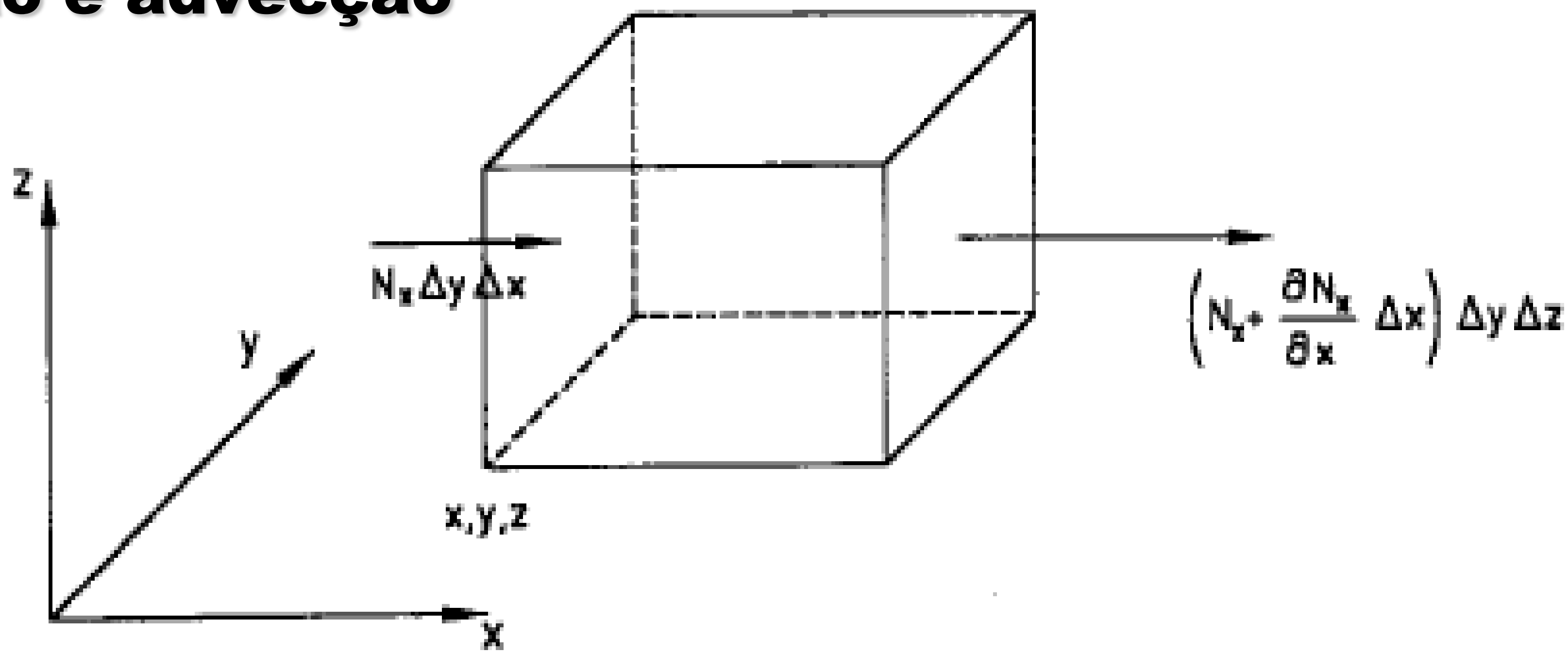
Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Tobias Bleninger

MECÂNICA DOS FLUIDOS AMBIENTAL II

Difusão e advecção



$$\begin{bmatrix}\text{net flux of mass}\\ (\text{In} - \text{Out})\end{bmatrix} + \begin{bmatrix}\text{Rate of production/}\\ \text{decay of mass}\\ \text{(chemical, physical,}\\ \text{biological...)}\end{bmatrix} = \begin{bmatrix}\text{Rate of mass}\\ \text{accumulation}\\ \text{within element}\end{bmatrix}$$

$$-\frac{\partial N_x}{\partial x}\Delta x\Delta y\Delta z - \frac{\partial N_y}{\partial y}\Delta x\Delta y\Delta z - \frac{\partial N_z}{\partial z}\Delta x\Delta y\Delta z + R\,\Delta x\Delta y\Delta z = \frac{\partial c}{\partial t}\Delta x\Delta y\Delta z$$

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Difusão e advecção

$$\begin{bmatrix} \text{net flux of mass} \\ (\text{In} - \text{Out}) \end{bmatrix} + \begin{bmatrix} \text{Rate of production /} \\ \text{decay of mass} \\ \text{(chemical, physical,} \\ \text{biological...)} \end{bmatrix} = \begin{bmatrix} \text{Rate of mass} \\ \text{accumulation} \\ \text{within element} \end{bmatrix}$$

$$-\frac{\partial N_x}{\partial x}\Delta x\Delta y\Delta z - \frac{\partial N_y}{\partial y}\Delta x\Delta y\Delta z - \frac{\partial N_z}{\partial z}\Delta x\Delta y\Delta z + R\,\Delta x\Delta y\Delta z = \frac{\partial c}{\partial t}\Delta x\Delta y\Delta z$$

$$\frac{\partial c}{\partial t} + \frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{\partial N_z}{\partial z} = \frac{\partial c}{\partial t} + \nabla\cdot\vec{N} = \frac{\partial c}{\partial t} + \nabla\cdot c\vec{q} - \nabla\cdot\left(D\nabla c\right) = R$$

Incompressivel (nabla*q)=0
D=const

$$\boxed{\frac{\partial c}{\partial t} + \vec{q}\cdot\nabla c = D\nabla^2 c + R}$$

Convective Diffusion Equation

Difusão e advecção

$$\begin{bmatrix} \text{net flux of mass} \\ (\text{In} - \text{Out}) \end{bmatrix} + \begin{bmatrix} \text{Rate of production /} \\ \text{decay of mass} \\ \text{(chemical, physical,} \\ \text{biological...)} \end{bmatrix} = \begin{bmatrix} \text{Rate of mass} \\ \text{accumulation} \\ \text{within element} \end{bmatrix}$$

$$\boxed{\frac{\partial c}{\partial t} + \vec{q} \cdot \nabla c = D \nabla^2 c + R}$$

Convective Diffusion Equation

$$\underbrace{\frac{\partial c}{\partial t}}_{\substack{\uparrow \\ \text{Temporal} \\ \text{change}}} + \underbrace{u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} + w \frac{\partial c}{\partial z}}_{\text{Advective transport}} = \underbrace{D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right)}_{\text{Diffusive transport}} + \underbrace{R}_{\substack{\uparrow \\ \text{Reaction}}}$$

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Balanço de massa

$$0 = \frac{\partial}{\partial t}\int_{V_c} \rho dV + \int_{S_c} \rho\left(\mathbf{v}\cdot\mathbf{n}\right) dS.$$

$$\frac{D\rho}{Dt} + \rho\boldsymbol{\nabla}\cdot\mathbf{v} = 0,$$

Balanço de massa de um soluto

$$\frac{\partial}{\partial t}\int_{V_c} C_A\rho dV + \int_{S_c} C_A\rho\left(\mathbf{v}\cdot\mathbf{n}\right) dS = -\int_{S_c}\left(\mathbf{j}\cdot\mathbf{n}\right) dS.$$

$$\frac{\partial C_A}{\partial t} + \left(\mathbf{v}\cdot\boldsymbol{\nabla}\right) C_A = D_{AB}\nabla^2 C_A.$$

