Reference, anaphora and deixis
in
predicate calculus and natural language

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1 Introduction

- Identification in structures for interpretation:
  - reference
  - anaphora
  - deixis
- Both in predicate calculus and natural language
- Nothing new, but some unexpressed options (explicitly, at least)
- "I reject the contention that an important theoretical difference exists between formal and natural languages." [8, p. 188]

2 First order language

2.1 Basic expressions

- Individual
  - constants: “$a, b, c, \ldots, t, a_1, b_1, \ldots, t_1, a_2, \ldots$” [11, p. 70]
  - variables: “$u, v, x, y, z, u_1, v_1, \ldots, z_1, u_2, \ldots$” [11, p. 71]
- Terms
  - individual constants and variables [11, p. 72]
- Predicate
  - constants: “$A, B, C, \ldots, T, A_1, B_1, \ldots, T_1, A_2, \ldots$” [11, p. 74]

2.2 Formation rules

- Atomic formulae

(1) If $\mathbf{P}$ is an n-ary predicate constant, and $\mathbf{t_1}, \ldots, \mathbf{t_n}$ are terms, then $\mathbf{Pt_1} \ldots \mathbf{t_n}$ is a formula [11, p. 79]

- Molecular formulae

(2) If α and β are formulae, so are $\neg\alpha$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \rightarrow \beta$, and $\alpha \leftrightarrow \beta$ [11, p. 86]

- General formulae

(3) If $\mathbf{x}$ is an individual variable, and α is a formula, so $\forall\mathbf{x}\alpha$ and $\exists\mathbf{x}\alpha$ are formulae too [11, p. 94]

2.3 Structure

- $\mathcal{L} = \{a, b, c, d, R, M, C, H, G\}$ [11, p. 159]
- Structure = Discourse Domain & Interpretation Function
- $\mathfrak{A} = \langle \mathcal{A}, [\![\ldots]\!]^{\mathfrak{A},\mathfrak{g}} \rangle$
- $\mathcal{A} = \{$Ana Maria, Conrado, Dorothee, Elisa, Felipe, Fernando, Gabriela, Juliana, Leila, Mariana, Sebastian, Veronika$\}$ (Mortari's nieces and nephews) [11, p. 159]
- $\mathfrak{g}$: arbitrary value assignment function; $\mathfrak{g}^{[\mathbf{x}/\mathfrak{i}]}$: any function $\mathfrak{g}$ in which the value assigned to $\mathbf{x}$ is $\mathfrak{i}$ [3, ps. 60–62]

- $[\![\ldots]\!]^{\mathfrak{A},\mathfrak{g}} =$

$$\left[\begin{array}{l} a \to \text{Ana Maria} \\ b \to \text{Juliana} \\ c \to \text{Sebastian} \\ d \to \text{Felipe} \\ R \to \{\text{Conrado}, \text{Felipe}, \text{Fernando}, \text{Sebastian}\} \\ M \to \{\text{Ana Maria}, \text{Dorothee}, \text{Juliana}, \text{Elisa}, \text{Leila}, \\ \quad\quad \text{Gabriela}, \text{Mariana}, \text{Veronika}\} \\ C \to \{\text{Leila}, \text{Gabriela}, \text{Mariana}\} \\ H \to \{\text{Veronika}, \text{Dorothee}, \text{Sebastian}\} \end{array}\right]$$

2.4 Interpretation

- Atomic formulae ([11, ps. 165–167] & [3, p. 73]):
  $[\![\mathbf{Pt_1}, \ldots, \mathbf{t_n}]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ iff $\langle [\![\mathbf{t_1}]\!]^{\mathsf{S},\mathfrak{g}}, \ldots, [\![\mathbf{t_n}]\!]^{\mathsf{S},\mathfrak{g}} \rangle \in [\![\mathbf{P}]\!]^{\mathsf{S},\mathfrak{g}}$
- Molecular formulae ([11, ps. 167–168] & [3, p. 73]):
  $[\![\mathbf{C}\ \alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T} | [\![\alpha\ \mathbf{C}\ \beta]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ iff
  - $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{F}$, when $\mathbf{C} = \neg$
  - $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ and $[\![\beta]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$, when $\mathbf{C} = \wedge$
  - $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ or $[\![\beta]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$, when $\mathbf{C} = \vee$
  - $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ or $[\![\beta]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{F}$, when $\mathbf{C} =\rightarrow$
  - $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}} = [\![\beta]\!]^{\mathsf{S},\mathfrak{g}}$, when $\mathbf{C} =\leftrightarrow$
- General formulae ([11, ps. 168–172] & [3, p. 74]):
  - $[\![\forall \mathbf{x} \alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ iff, for every $\mathfrak{i} \in \mathcal{A}$, $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}^{[\mathtt{x}/\mathtt{i}]}} = \mathtt{T}$
  - $[\![\exists \mathbf{x} \alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ iff, for some $\mathfrak{i} \in \mathcal{A}$, $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}^{[\mathtt{x}/\mathtt{i}]}} = \mathtt{T}$

