((A\ w)\ x))\ y))\end{array}}\,E\backslash
$$

- subfórmula a ser reduzida:

$$\exists y.((M\ y) \quad \wedge \quad \underbrace{(\lambda w.\forall x.((H\ x) \to ((A\ w)\ x))\ y)}_{\text{aqui}} \quad)$$

- redução-β:

$$\begin{array}{cccccccccc}
(& \lambda & w & . & \underbrace{\forall x.((H\ x) \to ((A\ w)\ x))} & y &) & & \\
| & | & \downarrow & | & \downarrow & \downarrow & | & & \\
(& \lambda & \upsilon & . & \varphi & \alpha &) & \Rightarrow & \varphi[\upsilon \mapsto \alpha] \\
 & & & & & & & & \downarrow \\
 & & & & & & & & \overbrace{\forall x.((H\ x) \to ((A\ y)\ x))}
\end{array}$$

- resultado:

$$\underbrace{\exists y.((M\ y) \quad \wedge \quad \underbrace{(\lambda w.\forall x.((H\ x) \to ((A\ w)\ x))\ y)}_{\forall x.((H\ x) \to ((A\ y)\ x))} \quad)}_{\exists y.((M\ y) \wedge \forall x.((H\ x) \to ((A\ y)\ x)))}$$

- incluindo no diagrama a segunda redução-β:

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{\text{todo homem}}{\begin{array}{c}S/(N\backslash S):\\ \lambda Q.\forall x.((H\ x)\to(Q\ x))\end{array}}D
\quad
\dfrac{\dfrac{\text{ama}}{\begin{array}{c}S:\\ ((A\ w)\ z)\end{array}}D}{\begin{array}{c}N\backslash S:\\ \lambda z.((A\ w)\ z)\end{array}}I\backslash^1
}{\begin{array}{l}S:(\lambda Q.\forall x.((H\ x)\to(Q\ x))\ \lambda z.((A\ w)\ z))\\ \quad =_{red.\beta}\ \forall x.((H\ x)\to(\lambda z.((A\ w)\ z)\ x))\\ \quad =_{red.\beta}\ \forall x.((H\ x)\to((A\ w)\ x))\end{array}}E/
}{S/N:\lambda w.\forall x.((H\ x)\to((A\ w)\ x))}I/^2
\quad
\dfrac{\text{uma mulher}}{\begin{array}{c}(S/N)\backslash S:\\ \lambda Q_1.\exists y.((M\ y)\wedge(Q_1\ y))\end{array}}D
}{\begin{array}{l}S:(\lambda Q_1.\exists y.((M\ y)\wedge(Q_1\ y))\ \lambda w.\forall x.((H\ x)\to((A\ w)\ x)))\\ \quad =_{red.\beta}\ \exists y.((M\ y)\wedge(\lambda w.\forall x.((H\ x)\to((A\ w)\ x))\ y))\\ \quad =_{red.\beta}\ \exists y.((M\ y)\wedge\forall x.((H\ x)\to((A\ y)\ x)))\end{array}}E\backslash
$$

4 Conclusão

- duas fórmulas diferentes:
  - $\forall x.((H\ x) \rightarrow \exists y.((M\ y) \wedge ((A\ y)\ x)))$
  - $\exists y.((M\ y) \wedge \forall x.((H\ x) \rightarrow ((A\ y)\ x)))$
- uma para cada interpretação
- ambas decorrentes unicamente dos procedimentos de análise

Referências

[1] Bob Carpenter. *Type-Logical Semantics.* The MIT Press, Cambridge, MA, 1997.