Balanço de quantidade de movimento

$$\frac{\partial}{\partial t}\int_{V_c} \mathbf{v}\rho dV + \int_{S_c} \mathbf{v}\rho\left(\mathbf{v}\cdot\mathbf{n}\right) dS = \int_{V_c} \rho\mathbf{g} dV + \int_{S_c}\left(\mathbf{T}\cdot\mathbf{n}\right) dS.$$

$$\rho\frac{D\mathbf{v}}{Dt} = -\boldsymbol{\nabla}\left(p + \rho g h\right) + \mu\nabla^2\mathbf{v}$$

Balanço de energia

$$\frac{\partial}{\partial t}\int_{V_c} e\rho dV + \int_{S_c} e\rho\left(\mathbf{v}\cdot\mathbf{n}\right) dS = -\int_{S_c}\left(\mathbf{q}\cdot\mathbf{n}\right) dS + \int_{S_c}\left[\left(\mathbf{T}\cdot\mathbf{v}\right)\cdot\mathbf{n}\right] dS$$

$$\rho\frac{\partial e}{\partial t} + \rho\left(\mathbf{v}\cdot\boldsymbol{\nabla}\right) e = \boldsymbol{\nabla}\cdot\left(\mathbf{T}\cdot\mathbf{v}\right) + \boldsymbol{\nabla}\cdot\left(\rho c_p \alpha\boldsymbol{\nabla} T\right)$$

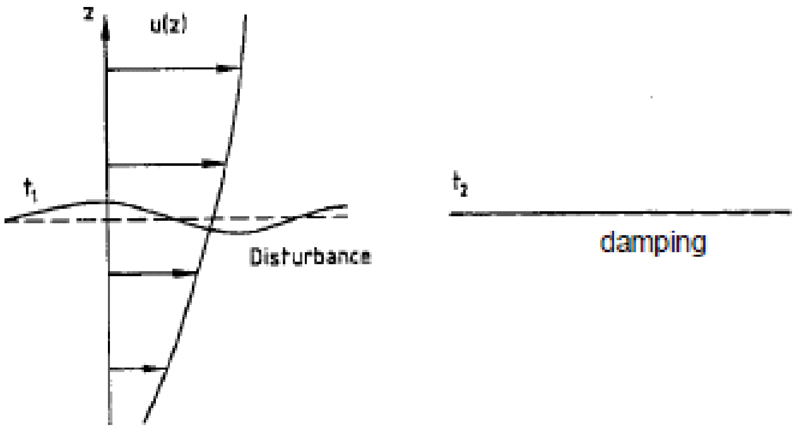
$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \left(\mathbf{v}\cdot\boldsymbol{\nabla}\right) T = \alpha\nabla^2 T.$$

Forma geral

$$\frac{\partial\eta}{\partial t} + \left(\mathbf{v}\cdot\boldsymbol{\nabla}\right)\eta - K\nabla^2\eta = f\left(x, y, z, t\right)$$

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

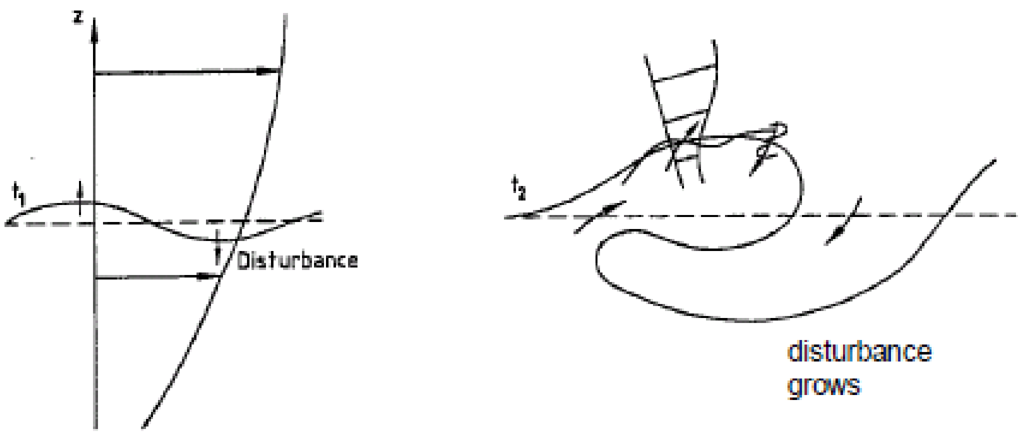
Turbulencia



Possibility 1:

- damping of disturbance (viscosity)
- stable flow

LAMINAR FLOW



Possibility 2:

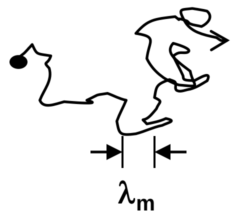
- insufficient damping
- large eddies
- secondary eddies

TURBULENT FLOW

Fonte: Socolofsky and Jirka, 2005

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

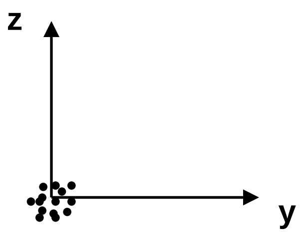
A) Molecular diffusion in stagnant ambient



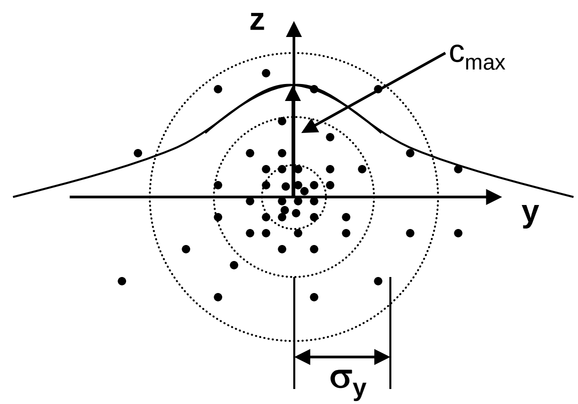
λ_m = free path length

u_m = molecule velocity

Particle mass M
t = 0



Particle cloud
t = t



$\sigma_y = \sqrt{2D_m t}$ standard deviation

D_m = mol. Diffusivity [m²/s]

