A KNAPSACK PROBLEM APPROACH FOR OPTIMAL ALLOCATION OF MAINTENANCE RESOURCES ON ELECTRIC POWER DISTRIBUTION NETWORKS

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Keywords: Reliability, Maintenance, Knapsack problem, Dynamic programming.

Abstract: The definitions of optimal preventive and corrective maintenance of electric power distribution networks can be seen as a special case of a knapsack problem. This paper proposes a dynamic programming approach to deal with this problem. The approach is developed for one or more years of planning horizon. Case studies compare the optimal dynamic programming approach with an heuristic method.

1 INTRODUCTION

The optimal allocation of maintenance resources on power distribution network must define the best compromise between investment and system reliability. Previous approaches relies on heuristic method to address this non-linear multicriteria optimisation problem.

The problem can be viewed as a special case of multidimensional knapsack problem (Martello and Toth, 1990). This paper relies on this interpretation in order to develop an exact optimisation procedure based on dynamic programming (DP).

Case studies compare the proposed approach with a previous heuristic algorithm developed to deal with the problem. Discussion of the case studies gives some insights into future developments of these ideas.

2 MAINTENANCE ON ELECTRIC POWER DISTRIBUTION NETWORKS

Electric power distribution networks are composed by electric circuits that transport electric energy from substations to the customers. The system reliability is measured by indices such as *System Average Interruption Frequency Index* (SAIFI) and *System Average Interruption Duration Index* (SAIDI) (Brown, 2002) which determine the network quality. After occurrence of a failure of one equipment, maintenance actions are employed in order to repair or replace this equipment. Since failures deteriorates the reliability indices, actions must be applied (Endrenyi and Anders, 2006; Bertling et al., 2007).

All preventive or corrective actions have a cost, therefore the objective of the optimisation problem is to minimize the cost of maintenance regarding safety values for the system reliability.

This work considers a radial network proposed by (Sittithumwat et al., 2004). This network is divided in sections defined by protection equipments such as break-fuses, switch-fuses and reclosers which seek to avoid the failure throughout the system distribution network. Besides, all equipments which compose the system are described into the optimisation model proposed.

2.1 Optimisation Model

The optimisation model proposed (Reis, 2007) presents an objective function to minimize the cost of preventive and corrective maintenance action with a reliability constraints (SAIFI). The SAIFI is calculated by following equation:

$$SAIFI^t = \frac{\sum_{s\in S}\lambda_s^t N_s}{NT}, \tag{1}$$

where S is the set of all sections, λ_s^t is the failure rate of section s in the period t, N_s is the number of customers in section s and NT is the total number of customers in the network.

Tadeu Bacalhau E., Luiz Usberti F., Lyra Filho C. and Cavellucci C. (2012).
A KNAPSACK PROBLEM APPROACH FOR OPTIMAL ALLOCATION OF MAINTENANCE RESOURCES ON ELECTRIC POWER DISTRIBUTION NETWORKS.
In *Proceedings of the 1st International Conference on Operations Research and Enterprise Systems*, pages 462-466
DOI: 10.5220/0003865504620466

The failure rate of section s in period t, λ_s^t, can be calculated by equations:

$$\sum_{n \in N_{k_e}} x_{en}^t = 1 \quad , \tag{2}$$

$$\lambda_e^t = \lambda_e^{(t-1)} \sum_{n \in N_{k_e}} \delta_{k_e n} x_{en}^t \tag{3}$$

and

$$\lambda_s^t = \lambda_s + \sum_{e \in E_s} \lambda_e^t \quad , \tag{4}$$

where λ_e^t is the failure rate for equipment e in the period t, N_{k_e} is the set of all preventive maintenance actions, $\delta_{k_e n}$ is the failure rate multiplier for equipment k_e for action level n and x_{en}^t is a boolean decision variable denoting whether the equipment e received ($x_{en}^t = 1$) or not ($x_{en}^t = 0$) maintenance level n in period t.

Finally, the optimisation model is described as:

$$\min_{x_{en}^t} \quad \sum_{t=1}^{HP} \left\{ \sum_{e \in E} \left[\sum_{n \in N_{k_e}} (p_{k_e n} x_{en}^t) + \lambda_e^t c_{k_e} \right] \times \alpha_t \right\}$$
$$s.t. \quad SAIFI^t \leq SAIFI_{perm} \qquad \forall t = 1, \ldots, HP,$$

where E is a set that contains all the equipment which can receive preventive maintenance, $SAIFI_{perm}$ is the maximum permitted value for SAIFI, $p_{k_e n}$ is the cost for action preventive level n for equipment k_e, c_{k_e} is the cost for action corrective level for equipment k_e and α_t is a parameter which is related to each period.

