

Tobias Bleninger

# **MECÂNICA DOS FLUIDOS AMBIENTAL I**

## Balço de massa

$$0 = \frac{\partial}{\partial t} \int_{V_c} \rho dV + \int_{S_c} \rho (\mathbf{v} \cdot \mathbf{n}) dS.$$

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0,$$

## Balço de massa de um soluto

$$\frac{\partial}{\partial t} \int_{V_c} C_A \rho dV + \int_{S_c} C_A \rho (\mathbf{v} \cdot \mathbf{n}) dS = - \int_{S_c} (\mathbf{j} \cdot \mathbf{n}) dS.$$

$$\frac{\partial C_A}{\partial t} + (\mathbf{v} \cdot \nabla) C_A = D_{AB} \nabla^2 C_A.$$

## Balço de quantidade de movimento

$$\frac{\partial}{\partial t} \int_{V_c} \mathbf{v} \rho dV + \int_{S_c} \mathbf{v} \rho (\mathbf{v} \cdot \mathbf{n}) dS = \int_{V_c} \rho \mathbf{g} dV + \int_{S_c} (\mathbf{T} \cdot \mathbf{n}) dS.$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla (p + \rho g h) + \mu \nabla^2 \mathbf{v}$$

## Balço de energia

$$\rho \frac{\partial e}{\partial t} + \rho (\mathbf{v} \cdot \nabla) e = \nabla \cdot (\mathbf{T} \cdot \mathbf{v}) + \nabla \cdot (\rho c_p \alpha \nabla T)$$

$$\frac{\partial}{\partial t} \int_{V_c} e \rho dV + \int_{S_c} e \rho (\mathbf{v} \cdot \mathbf{n}) dS = - \int_{S_c} (\mathbf{q} \cdot \mathbf{n}) dS + \int_{S_c} [(\mathbf{T} \cdot \mathbf{v}) \cdot \mathbf{n}] dS$$

## Forma geral

$$\frac{\partial \eta}{\partial t} + (\mathbf{v} \cdot \nabla) \eta - K \nabla^2 \eta = f(x, y, z, t)$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T = \alpha \nabla^2 T,$$

# Equação de difusão

Concentração:

$$C = \frac{M_{\text{substância}}}{V_{\text{total}}}$$

Fluxo de massa:

$$j_x = -D \frac{\partial C}{\partial x}$$

D: difusividade molecular de A em B

Compare:

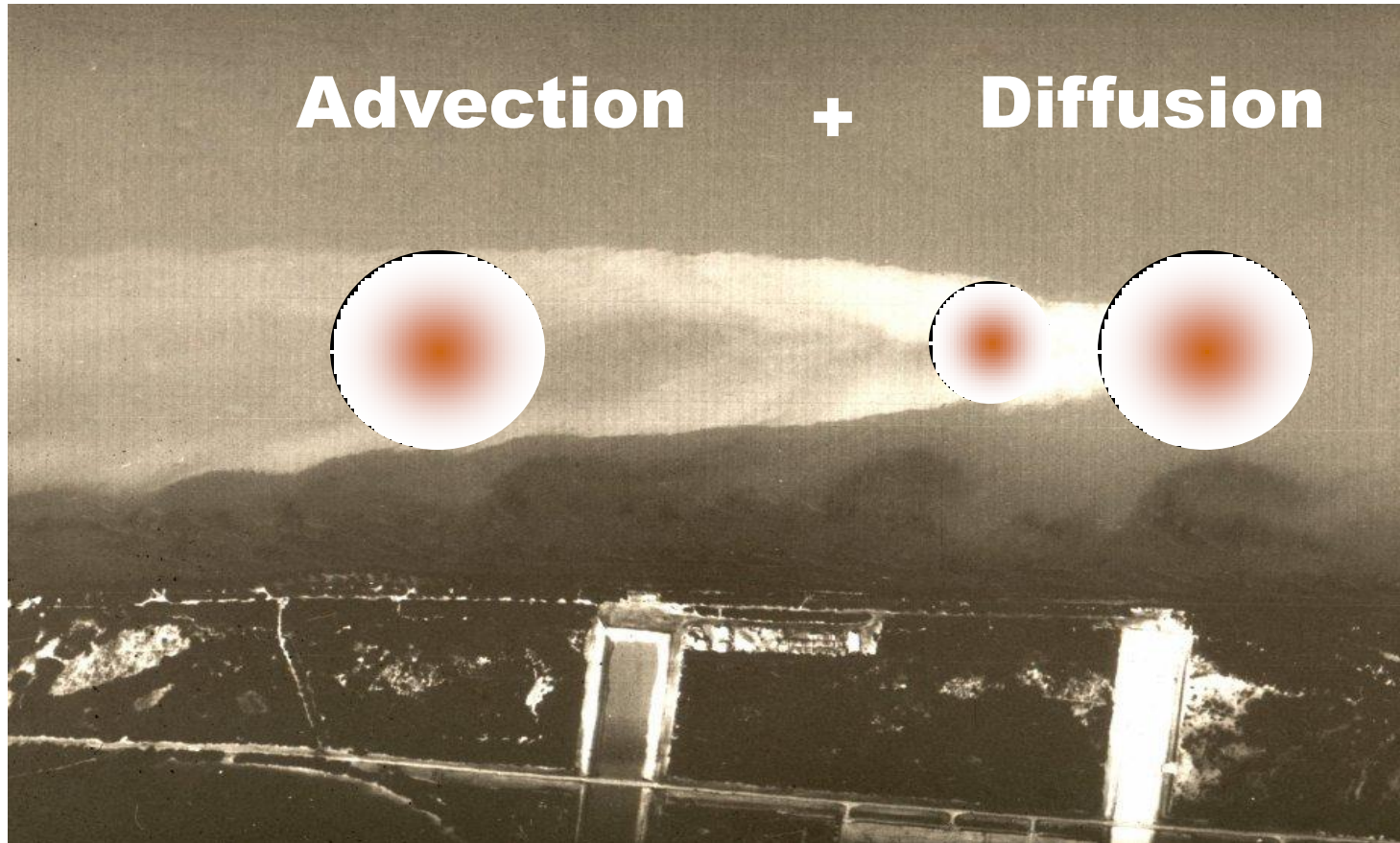
- Fluxo de calor:  $q_x = -\lambda \frac{\partial T}{\partial x}$

- Fluxo de quantidade de movimento (impulso):