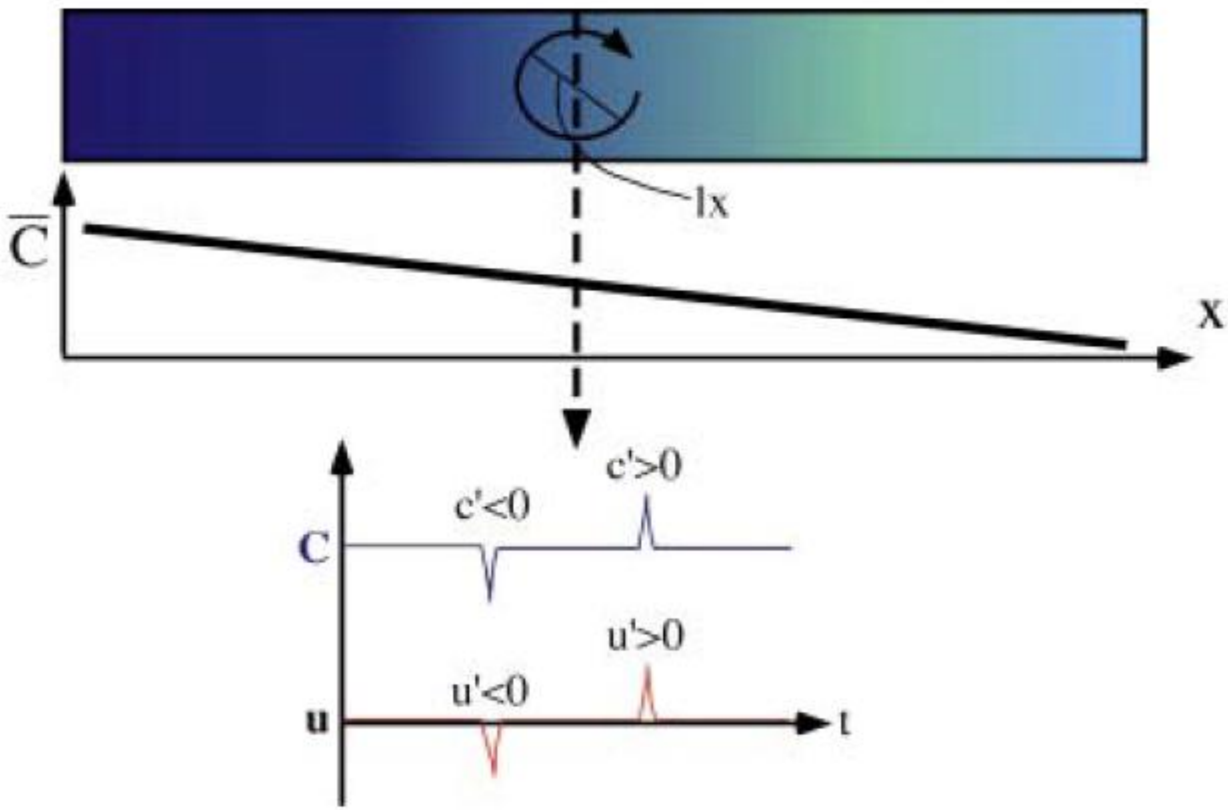
E.g.: $D_m = 2 \times 10^{-9}$ m²/s salt in water

$D_m \sim \lambda_m u_m$ Mass diffusion as sum of singular movements

Ficks law

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Difusao turbulenta



Fonte: MIT, 2010,
http://www.universia.com.br/mit/curso.jsp?codcurso=15

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

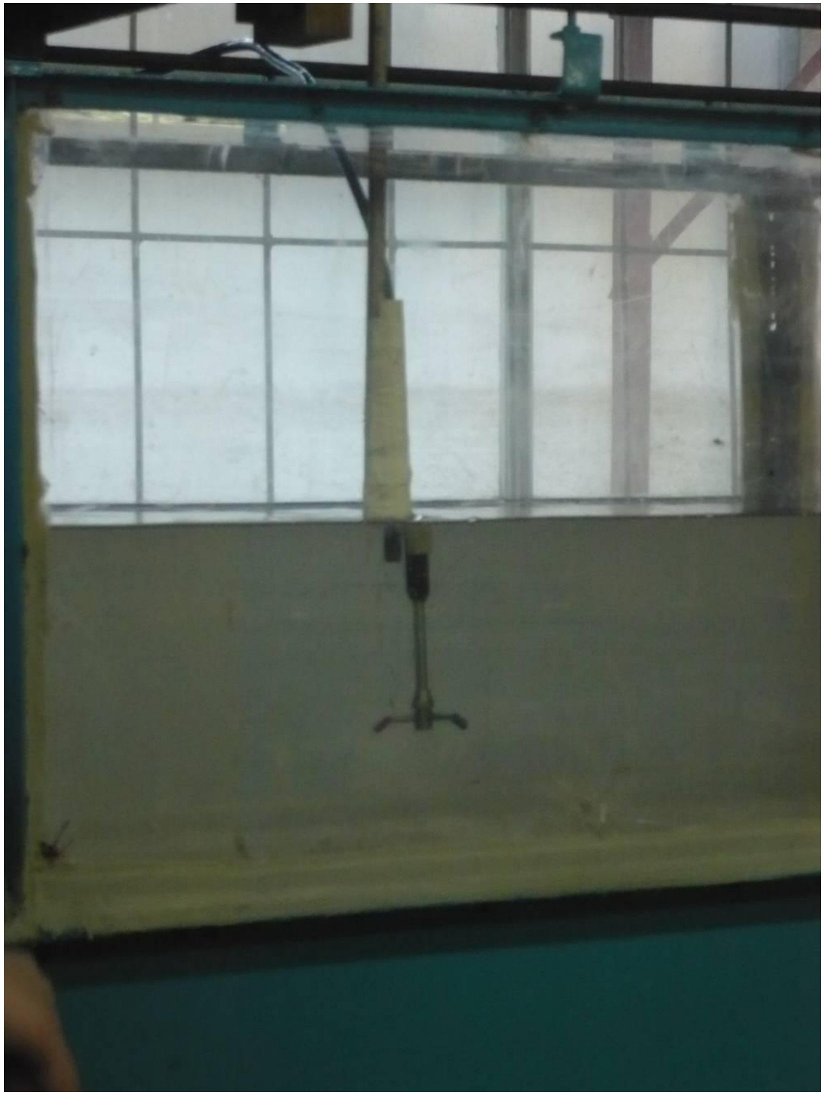
Laboratorio didático de mecânica dos fluidos

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Laboratorio didático de mecânica dos fluidos

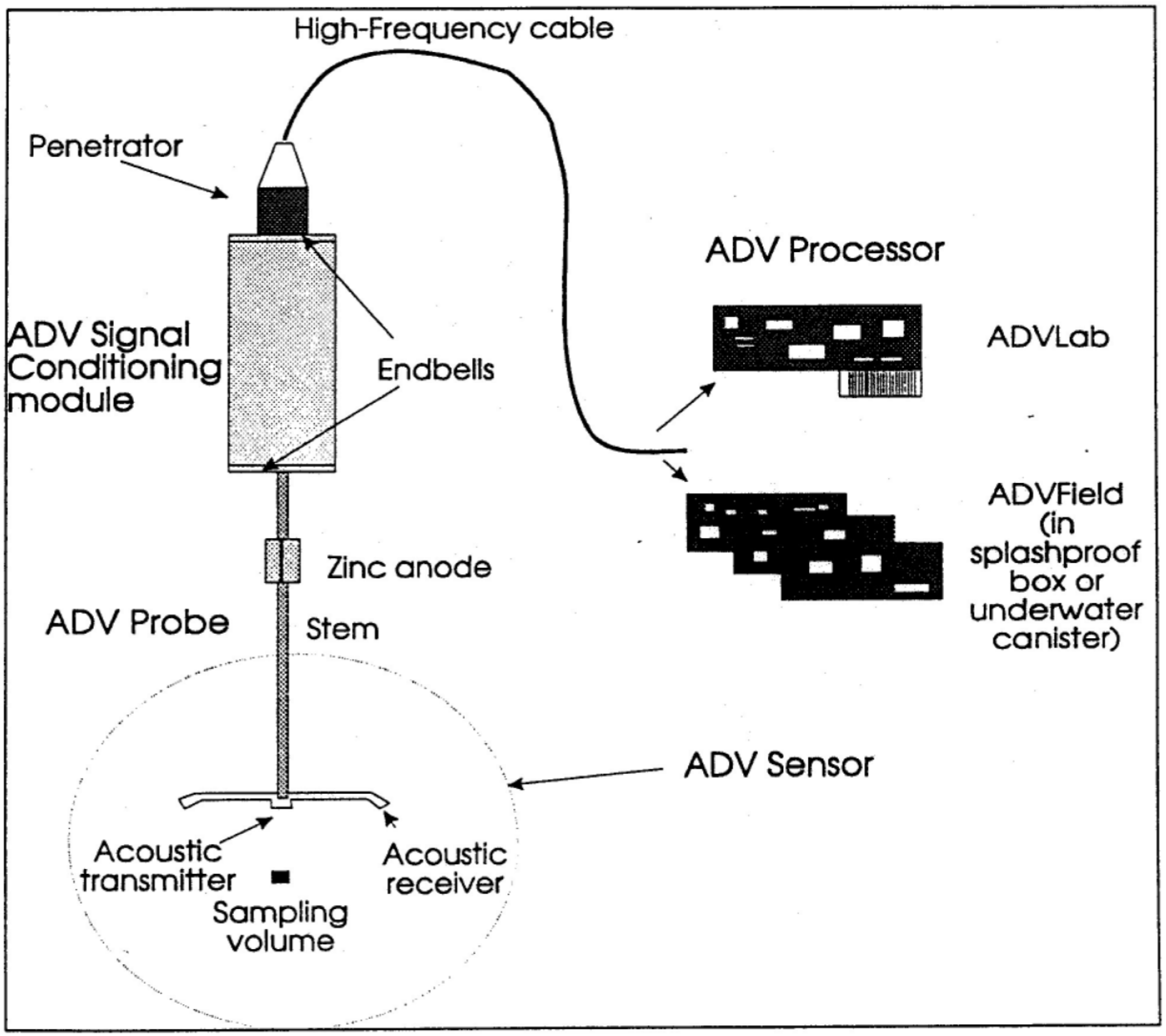
Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Medições de velocidades com ADV (Acoustic Doppler Velocimeter)



Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

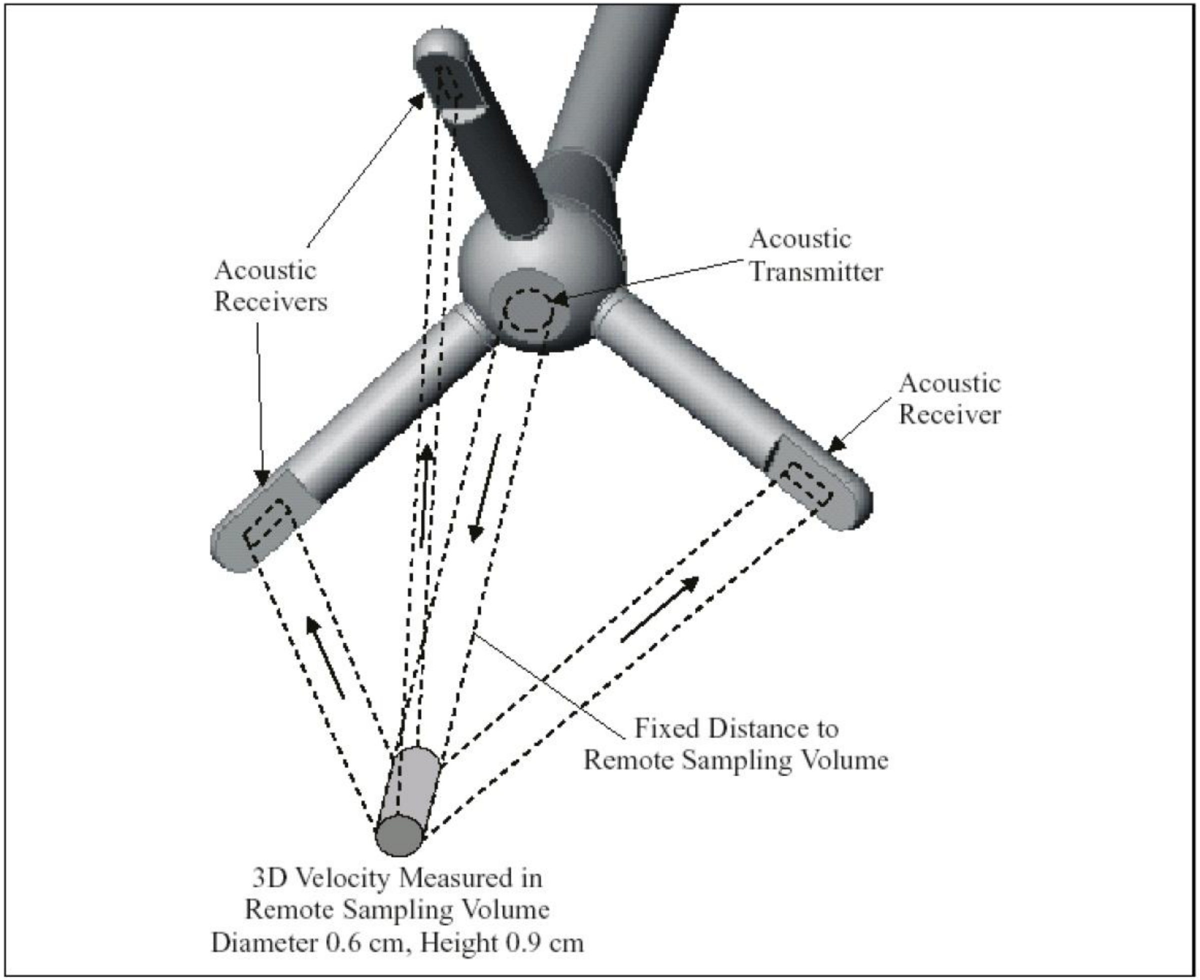
Medições de velocidades com ADV (Acoustic Doppler Velocimeter)



Fonte: www.sontek.com

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

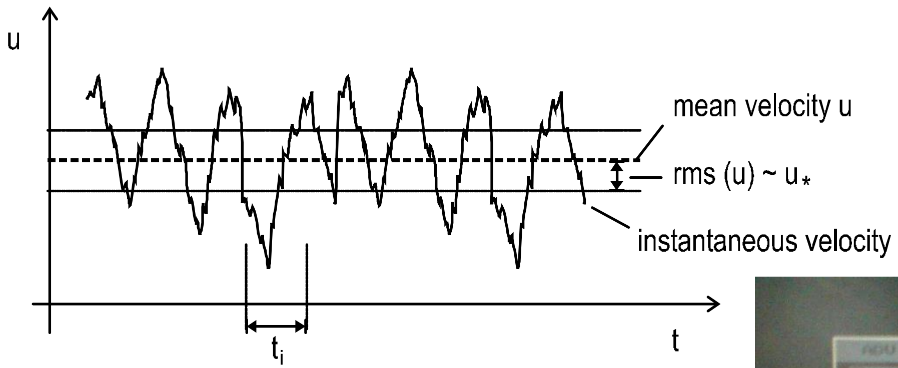
Medições de velocidades com ADV (Acoustic Doppler Velocimeter)