3 KNAPSACK PROBLEM

Since the problems were developed for more than one year of planning horizon we are going to consider the multidimensional knapsack problem (MKP) (Martello and Toth, 1990). The MKP could be defined as a set $N = \{1, \ldots, n\}$ of items that should be packed in a set $M = \{1, \ldots, m\}$ of knapsacks with given capacities, $b_{0,i}$ $i \in M$. Associated with every item $j \in N$ there is a value c_j and a weight a_{ij}, which is the amount of resource used by the item j in the ith knapsack. The goal is to find a subset of the items that yield the maximum value subject to the capacity constraints of the knapsacks. Therefore, a formulation for MKP can be defined as:

$$\begin{aligned} F_n(b) = \max \quad & \sum_{j=1}^{n} c_j x_j, \\ s.\,to: \quad & \sum_{j=1}^{n} a_{ij} x_j \leq b_{0,i} \quad i \in M \\ & x_j \in \{0,1\}, \quad j \in N, \end{aligned} \tag{5}$$

where a_{ij}, c_j, $b_{0,i} \geq 0$ for $i = 1, \ldots, m$ and $j = 1, \ldots, n$.

3.1 Knapsack Problem on Dynamic Programming

A MKP can be solved via dynamic programming, that chooses items with highest costs c_j and with volumes v_j that do not exceed the knapsack capacity V (Puchinger et al., 2010). The following equation shows the dynamic programing for the binary knapsack problem:

$$\begin{aligned} \textit{Get} \quad & F_n(V_0) \\ \textit{Where} \quad & F_k(V) = \max\{F_{k-1}(V), F_{k-1}(V - v_k) + c_k\} \\ \textit{With} \quad & F_0(V) = 0 \ \forall v \end{aligned} \tag{6}$$

To determine the optimal solution we should create an indicator I_k that is equal 0 if $F_k(V) = F_{k-1}(V)$ and 1 otherwise. After that we analyze all indicators from I_n up to I_1. If the indicator $I_k = 0$ then $x_k^* = 0$, else $x_k^* = 1$.

4 ADAPTED KNAPSACK PROBLEM

As from the optimisation model and the presented knapsack problem we can formulate an adapted model for the maintenance problem. First, we are going to present the knapsack problem on dynamic programming for one year of planning horizon. In this model we should define the parameter $SAIFI_{perm}$ as the knapsack capacity V and define $\delta_{k_e}^{sm}$ and $\delta_{k_e}^{cm}$ as the failure rate multipliers for equipment k_e in the absence and occurrence of preventive maintenance respectively.

$$\begin{aligned} \textit{Get} \quad & F_n(V_0) \\ \textit{Where} \quad & F_{k_e}(V) = \min\Big\{ F_{k_e-1}(V - v_{k_e}^{cm}) + p_{k_e} + \\ & \Big((\lambda_{k_e-1} \delta_{k_e}^{cm}) c_{k_e} \Big), \\ & F_{k_e-1}(V - v_{k_e}^{sm}) + \Big((\lambda_{k_e-1} \delta_{k_e}^{sm}) c_{k_e} \Big) \Big\} \\ \textit{With} \quad & F_0(V) = 0 \ \forall \ V, \end{aligned} \tag{7}$$

where p_{k_e} is the maintenance preventive cost for equipment k_e, c_{k_e} is the maintenance corrective cost for equipment k_e, $v_{k_e}^{cm}$ is the reliability volume calculated for equipment k_e which received preventive maintenance and $v^{sm}_{k_e}$ is the reliability volume for equipment k_e which not received preventive maintenance.

The reliability volumes can be defined as the following equations:

$$\begin{array}{l} v^{cm}_{k_e} = \frac{(\lambda_{k_e-1}\delta^{cm}_{k_e})N_s}{NT}, \\ \text{if maintenance is performed, and} \\ \\ v^{sm}_{k_e} = \frac{(\lambda_{k_e-1}\delta^{sm}_{k_e})N_s}{NT}, \\ \text{if maintenance is not performed,} \end{array} \tag{8}$$

where N_s is the number of customers in section s.