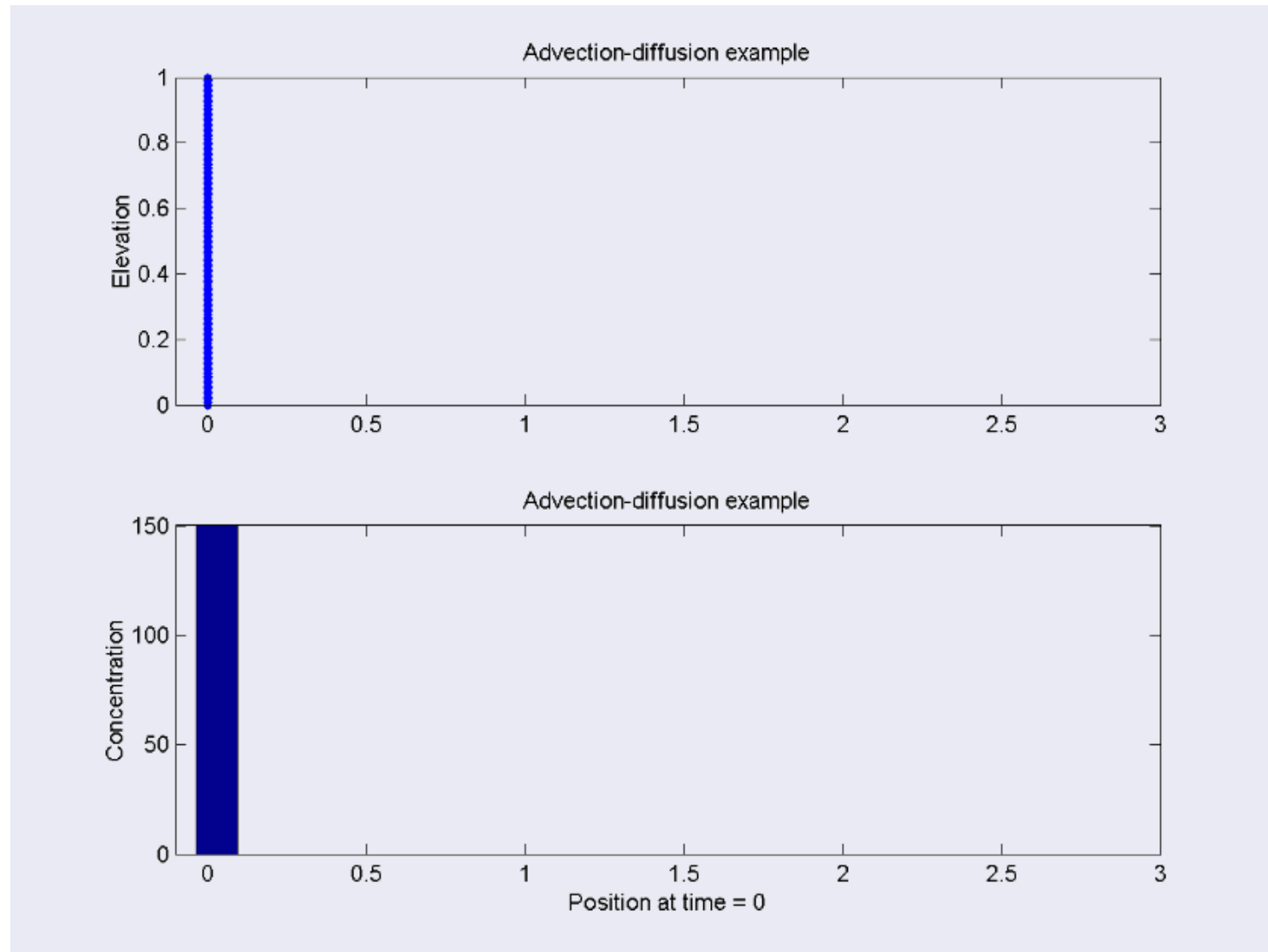
$$\tau = \mu \frac{dV}{dy} = \rho \nu \frac{dV}{dy} = \nu \frac{d\rho V}{dy}$$

**Impulso**

# Diffusion and Advection

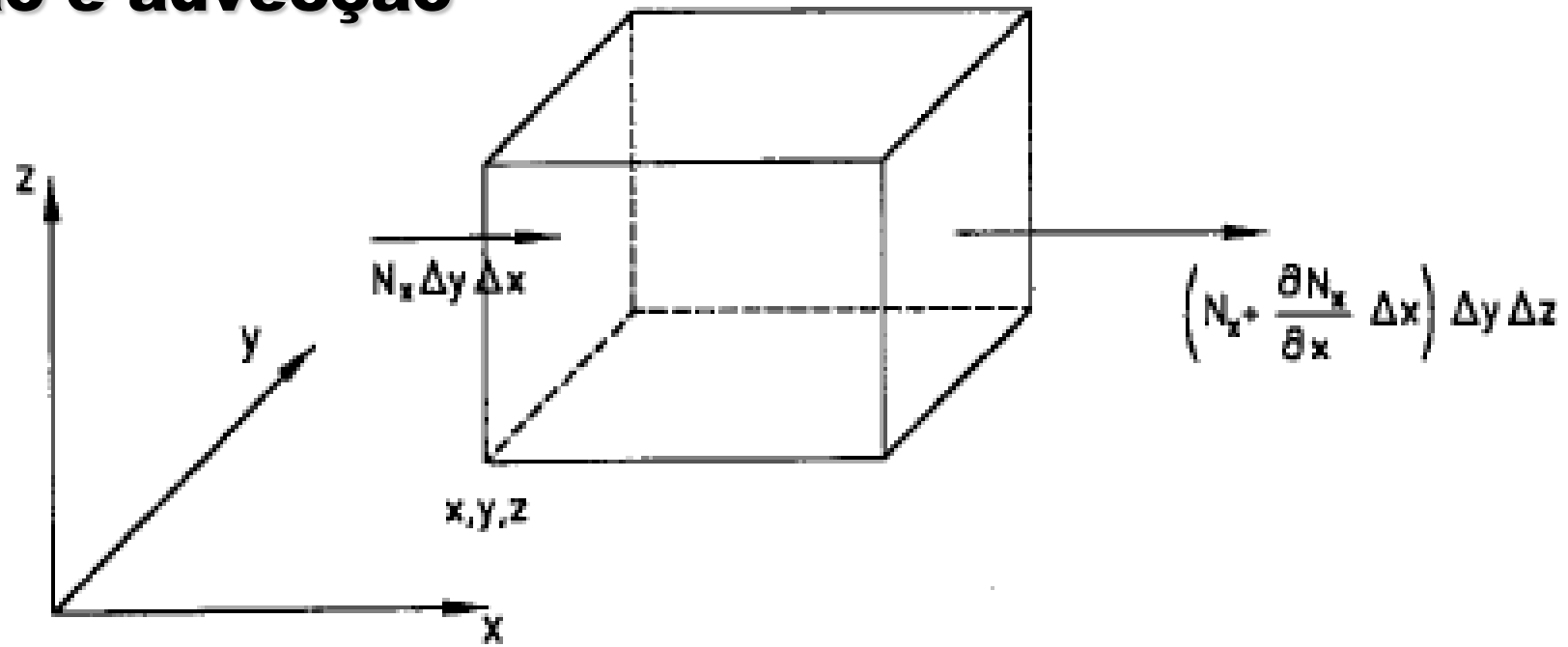


# Solução ilustrativa



Fonte: Socolofsky and Jirka, 2005

# Difusão e advecção



$$\left[ \begin{array}{c} \text{net flux of mass} \\ \text{(In - Out)} \end{array} \right] + \left[ \begin{array}{c} \text{Rate of production/} \\ \text{decay of mass} \\ \text{(chemical, physical,} \\ \text{biological...)} \end{array} \right] = \left[ \begin{array}{c} \text{Rate of mass} \\ \text{accumulation} \\ \text{within element} \end{array} \right]$$

$$-\frac{\partial N_x}{\partial x} \Delta x \Delta y \Delta z - \frac{\partial N_y}{\partial y} \Delta x \Delta y \Delta z - \frac{\partial N_z}{\partial z} \Delta x \Delta y \Delta z + R \Delta x \Delta y \Delta z = \frac{\partial c}{\partial t} \Delta x \Delta y \Delta z$$

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$$\frac{\partial c}{\partial t} + \frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{\partial N_z}{\partial z} = \frac{\partial c}{\partial t} + \nabla \cdot \bar{N} = \frac{\partial c}{\partial t} + \nabla \cdot c \bar{q} - \nabla \cdot (D \nabla c) = R$$

Incompressível ( $\nabla \cdot \bar{q} = 0$ )  
 $D = \text{const}$

$$\frac{\partial c}{\partial t} + \bar{q} \cdot \nabla c = D \nabla^2 c + R$$

**Convective Diffusion Equation**

# Difusão e advecção

$$\left[ \begin{array}{c} \text{net flux of mass} \\ \text{(In - Out)} \end{array} \right] + \left[ \begin{array}{c} \text{Rate of production /} \\ \text{decay of mass} \\ \text{(chemical, physical,} \\ \text{biological...)} \end{array} \right] = \left[ \begin{array}{c} \text{Rate of mass} \\ \text{accumulation} \\ \text{within element} \end{array} \right]$$

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**Convective Diffusion Equation**

$$\frac{\partial c}{\partial t} + \underbrace{u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} + w \frac{\partial c}{\partial z}}_{\text{Advective transport}} = D \underbrace{\left( \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right)}_{\text{Diffusive transport}} + \underbrace{R}_{\text{Reaction}}$$

↑  
 Temporal change



# Deducao com equ. integrais

No quadro

## Balço de massa

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# Equação de difusão

## Equação de difusão 1D sem adveccao:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} + R(x, t)$$

$R(x,t)$ : fonte ou sumidor (reações, transformações)