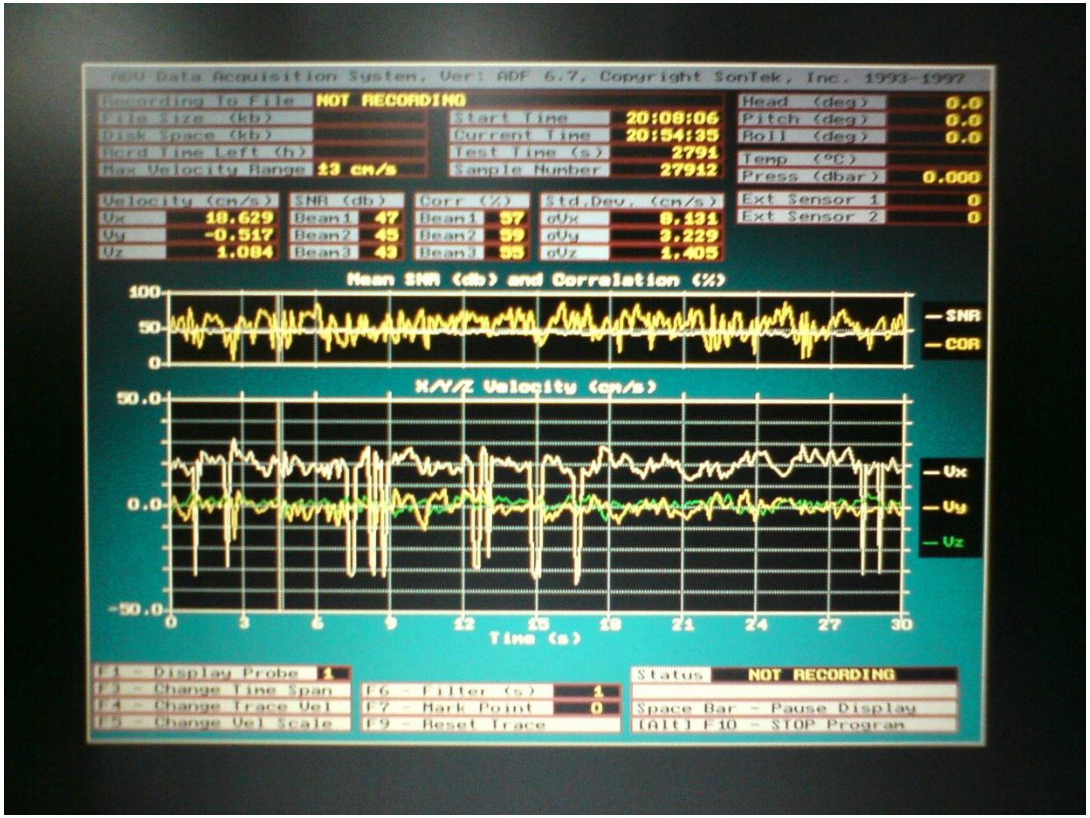


Fonte: www.sontek.com

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

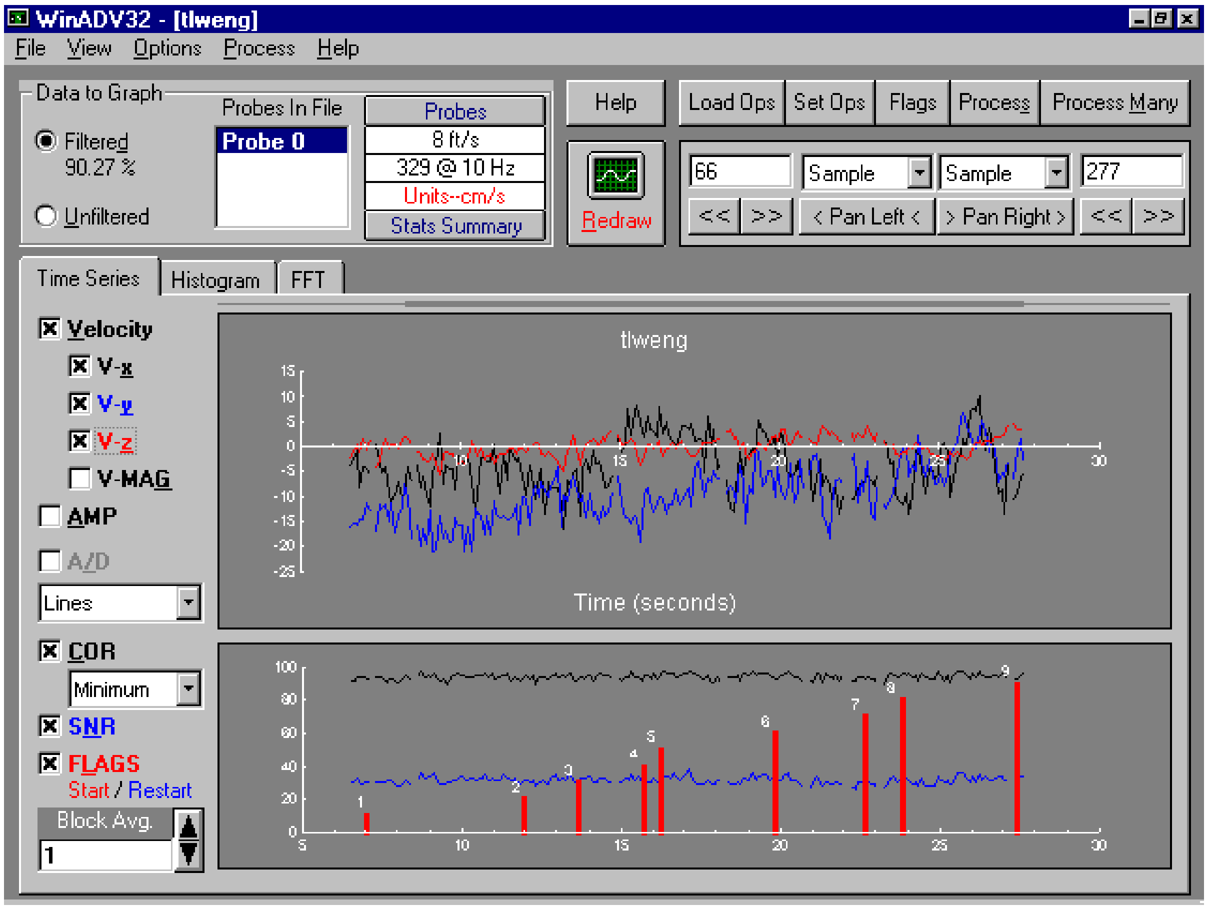
Medições de velocidades com ADV (Acoustic Doppler Velocimeter)





Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

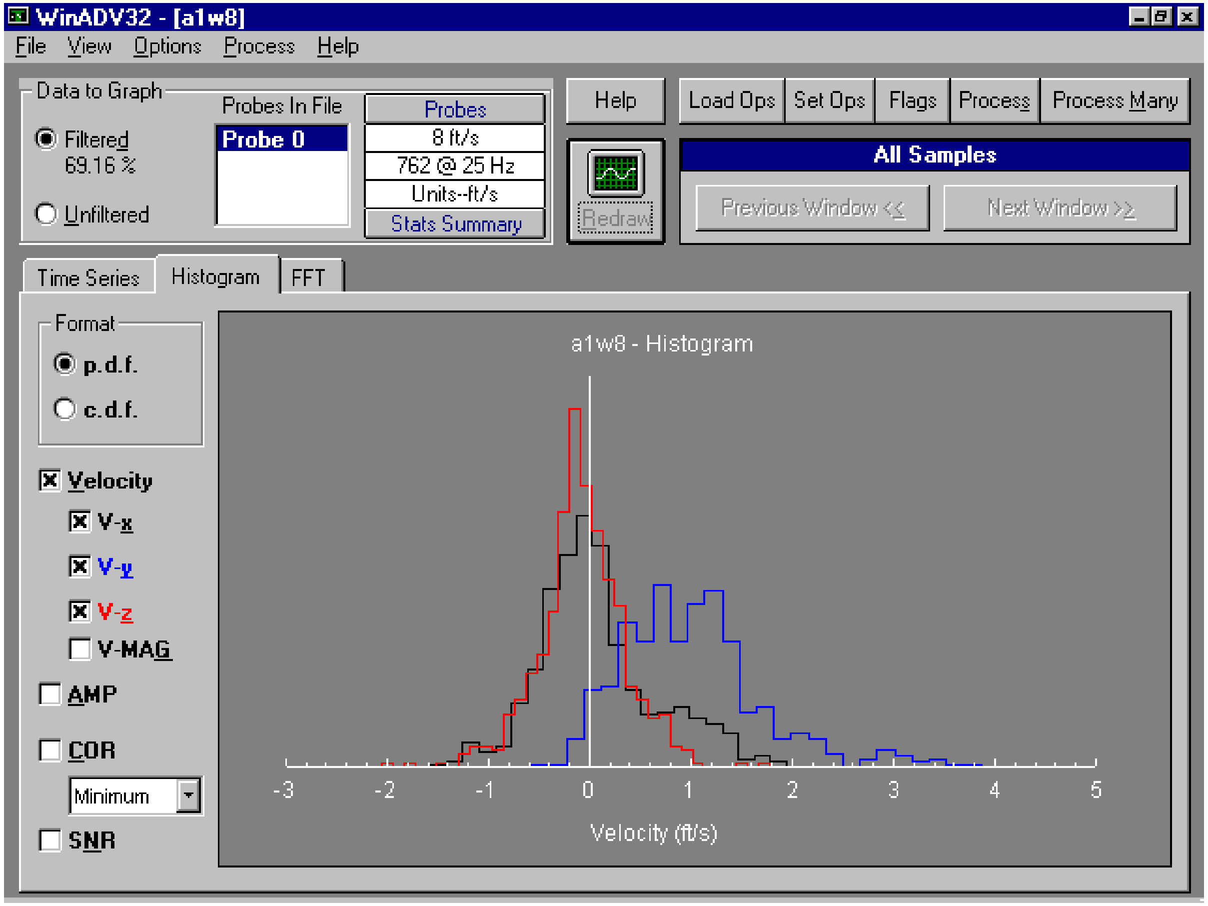
Medições de velocidades com ADV (Acoustic Doppler Velocimeter)