2.5 Identifications

- Reference: $[\![\ldots]\!]^{\mathsf{S}}$

$$[\![Mb]\!]^{\mathfrak{A},\mathfrak{g}} = \mathtt{T} \quad \text{iff} \quad \underbrace{[\![b]\!]^{\mathfrak{A},\mathfrak{g}}}_{\text{Juliana}} \quad \in \quad \underbrace{[\![M]\!]^{\mathfrak{A},\mathfrak{g}}}_{\text{the girls}}$$

- Deixis: arbitrary $\mathfrak{g}$

$$[\![Mx]\!]^{\mathfrak{A},\mathfrak{g}} = \mathtt{T} \quad \text{iff} \quad \underbrace{[\![x]\!]^{\mathfrak{A},\mathfrak{g}}}_{\mathfrak{g}(\mathbf{x})} \quad \in \quad \underbrace{[\![M]\!]^{\mathfrak{A},\mathfrak{g}}}_{\text{the girls}}$$

- Anaphora: fixed $\mathfrak{g}^{[\mathbf{x}/\mathtt{i}]}$

$$[\![\exists x(Rx \wedge Hx)]\!]^{\mathfrak{A},\mathfrak{g}} = \mathtt{T} \text{ iff, for some } \mathtt{i},$$

$$\underbrace{\underbrace{[\![x]\!]^{\mathfrak{A},\mathfrak{g}^{[x/\mathtt{i}]}}}_{\underbrace{\mathfrak{g}^{[x/\mathtt{i}]}(x)}_{\text{Sebastian}}} \in \underbrace{[\![R]\!]^{\mathfrak{A},\mathfrak{g}^{[x/\mathtt{i}]}}}_{\text{the boys}} \cap \underbrace{[\![H]\!]^{\mathfrak{A},\mathfrak{g}^{[x/\mathtt{i}]}}}_{\text{lives in Heidelberg}}}$$

3 Natural language

3.1 Basic expressions

- Individual
  - constants: Pedro, Maria, Antônio, Sócrates, ...
  - variables: ele, ela, ...
- Predicate
  - constants: é jogador de futebol, é filósofo

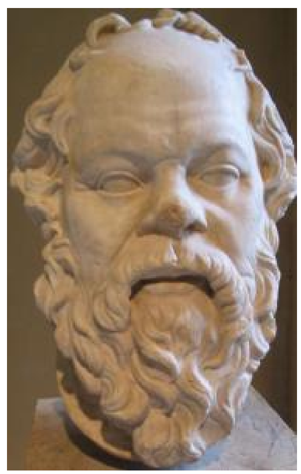
3.2 Formation rules

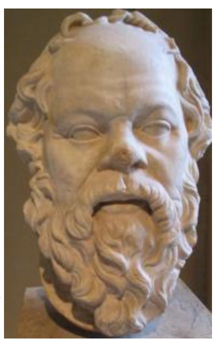
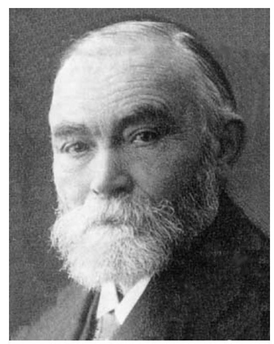
(1) If **P** is a constant predicate and **t** is a term, then ⌜**t P**⌝ and ⌜**t** não **P**⌝ are sentences

(2) If α and β are sentences, so is ⌜α e β⌝

3.3 Structures

- $[\![\text{Sócrates}]\!]^{\mathfrak{I},\mathfrak{g}} =$ 

- $[\![\text{Sócrates}]\!]^{\mathfrak{F},\mathfrak{g}} =$ 

- $[\![\text{é jogador de futebol}]\!]^{\mathfrak{I},\mathfrak{g}} = [\![\text{é jogador de futebol}]\!]^{\mathfrak{F},\mathfrak{g}} =$
  $\{$, , $, \ldots\}$
- $[\![\text{é filósofo}]\!]^{\mathfrak{F},\mathfrak{g}} = [\![\text{é filósofo}]\!]^{\mathfrak{I},\mathfrak{g}} =$
  $\{$, , $, \ldots\}$

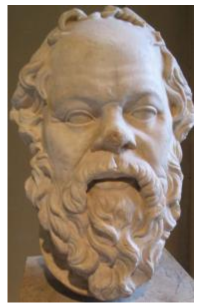
3.4 Interpretation

- $[\![\mathbf{t}\ \mathbf{P}]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ iff $[\![\mathbf{t}]\!]^{\mathsf{S},\mathfrak{g}} \in [\![\mathbf{P}]\!]^{\mathsf{S},\mathfrak{g}}$
- $[\![\mathbf{t}\ \text{não}\ \mathbf{P}]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ iff $[\![\mathbf{t}\ \mathbf{P}]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{F}$
- $[\![\alpha\ \text{e}\ \beta]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ iff $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ and $[\![\beta]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$