# Soluções

- **Métodos analíticos para sistemas simplificados**

- Tipo: Navier-Stokes para escoamento laminar, sem atrito, 1D, entre placas (Couette, Pouseuille)
- → **Perfis de velocidade (quais)**
- → **Perfil de tensão**
- → **Forças**

- **Métodos numéricos para sistemas complexos**

- Aproximar derivadas com diferenças finitas, volumes finitos ou elementos finitos

- **Métodos experimentais**

# Equação de calor

Equação do calor unidimensional (ec1):

$$q_x = -\lambda \frac{\partial T}{\partial x} \quad \frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial x^2} + F(x, t) \quad \alpha^2 = \frac{\lambda}{c_p \rho}$$

- $0 < x < l$  ,  $t > 0$
- **$\lambda$ : condutividade térmica,  $c$ : calor específico a p const.,  $\rho$ : densidade,  $\alpha$ : difusividade térmica**

Compare

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} + R(x, t)$$

# Propriedades de Fluidos

a) Água sem substâncias dissolvidas							
temperatura [°C]	0	4	10	20	30	50	100
Massa específica/densidade	999,8	1000	999,7	998,3	995,7	988	958,1
Viscosidade cinemática $\nu$ [ $10^{-6} \text{m}^2/\text{s}$ ]	1,780	1,584	1,300	1,006	0,805	0,556	0,294
Calor específico $c_p$ [J/kgK]	4217	4205	4192	4182	4178	4180	4216
Condutividade de calor $\alpha_t$ [ $10^{-6} \text{m}^2/\text{s}$ ]	0,135	-	-	0,143	-	-	0,168
Modulo de elasticidade $E$ [ $10^9 \text{Pa}$ ]	1,964	-	2,092	2,197	2,233	2,264	2,041
Pressão de vapor $p_d$ [hPa]	6,11	8,13	12,27	23,37	42,41	123,35	1013,3
tensão superficial $\sigma$ [N/m]	0,0756	0,0749	0,0742	0,0728	0,0712	0,0679	0,0589
b) Ar com pressão atmosférica 1013 hPa Elastizitätsmodul $E = 1,42 \cdot 10^9 \text{ Pa}$ bei adiabatischen Verhältnissen							
temperatura [°C]	0	4	10	20	30	50	100
Massa específica [ $\text{m}^3$ ]	1,293	1,274	1,247	1,205	1,165	1,092	0,946
Viscosidade cinemática $\nu$ [ $10^{-6} \text{m}^2/\text{s}$ ]	13,28	13,64	14,18	15,10	16,03	17,86	23,15
Calor específico $c_p$ [J/kgK]	1006	-	-	1005	-	-	-
Condutividade de calor $\alpha_t$ [ $10^{-6} \text{m}^2/\text{s}$ ]	18,49	-	-	21,19	-	-	-

Fonte: Jirka, 2007, <http://digbib.ubka.uni-karlsruhe.de/volltexte/1000007165>

Define-se a massa específica de um sistema como:

$$\rho = \frac{M}{V} , \quad (2.36)$$

e o volume específico é o inverso de  $\rho$ :

$$v = \frac{V}{M} . \quad (2.37)$$

Conforme mencionado anteriormente, num meio contínuo imagina-se que existe de um sistema termodinâmico em torno (na vizinhança) de cada ponto  $(x, y, z)$  do espaço. Neste caso, a energia interna específica (energia interna por unidade de massa)  $u$  será uma função de  $T$  e de  $v$ :

$$u = u(T, v) . \quad (2.38)$$

O calor específico a volume constante de uma substância é definido como:

$$c_v = \frac{\partial u}{\partial T} . \quad (2.39)$$

# Calor específico c

Substance	Phase	(mass) specific heat capacity $c_p$ or $c_m$ $J \cdot g^{-1} \cdot K^{-1}$	Constant pressure molar heat capacity $C_{p,m}$ $J \cdot mol^{-1} \cdot K^{-1}$	Constant volume molar heat capacity $C_{v,m}$ $J \cdot mol^{-1} \cdot K^{-1}$	Volumetric heat capacity $C_v$ $J \cdot cm^{-3} \cdot K^{-1}$	Constant vol. atom-molar heat capacity in units of R $C_{v,m(atom)}$ $atom \cdot mol^{-1}$
Air (Sea level, dry, 0 °C (273.15 K))	gas	1.0035	29.07	20.7643	0.001297	~ 1.25 R
Air (typical room conditions <sup>A</sup> )	gas	1.012	29.19	20.85	0.00121	~ 1.25 R
Aluminium	solid	0.897	24.2		2.422	2.91 R
Ammonia	liquid	4.700	80.08		3.263	3.21 R
Animal tissue (incl. human) <sup>[24]</sup>	mixed	3.5			3.7*	
Antimony	solid	0.207	25.2		1.386	3.03 R
Argon	gas	0.5203	20.7862	12.4717		1.50 R
Arsenic	solid	0.328	24.6		1.878	2.96 R

rotomolding grade)	solid	2.3027				
Silica (fused)	solid	0.703	42.2		1.547	1.69 R
Silver <sup>[25]</sup>	solid	0.233	24.9		2.44	2.99 R
Sodium	solid	1.230	28.23			3.39 R
Steel	solid	0.466				
Tin	solid	0.227	27.112			3.26 R
Titanium	solid	0.523	26.060			3.13 R
Tungsten <sup>[25]</sup>	solid	0.134	24.8		2.58	2.98 R
Uranium	solid	0.116	27.7		2.216	3.33 R
Water at 100 °C (steam)	gas	2.080	37.47	28.03		1.12 R
Water at 25 °C	liquid	4.1813	75.327	74.53	4.1796	3.02 R
Water at 100 °C	liquid	4.1813	75.327	74.53	4.2160	3.02 R
Water at -10 °C (ice) <sup>[24]</sup>	solid	2.11	38.09		1.938	1.53 R
Zinc <sup>[25]</sup>	solid	0.387	25.2		2.76	3.03 R
Substance	Phase	(mass) specific heat capacity $c_p$ or $c_m$ $J \cdot g^{-1} \cdot K^{-1}$	Constant pressure molar heat capacity $C_{p,m}$ $J \cdot mol^{-1} \cdot K^{-1}$	Constant volume molar heat capacity $C_{v,m}$ $J \cdot mol^{-1} \cdot K^{-1}$	Volumetric heat capacity $C_v$ $J \cdot cm^{-3} \cdot K^{-1}$	Constant vol. atom-molar heat capacity in units of R $C_{v,m(atom)}$ $atom \cdot mol^{-1}$