Fonte: http://**www.usbr.gov/**pmts/hydraulics_lab/twahl/winadv/wadv-ex.html

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

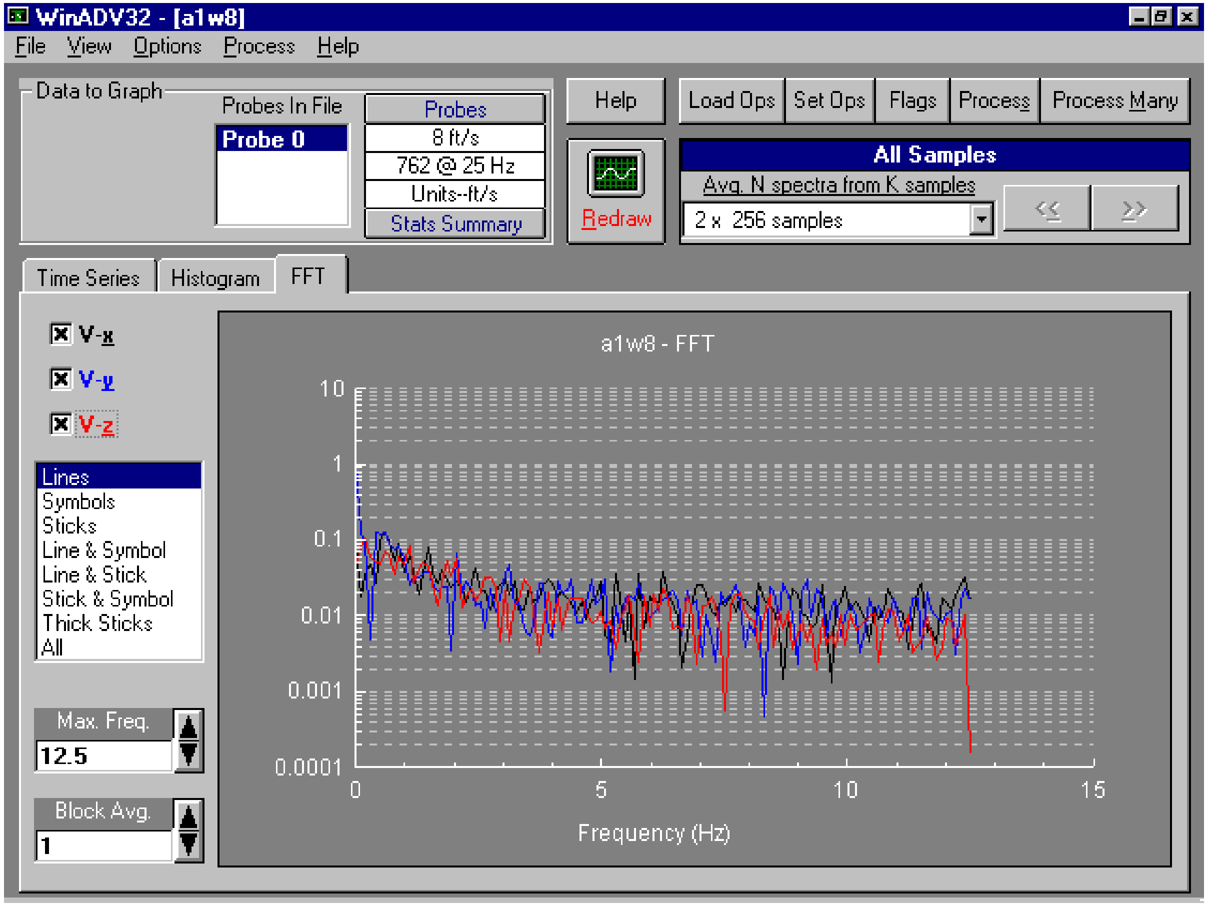
Medições de velocidades com ADV (Acoustic Doppler Velocimeter)



Fonte: http://**www.usbr.gov/**pmts/hydraulics_lab/twahl/winadv/wadv-ex.html

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

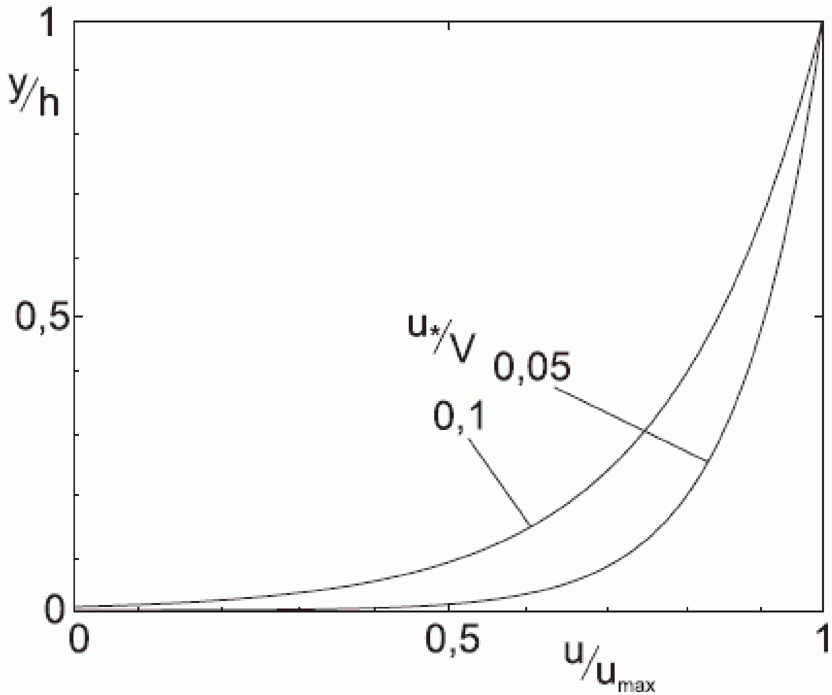
Medições de velocidades com ADV (Acoustic Doppler Velocimeter)