The problem was divided in two subproblems describing the possible choices of maintenance action. It means that we can write the number of subproblem such as 2^{HP} where HP is the number of years of planning horizon. Besides, the number of knapsacks is going to be exactly the number of years of this planning. Since that the failure rates are dependent year to year, we must have a different knapsack for each year. Therefore, to develop a adapted model for more than one year we must apply the multidimensional knapsack problem to the problem. Likewise, we can denote this idea for two years of planning horizon.

$$\begin{array}{rl} \textit{Get} & F_n(V^1_0, V^2_0) \\ \textit{Where} & F_{k_e}(V^1, V^2) = \\ & \min\Big\{F_{k_e-1}(V^1 - v^{cm}_{1,k_e}, V^2 - v^{cm}_{2,k_e}) \\ & +(2p_{k_e}) + \big(((\lambda_{k_e-1}\delta^{cm}_{k_e}) + (\lambda_{k_e-1}\delta^{cm}_{k_e})^2)c_{k_e}\big), \\ & F_{k_e-1}(V^1 - v^{cm}_{1,k_e}, V^2 - v^{sm}_{2,k_e}) \\ & +(p_{k_e}) + \big(((\lambda_{k_e-1}\delta^{cm}_{k_e}) + (\lambda_{k_e-1}\delta^{cm}_{k_e}\delta^{sm}_{k_e}))c_{k_e}\big), \\ & F_{k_e-1}(V^1 - v^{sm}_{1,k_e}, V^2 - v^{cm}_{2,k_e}) \\ & +(p_{k_e}) + \big(((\lambda_{k_e-1}\delta^{sm}_{k_e}) + (\lambda_{k_e-1}\delta^{sm}_{k_e}\delta^{cm}_{k_e}))c_{k_e}\big), \\ & F_{k_e-1}(V^1 - v^{sm}_{1,k_e}, V^2 - v^{sm}_{2,k_e}) \\ & + \big(((\lambda_{k_e-1}\delta^{sm}_{k_e}) + (\lambda_{k_e-1}\delta^{sm}_{k_e})^2)c_{k_e}\big)\Big\}, \\ \textit{With} & F_0(V^1, V^2) = 0 \; \forall \, V^1, V^2, \end{array} \tag{9}$$

where v^{cm}_{1,k_e} is the volume of reliability for equipment k_e which received preventive maintenance at year one, v^{cm}_{2,k_e} is the volume of reliability for equipment k_e which received preventive maintenance at year two, v^{sm}_{1,k_e} is the volume of reliability for equipment k_e which not received preventive maintenance at year one, v^{sm}_{2,k_e} is the volume of reliability for equipment k_e which not received preventive maintenance at year two, V^1 is knapsack for first year and V^2 is knapsack for second year.

However, is important to note that the volume of reliability calculated to the second year depends on the choice taken on previous year and the knapsack of the second year should tolerate both volume of reliability calculated for each year. Thus, the volume of reliability in this case must be calculated as follows:

$$\begin{array}{l} v^{cm}_{2,k_e} = v^{sm}_{1,k_e}\delta^{cm}_{k_e} = \left(\frac{(\lambda_{k_e-1}(\delta^{sm}_{k_e}\delta^{cm}_{k_e}))\ N_s}{NT}\right), \\ \text{if not realized maintenance at year one or} \\ v^{cm}_{2,k_e} = v^{cm}_{1,k_e}\delta^{cm}_{k_e} = \left(\frac{(\lambda_{k_e-1}(\delta^{cm}_{k_e}))^2\ N_s}{NT}\right), \\ \text{if realized maintenance at year one and} \\ v^{sm}_{2,k_e} = v^{sm}_{1,k_e}\delta^{sm}_{k_e} = \left(\frac{(\lambda_{k_e-1}(\delta^{sm}_{k_e}))^2\ N_s}{NT}\right), \\ \text{if not realized maintenance at year one or} \\ v^{sm}_{2,k_e} = v^{cm}_{1,k_e}\delta^{sm}_{k_e} = \left(\frac{(\lambda_{k_e-1}(\delta^{cm}_{k_e}\delta^{sm}_{k_e}))\ N_s}{NT}\right), \\ \text{if realized maintenance at year one.} \end{array} \tag{10}$$

Finally, to more years of planning horizon we proceed in the same way. We increase a knapsack for each year added and the volume of equipments continues being calculated depending of choices taken in previous years.

5 CASE STUDIES

These case studies rely on a comparison between the dynamic programming approach (DPA) and a heuristic method previously developed. This heuristic method is a state space search which consist in combine the depth search with simulated annealing (DSA) (Bacalhau, 2009).

We have created three instances for these case studies. All instances were executed for one year of planning horizon and for each instance five values of reliability constraints were chosen through the following equation:

$$SAIFI_{\beta} = SAIFI_{min} + (SAIFI_{max} - SAIFI_{min}) \times \beta \tag{11}$$

where $SAIFI_{min}$ is minimum value that can be calculated for reliability indices, $SAIFI_{max}$ is maximum value that can be calculated for reliability indices and β is $0.2,\ 0.4,\ 0.6,\ 0.8$ and 1.0.