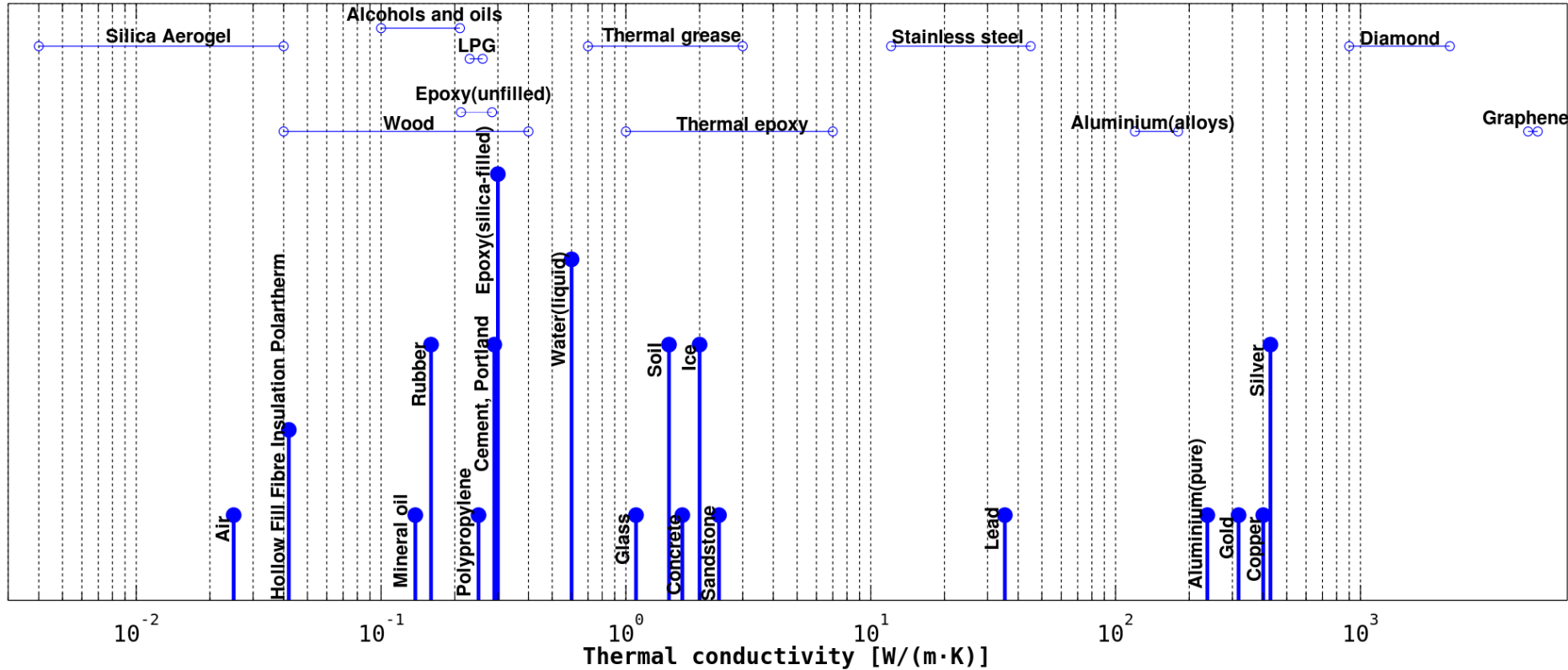
# Calor específico $c$ de sólidos

Substance	Phase	$c_p$ J/(g·K)
<a href="#">Asphalt</a>	solid	0.920
<a href="#">Brick</a>	solid	0.840
<a href="#">Concrete</a>	solid	0.880
<a href="#">Glass, silica</a>	solid	0.840
<a href="#">Glass, crown</a>	solid	0.670
<a href="#">Glass, flint</a>	solid	0.503
<a href="#">Glass, pyrex</a>	solid	0.753
<a href="#">Granite</a>	solid	0.790
<a href="#">Gypsum</a>	solid	1.090
<a href="#">Marble, mica</a>	solid	0.880
<a href="#">Sand</a>	solid	0.835
<a href="#">Soil</a>	solid	0.800
<a href="#">Sulphur Hexafluoride</a>	gas	0.664
<a href="#">Wood</a>	solid	1.7 (1.2 to 2.3)
Substance	Phase	$c_p$ J/(g·K)

# Conductividade de calor $\lambda$ (thermal conductivity)



Experimental values of thermal conductivity



# Diffusividade termica

$$\alpha^2 = \frac{\lambda}{c_p \rho}$$

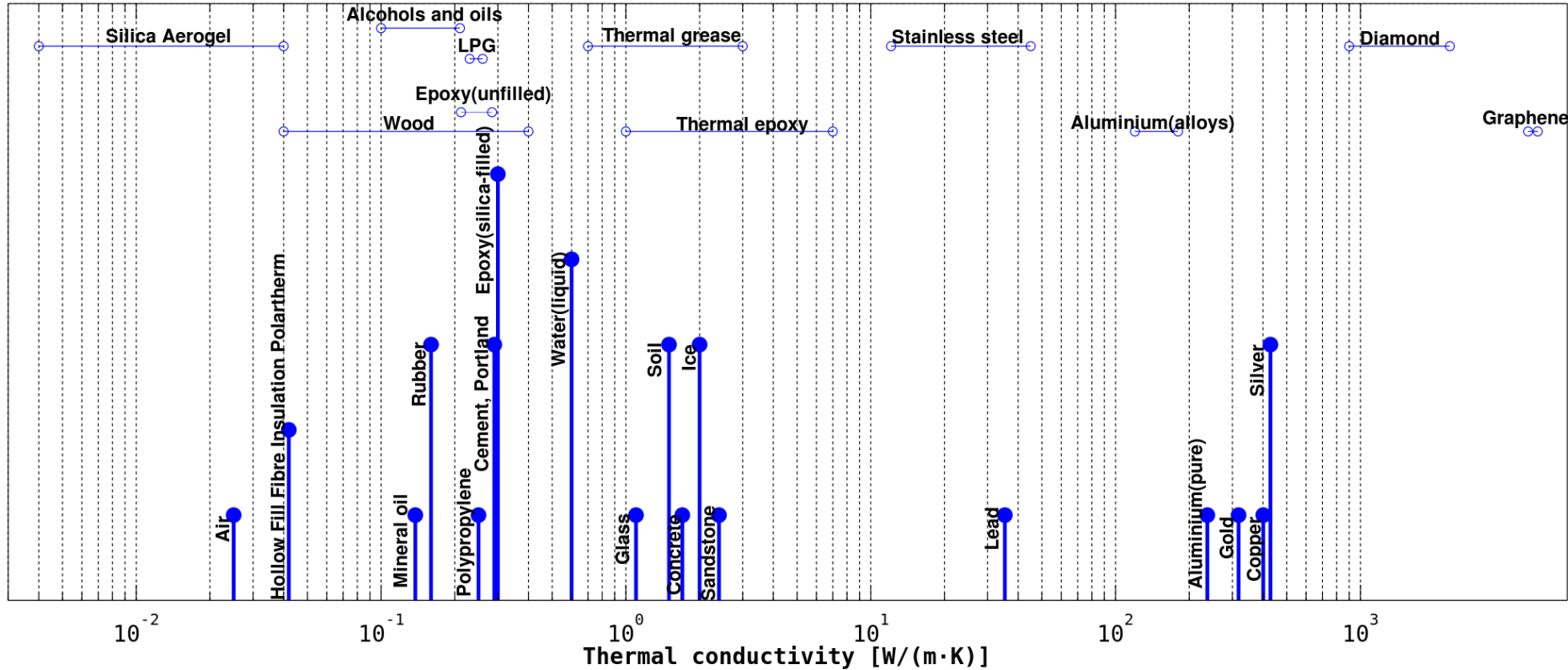
Thermal diffusivity of selected materials and substances[8]

Material	Thermal diffusivity (m <sup>2</sup> /s)	Thermal diffusivity (mm <sup>2</sup> /s)
Pyrolytic graphite, parallel to layers	1.22 × 10 <sup>-3</sup>	1220
Silver, pure (99.9%)	1.6563 × 10 <sup>-4</sup>	165.63
Gold	1.27 × 10 <sup>-4</sup> [9]	127
Copper at 25°C	1.11 × 10 <sup>-4</sup> [10]	111
Aluminium	8.418 × 10 <sup>-5</sup>	84.18
Al-10Si-Mn-Mg (Silafont 36) at 20°C	74.2 × 10 <sup>-6</sup> [11]	74.2
Aluminum 6061-T6 Alloy	6.4 × 10 <sup>-5</sup> [9]	64
Al-5Mg-2Si-Mn (Magsimal-59) at 20°C	44.0 × 10 <sup>-6</sup> [12]	44.0
Steel, 1% carbon	1.172 × 10 <sup>-5</sup>	11.72
Steel, stainless 304A at 27°C	4.2 × 10 <sup>-6</sup> [9]	4.2
Steel, stainless 310 at 25°C	3.352 × 10 <sup>-6</sup> [13]	3.352
Tin	4.0 × 10 <sup>-5</sup> [9]	40
Brick, common	5.2 × 10 <sup>-7</sup>	0.52
Brick, adobe	2.7 × 10 <sup>-7</sup>	0.27
Glass, window	3.4 × 10 <sup>-7</sup>	0.34
Rubber	1.3 × 10 <sup>-7</sup> [citation needed]	0.13
Nylon	9 × 10 <sup>-8</sup>	0.09
Wood (Yellow Pine)	8.2 × 10 <sup>-8</sup>	0.082
Oil, engine (saturated liquid, 100 °C)	7.38 × 10 <sup>-8</sup>	0.0738