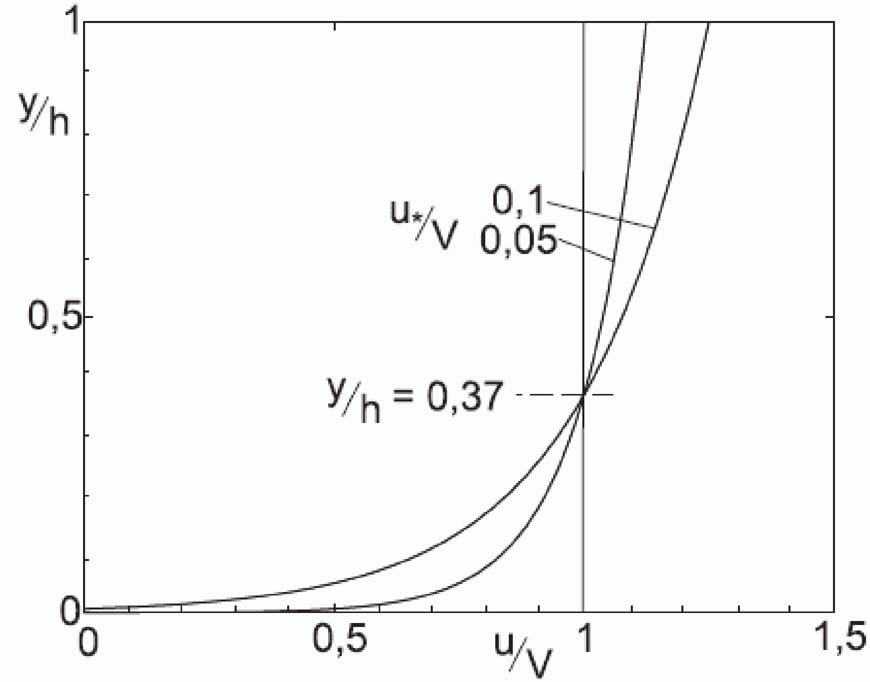
Fonte: http://**www.usbr.gov/**pmts/hydraulics_lab/twahl/winadv/wadv-ex.html

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Perfil de velocidade



(a) Normalisiert mit $u_{\max}$



(b) Normalisiert mit V

Abb. 2.3: Logarithmisches Profil für turbulente breite Gerinneströmungen (vereinfachte Gl. (2.11)) normiert durch (a) maximale Geshwindigkeit $u_{\max}$ und (b) mittlere Geschwindigkeit V, in Abhängigkeit vom Turbulenzparameter u_*/V.

Fonte: Jirka e Lang, 2009

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Flutuações turbulentas

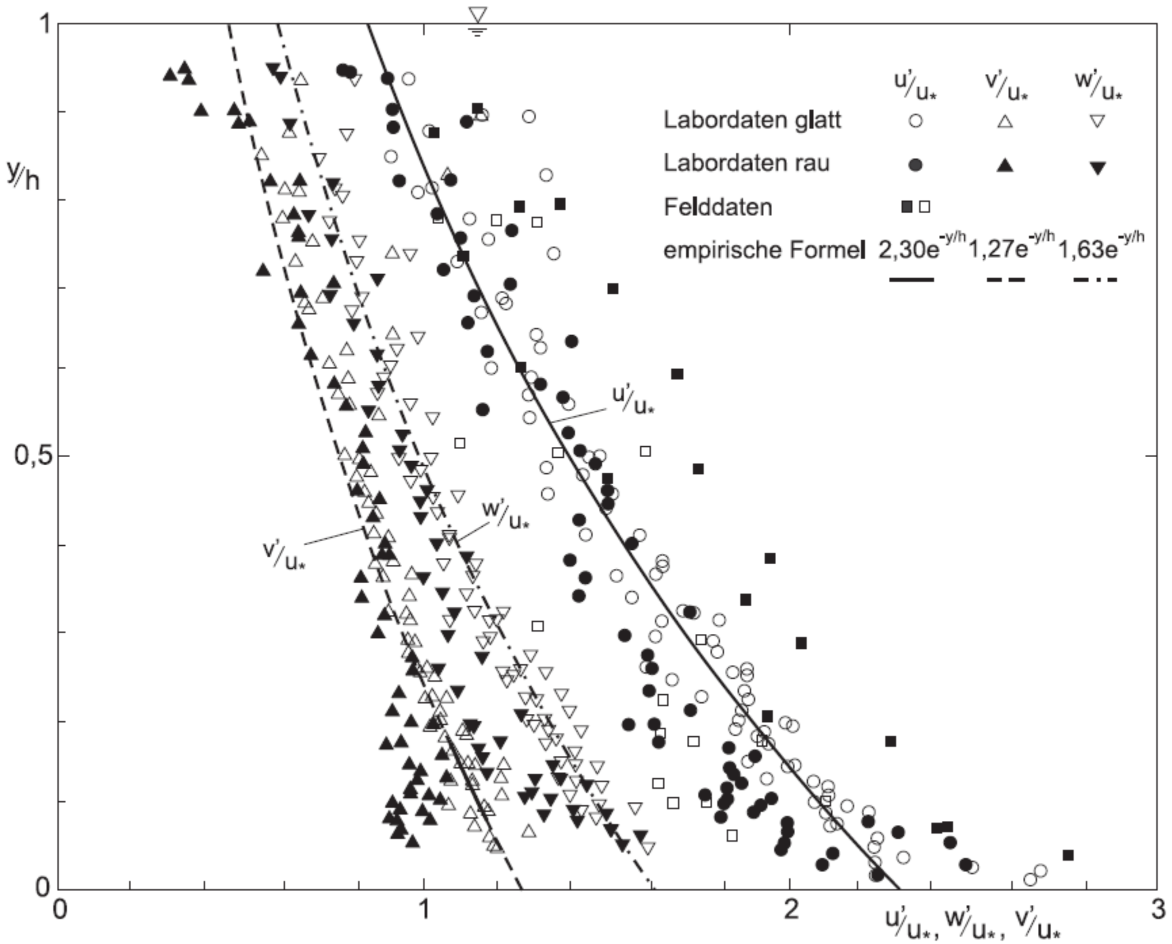
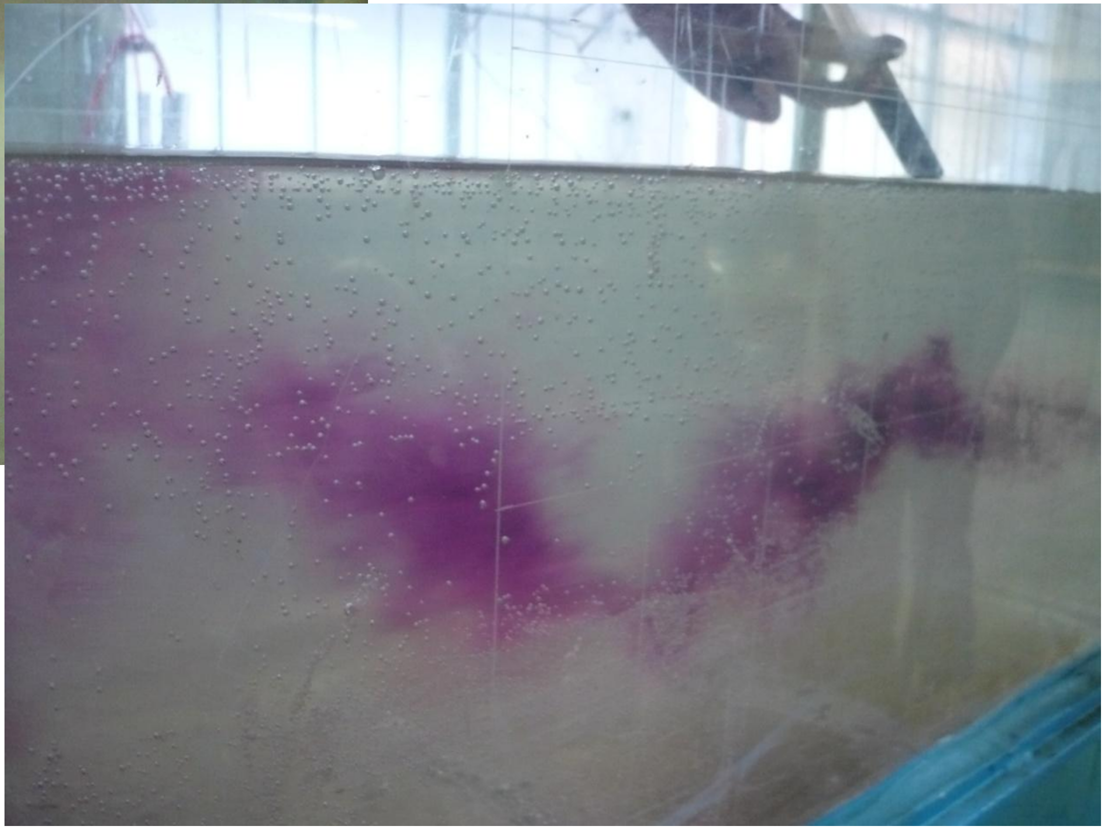