4 Reference problem

- $[\![\text{Sócrates é filósofo e Sócrates é jogador de futebol}]\!]^{\mathfrak{F},\mathfrak{g}} = \mathtt{F}$
- $[\![\text{Sócrates é filósofo e Sócrates é jogador de futebol}]\!]^{\mathfrak{I},\mathfrak{g}} = \mathtt{F}$
- Since interpretations are functions, there is no $\mathfrak{H}$ such that

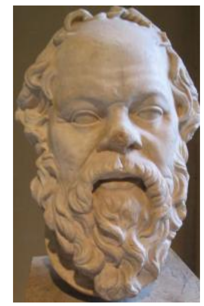
$[\![\text{Sócrates}]\!]^{\mathfrak{H},\mathfrak{g}} =$ 

$[\![\text{Sócrates}]\!]^{\mathfrak{H},\mathfrak{g}} =$ 

- But intuitively "Sócrates é filósofo e Sócrates é jogador de futebol" is a true sentence

4.1 Tradicional solution

- Different names with (accidentally) same phonetic realization

$[\![\text{Sócrates}_2]\!]^{\mathfrak{H},\mathfrak{g}} =$  $[\![\text{Sócrates}_1]\!]^{\mathfrak{H},\mathfrak{g}} =$ 

- The sentence "Sócrates é filósofo e Sócrates é jogador de futebol" is interpreted as

$$[\![\text{Sócrates}_1 \text{ é filósofo e Sócrates}_2 \text{ é jogador de futebol}]\!]^{\mathfrak{H},\mathfrak{g}} = \mathtt{T}$$

4.2 Modern solution

$\Rightarrow$ $[\![\alpha \text{ e } \beta]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ iff $[\![\alpha]\!]^{\mathsf{S},\mathfrak{g}} = \mathtt{T}$ and $[\![\beta]\!]^{\mathsf{T},\mathfrak{g}} = \mathtt{T}$

- $[\![$Sócrates é filósofo e Sócrates é jogador de futebol$]\!]^{\mathfrak{F},\mathfrak{g}} = \mathtt{T}$
  - $[\![$Sócrates é filósofo$]\!]^{\mathfrak{F},\mathfrak{g}} = \mathtt{T}$
  - $[\![$Sócrates é jogador de futebol$]\!]^{\mathfrak{I},\mathfrak{g}} = \mathtt{T}$
- $[\![$Sócrates é filósofo e Sócrates é jogador de futebol$]\!]^{\mathfrak{I},\mathfrak{g}} = \mathtt{F}$
  - $[\![$Sócrates é filósofo$]\!]^{\mathfrak{I},\mathfrak{g}} = \mathtt{F}$
  - $[\![$Sócrates é jogador de futebol$]\!]^{\mathfrak{I},\mathfrak{g}} = \mathtt{T}$

  (There is no restriction for $\mathsf{S} = \mathsf{T}$)
- Dynamic semantics [4, 5, 12, 1, 13, 2]

5 Deixis & anaphora

"Sócrates é jogador de futebol. Ele é filósofo."

- Sócrates$_j$ é jogador de futebol. Ele$_j$ é filósofo.
  - anaphora: $[\![\text{ele}]\!]^{\mathfrak{I},\mathfrak{g}} = [\![\text{Sócrates}]\!]^{\mathfrak{I},\mathfrak{g}}$
  - only when $\mathfrak{g}^{[\text{ele}/\mathfrak{i}]}$, and $\mathfrak{i} =$ 

- Sócrates$_j$ é jogador de futebol. Ele$_f$ é filósofo.
  - deixis: $[\![\text{ele}]\!]^{\mathfrak{I},\mathfrak{g}}$, for any arbitrary $\mathfrak{g}$

6 Deixis versus anaphora

- Todo jogador de futebol que é filósofo morre pela bebida.
  - ok: $\forall x((Sx \wedge Px) \rightarrow Dx)$
  - no: $\forall x((Sx \wedge \exists yPy) \rightarrow Dx)$
- ?Todo jogador de futebol que ele é filósofo morre pela bebida.
  - not standard: $\forall x((Sx \wedge Px) \rightarrow Dx)$
  - no: $\forall x((Sx \wedge \exists yPy) \rightarrow Dx)$
- Todo jogador de futebol tal que ele é filósofo morre pela bebida.
  - ok: $\forall x((Sx \wedge Px) \rightarrow Dx)$
  - no: $\forall x((Sx \wedge \exists yPy) \rightarrow Dx)$

7 Token-reflexivity

- No need for ostention or salience
- First person pronoun "eu" (I): constant or variable?
  - Acording to Kaplan [6, p. 542], it is a constant
  - But there is a long tradition in Linguistics classifying it as a pronoun, which suggests that it is a variable
  - Is there any linguistic evidence for the choice?

8 Conclusions

- No need for a special pragmatic language [10, 7]: the choosing of appropriate structure and attribution are already pragmatics
- Reference by interpretation relative to structure ($[\![\ldots]\!]^{\mathfrak{A}}$)
- Deixis by arbitrary assignment ($\mathfrak{g}$)
- Anaphora by bound assignment ($\mathfrak{g}^{\mathrm{x}/\mathrm{i}}$)
- Indexical: constant or variable?

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