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# Conductividade de calor $\lambda$ (thermal conductivity)

Experimental values of thermal conductivity



# Diffusividade termica

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# Definição do problema de valor de contorno 1

## Equação do calor

- $0 < x < l$  ,  $t > 0$

homogêneo

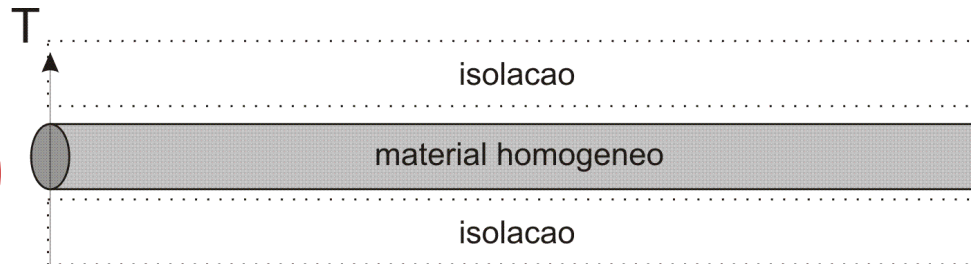
cc1:

$$\frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial x^2} + \mathbf{0}$$

cc2:

Dirichlet  
(T fixo)

$$\left. \begin{array}{l} T(0,t) \\ \text{ou} \\ \frac{\partial T}{\partial x}(0,t) \end{array} \right\} = \mathbf{0}$$

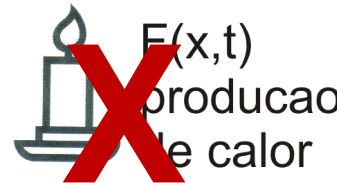


$$\left. \begin{array}{l} T(l,t) \\ \text{ou} \\ \frac{\partial T}{\partial x}(l,t) \end{array} \right\} = \mathbf{0}$$

von Neumann  
(fluxo fixo)

condição inicial (ci):

$T(x,0) = \varphi(x)$   
distr. inicial

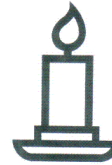
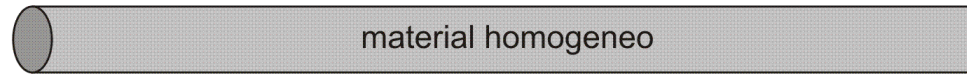


$$\varphi(0) = \psi_1(0)$$

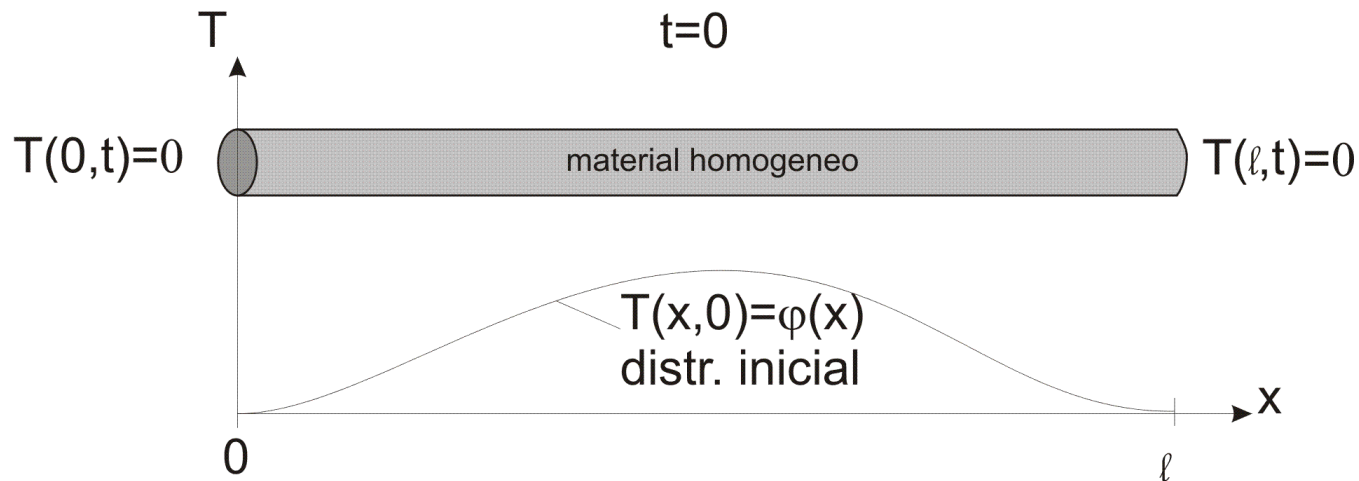
$$\varphi(l) = \psi_2(0)$$

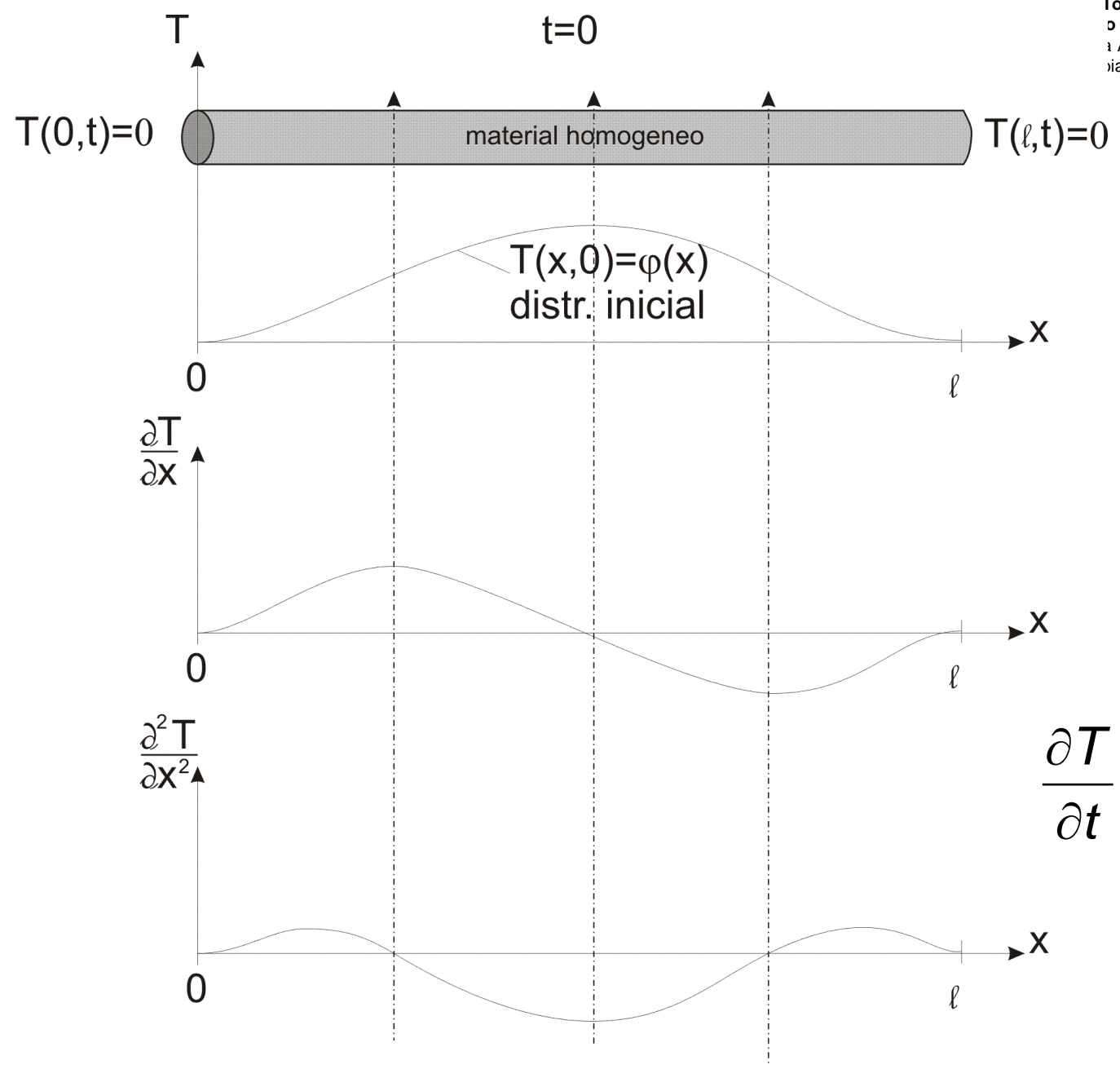
# Equação do calor homogênea: Ilustração do problema de valor de contorno 1