An instance with 30 equipments was tested, in the attempt to show the efficiency of the DPA method with a limited number of options of optimisation. Table 1 illustrates the optimal results obtained for this instance:

Table 1: Results - Instance with 30 equipments.

	DPA		**DSA**		
SAIFI	**Cost** (x 1000)	**Time** (s)	**Cost** (x 1000)	**Time** (s)	**Profit** (%)
0.3476	**10.076**	**0.156**	11.345	1.201	11.14
0.3819	**6.821**	0.702	7.114	**0.296**	4.11
0.4163	**4.785**	1.622	4.815	**0.218**	0.61
0.4506	**3.369**	2.854	**3.369**	**0.140**	0
0.4849	**2.414**	4.633	**2.414**	**0.171**	0

The best results are described by numbers in bold. The DPA method performed better when the reliability constraints were tighter.

After that, we have increased the number of equipments, trying to show the robustness of the DPA method for cases where the optimisation procedure is more complex. We have created an instance with 300 equipments. The Table 2 illustrates the optimal results obtained for this instance:

Table 2: Results - Instance with 300 equipments.

	DPA		DSA		
SAIFI	Cost (x 1000)	Time (s)	Cost (x 1000)	Time (s)	Profit (%)
2.9757	**80.498**	**11.528**	107.627	290.825	25.20
3.2543	**53.466**	**52.244**	54.908	250.846	2.61
3.5330	**33.562**	**124.738**	35.879	1295.747	6.45
3.8117	**24.962**	**242.035**	26.664	1681.954	6.38
4.4068	**19.169**	398.083	22.214	**231.554**	13.70

The DPA method performed better in all cases and the computational time is better in all except one case. These results show the efficiency of the approach in cases with a larger number of equipments.

Following this idea, we have created an instance with 400 equipments. The idea was to show the growth of computational time. In the Table 3 we can see the results for this instance.

Table 3: Results - Instance with 400 equipments.

	DPA		DSA		
SAIFI	Cost (x 1000)	Time (s)	Cost (x 1000)	Time (s)	Profit (%)
3.9625	**106.270**	**25.755**	142.048	7258.345	25.18
4.3336	**70.830**	**111.821**	72.987	705.412	2.95
4.7046	**44.351**	**255.748**	47.611	2240.451	6.84
5.0757	**33.111**	**446.880**	38.455	1290.745	13.89
5.4468	**24.437**	**712.909**	28.601	5070.507	14.55

The performance obtained in this case is similar with the previous results, but the computational time obtained by DPA method is much lower.

6 DISCUSSION

We have done case studies in order to make a comparison between dynamic programming approach and a heuristic method.

Three examples of networks were executed using the radial network mentioned. One of them was created with a reduced number of equipments and two of them with a large number of equipments.

The DPA method got a cost profit in all instance where the reliability constraints were tighter, taking up to 25% profit in large instances cases.

In instances with a small number of equipments, the heuristic method got the best results for computational time and obtained the same results when the reliability constraints were looser. However, when the number of equipments increased the computational time of the heuristic method was much greater than the DPA method.

For two-years of planning horizon the results were promising as well, however the algorithm has increased its computational resources since the *bellman's principle* leads to a combinatorial explosion of the problem (Bellman, 2003). For this reason, we have done some approximations of parameters into the procedure. In some cases, these approximations lead the algorithm to produce a non-optimal solution.

However, some alternatives could be studied to apply this approach, trying to reduce the complexity and the computational time for this problem. With these approximations, the dynamic programming still provide good quality solutions, although it may lose the optimality guarantee.

7 CONCLUSIONS

We have developed a knapsack problem approach using dynamic programming for the problem of preventive maintenance on power distribution networks. The approach was studied for one and two years of planning horizon and its optimisation model for knapsack problem adapted was presented.

Cases studies were conducted followed by a discussion about the results obtained for three examples of radial networks developed.

The results obtained by dynamic programming approach were promising in relation to a presented heuristic method. In all cases the approach had a better performance, but the best results of cost and computational time were obtained when the reliability constraints were tighter and the number of equipments larger.

A discussion was produced from the case studies. The results were analysed proving the robustness of the approach. Besides, results for more than one year of planning horizon were discussed, highlighting some alternatives for the problem complexity.

ACKNOWLEDGEMENTS

This work has been supporting by CNPq and CAPES.

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