$t < 0$

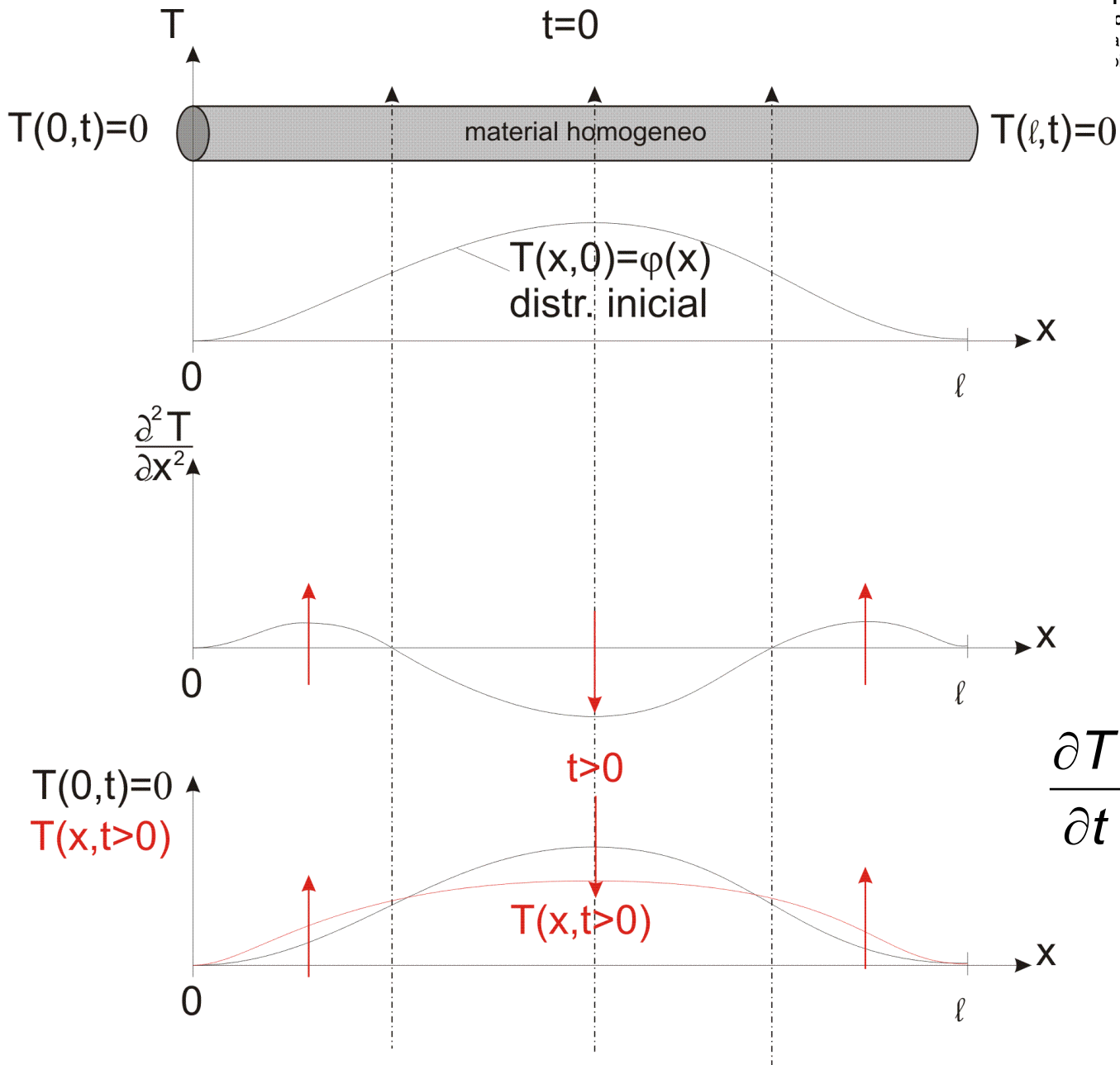


$t = 0$









# Equação do calor homogênea: Solução do problema de contorno 1

## Matemática!

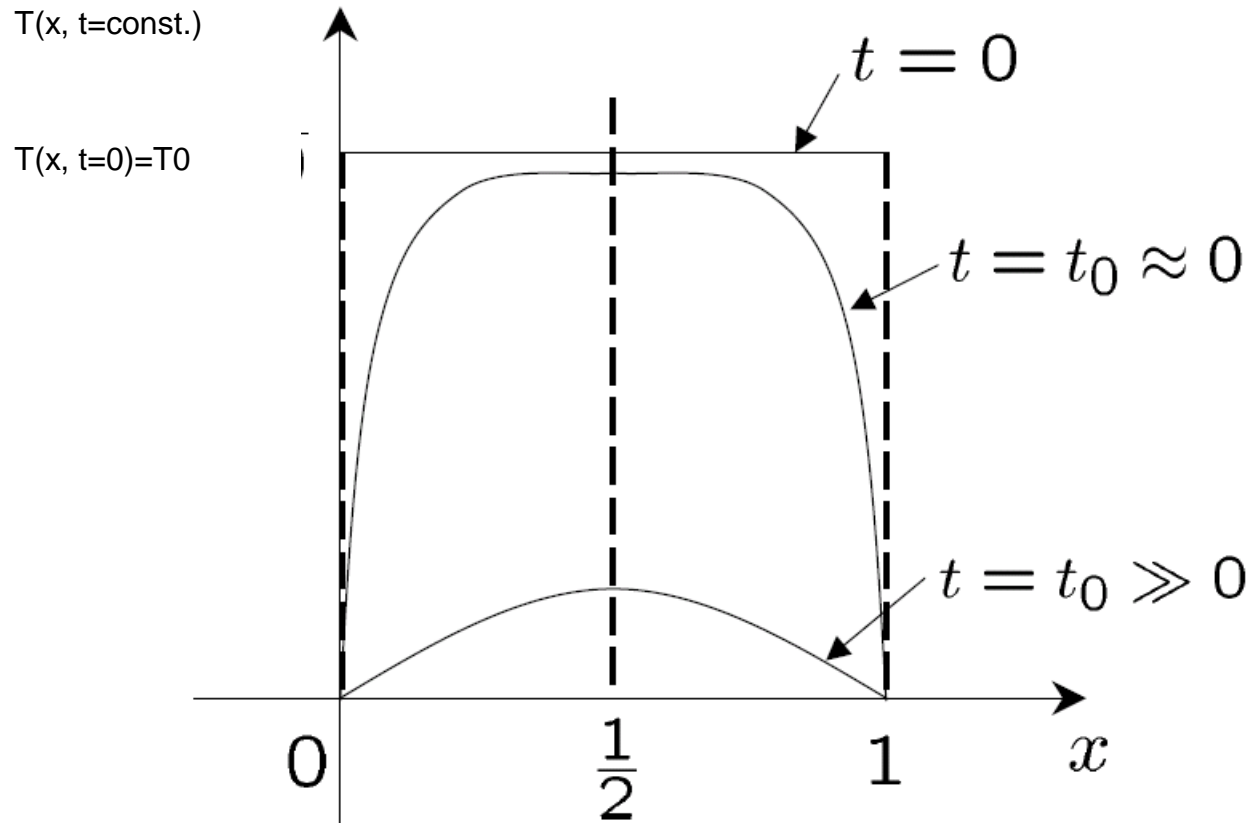
$$T(x, t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{l}\right) \exp\left(\frac{-n^2\pi^2\alpha^2 t}{l^2}\right)$$

### Observações:

- falta mostrar convergência e continuidade, mas se pode mostrar
- $t \rightarrow \infty \rightarrow T(x, t) \rightarrow 0$  com limites maiores
- assim  $\lambda_n = -n^2\pi^2/l^2$ ,  $n=1, 2, \dots$  são auto valores com a
- auto função

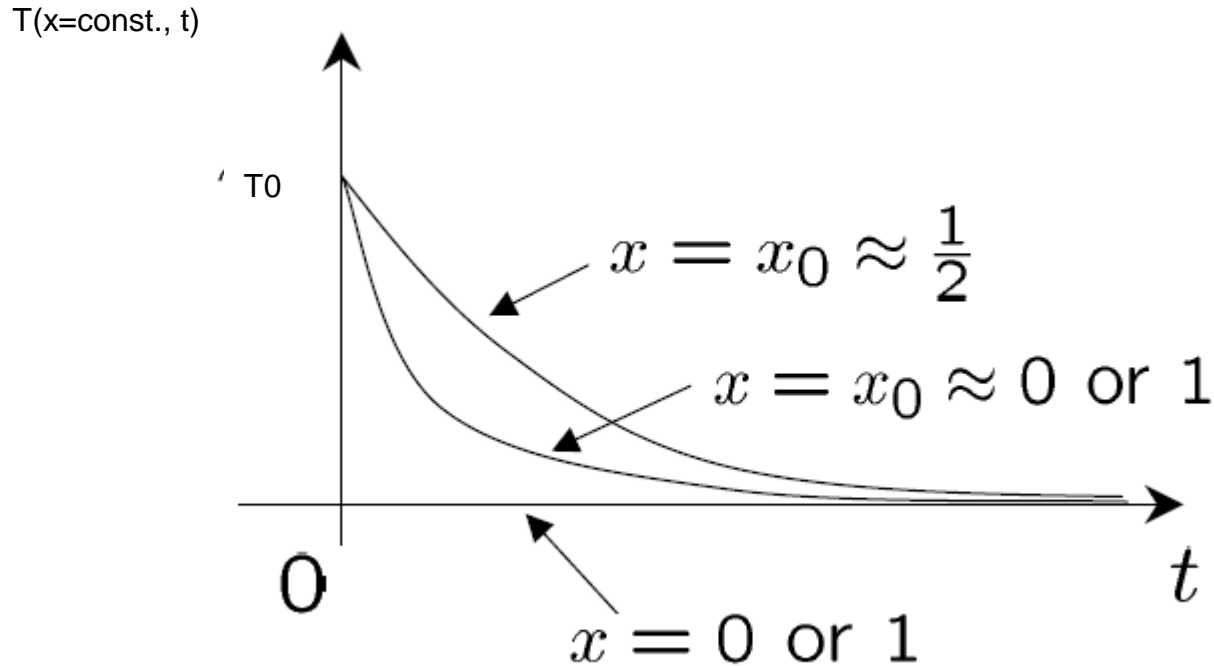
$$u_n(x) = c_n \sin\left(\frac{n\pi x}{l}\right)$$

# Equação do calor homogênea: Solução do problema de contorno 1 $T(x, t = \text{const.})$



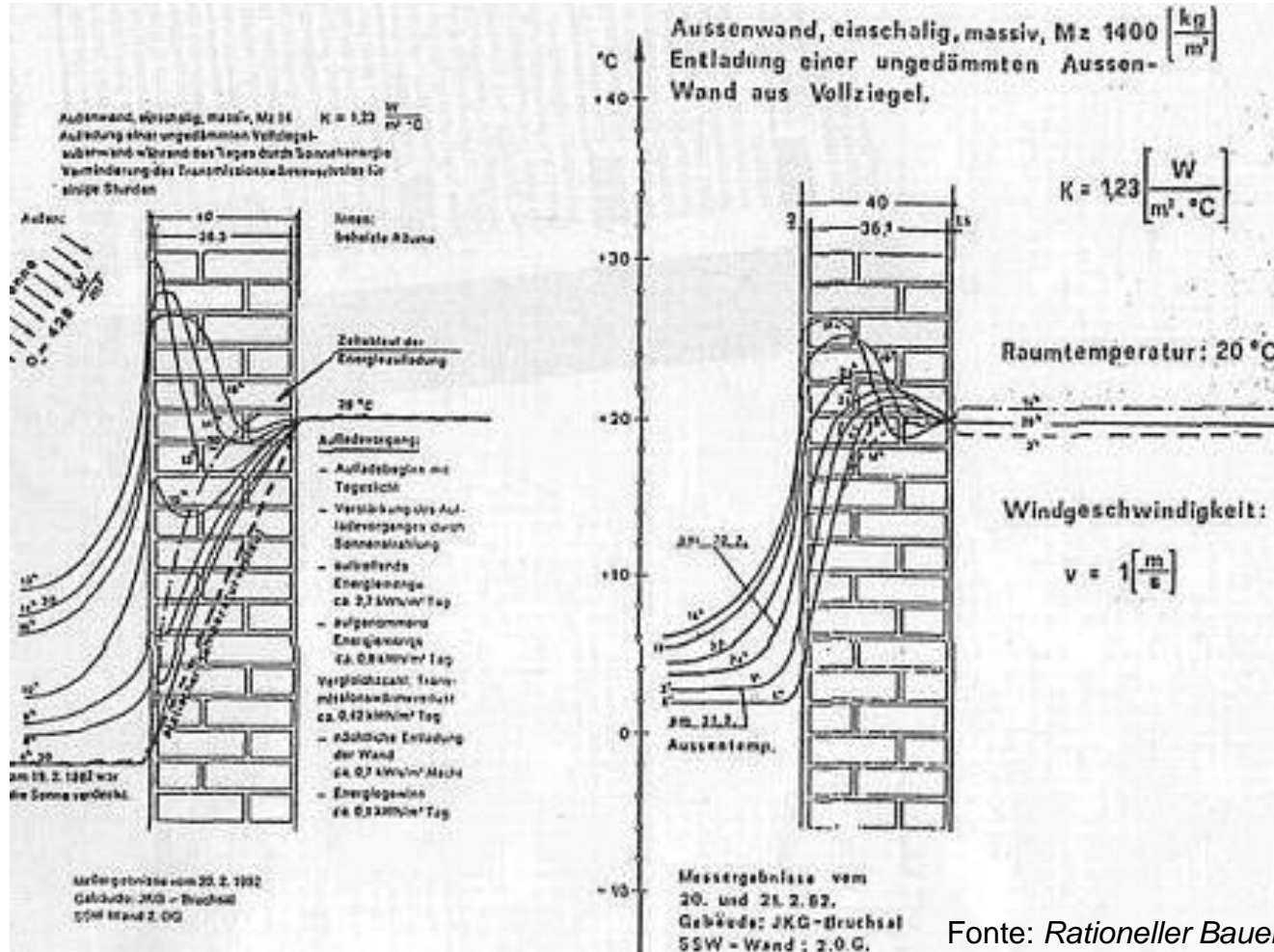
Fonte: Hancock, 2006

# Equação do calor homogênea: Solução do problema de contorno 1 $T(x=\text{const.}, t)$



Fonte: Hancock, 2006

# Equação do calor : Aplicações

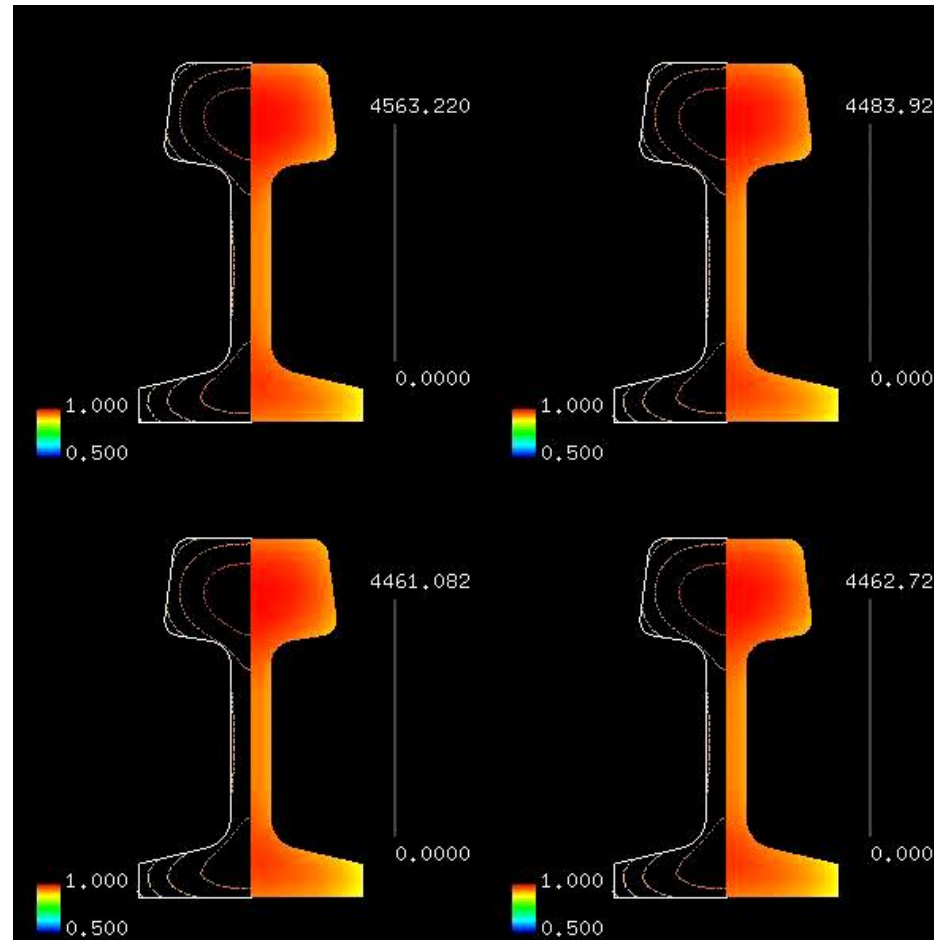


Fonte: *Rationeller Bauen*, 1983

Medições de temperatura numa parede em inverno (nublado).

$T_{ar,amb} = 6-10^{\circ}\text{C}$ ,  $T_{par,amb} = 14-30^{\circ}\text{C}$ ,  $T_{par,int.} = T_{ar,int.} = 20^{\circ}\text{C}$   
 $T_{ar,amb} < T_{par} \rightarrow$  não ocorre condensação. Obs.: distribuição permanente não corresponde com medições!

## Equação do calor : Aplicações



Modelagem da refrigeração controlada de barras de aço (trilha) no processo de produção. Objetivo: definir condições de contorno (refrigeração controlada) e/ou condições iniciais (produção controlada) para obter uma distribuição de temperatura equilibrada para cada momento. Fonte: Dipl. Math. Jens Saak

# Equação de difusão

Concentração:

$$C = \frac{M_{\text{substância}}}{V_{\text{total}}}$$

Fluxo de massa:

$$j_x = -D \frac{\partial C}{\partial x}$$

- **D: difusividade molecular de A em B**

**Equação de difusão:**

$$\frac{\partial C}{\partial t} = -D \frac{\partial^2 C}{\partial x^2} + F(x, t) \quad \left( \text{compare: } \frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial x^2} + F(x, t) \right)$$

- $F(x, t)$ : fonte ou sumidor (reações, transformações)

# Equação de difusão homogênea:

**Matemática!**

$$C(x, t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{\ell}\right) \exp\left(\frac{-n^2\pi^2 Dt}{\ell^2}\right)$$



# Soluções da equação de difusão

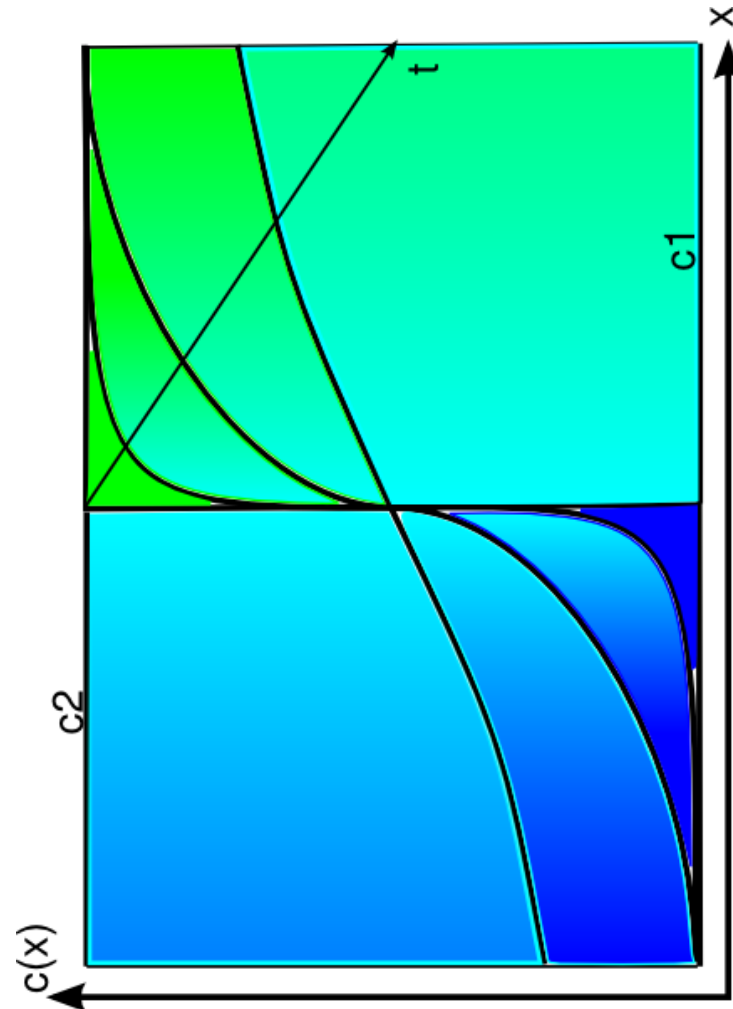
## Mistura de fluidos com concentrações diferentes (experimento)

- $F(x,t) = 0$
- Condições iniciais  $C(x,0) = C_0 = \varphi(\mathbf{x})$ 
  - transparente: água com  $C_{\text{sal}} = 0$
  - colorido: água com  $C_{\text{sal}} = C_0$
- Condições de contorno:
  - superfície:  $C(\infty,t) = 0$
  - fundo:  $C(-\infty,t) = C_0$
- Solução (análogo eq. do calor)

$$c(x, t) = \frac{c_0}{2\sqrt{\pi Dt}} \int_0^{\infty} dx' \exp\left(-\frac{(x-x')^2}{4Dt}\right) = \frac{c_0}{2} \left(1 + \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right)$$

# Soluções da equação de difusão

## Mistura de fluidos com concentrações diferentes (experimento)



# Referências

- Boyce, W. E., DiPrima, R. C., 1998, “Equações Diferenciais Elementares e Problemas de Valores de Contorno”, traduzido do inglês por Horacio Macedo, LTC - Livros Técnicos e Científicos Editora S.A., Rio de Janeiro, ISBN 85-216-1131-5**
- Kreyszig, E., 1999, “Advanced Engineering Mathematics”, John Wiley & Sons, Inc., New York, ISBN 0-471-15496-2**
- Hancock, M. 2006, “Linear Partial Differential Equations”, MIT Open Course Ware, Massachusetts Institute of Technology, <http://ocw.mit.edu/courses/mathematics/18-303-linear-partial-differential-equations-fall-2006/>**
- Robert E. Terrell, 2011, “PDE applets heat equation in 1d and 2d“, Cornell University, Department of Mathematics, <http://www.math.cornell.edu/~bterrell/>**

# Referencias

- **Materiais adicionais do MIT open course ware (*ingles*) da disciplina [Transport Processes in the Environment](#)**
- **Materiais adicionais do MIT open course ware (*portugues*) da disciplina [Processos de Transporte no meio ambiente](#)**
- [Referencias recomendadas do MIT](#)
- [Texas A&M course notes](#). Veja especialmente o "[course script](#)" que tambem contem excercisios.
- [Materiais multimídias na hidráulica](#)