Abb. 2.4: Profile der Fluktuationsgeschwindigkeiten u',v' und w' normiert durch die Reibungsgeschwindigkeit u_* in turbulenten Gerinneströmungen (Daten nach Nezu und Nakagawa, 1993)

Jirka and Lang, 2009

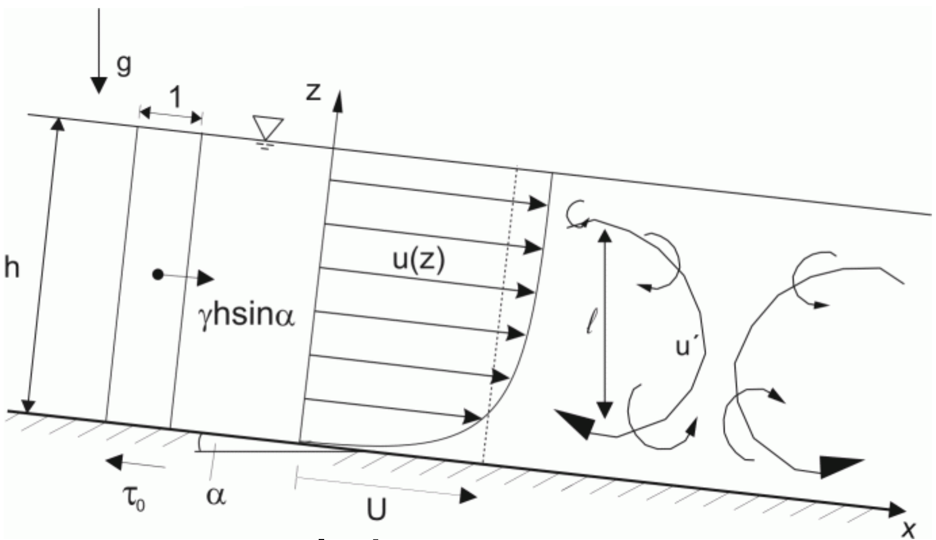
Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Visualização do escoamento e processos de mistura

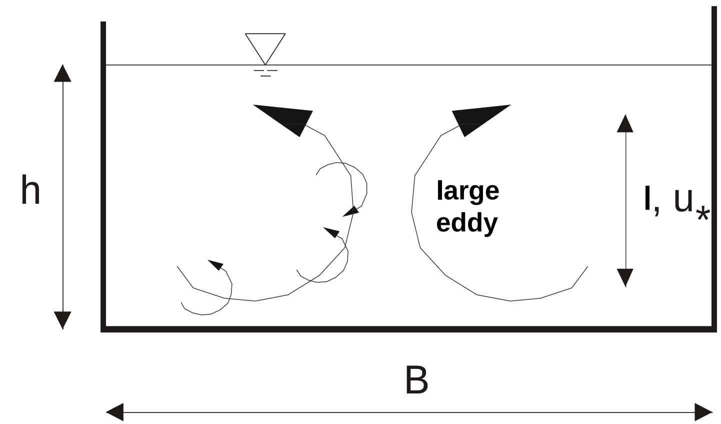


Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

B) Turbulent Diffusion in river



U = average velocity



large river B/h > (5 - 10)

- Eddy generation due to roughness:

$\boldsymbol{\tau_0} = \rho g h \sin\alpha \approx \rho g h I = g h I$

I = slope $\approx \sin\alpha$

$$\sqrt{\frac{\tau_0}{\rho}} = u_* = \sqrt{ghI}$$

$\mathbf{u_*} = \sqrt{ghI}$, Friction velocity,

typical $u_* = (0{,}05 – 0{,}1)U$

- Turbulent eddy spectrum:

u´ = eddy velocity

l $\approx$ **h** = eddy size

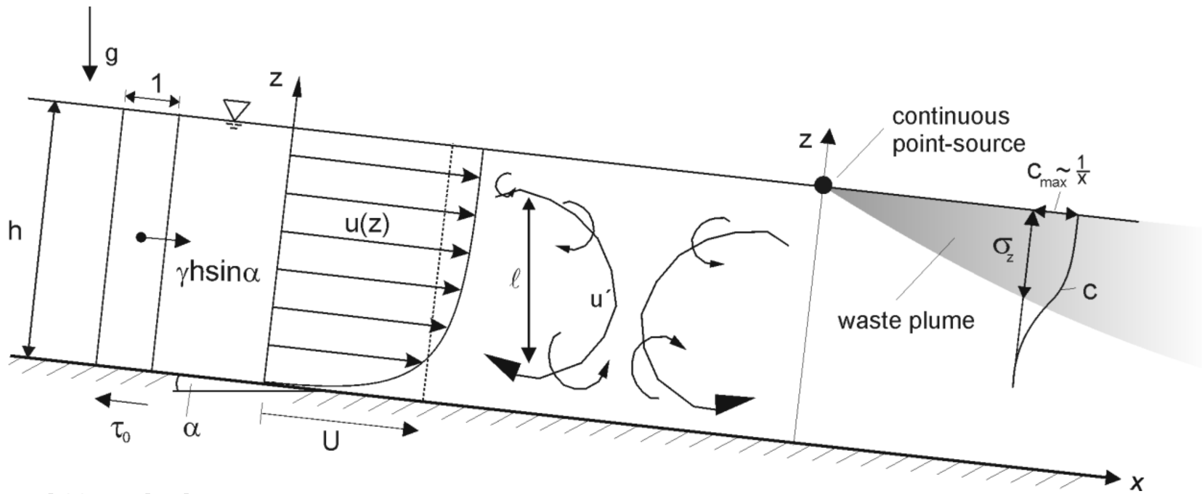
- Duffusion analogy ($D_m \sim \lambda_m u_m$):

E $\approx$ h u_* turbulent Diffusivity [m²/s]

E = 0,5(+/-50%) u_* h for rivers

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Vertical mixing (Advection + Diffusion)



- vertical Diffusivity: $E_z = \alpha_z u_* h$ $\alpha_z = 0{,}07$ +/- 50%
- vertical distribution: $\sigma_z = \sqrt{2E_z t} = \sqrt{2E_z \frac{x}{U}}$ **$t = \frac{x}{U}$ = „flowtime“ (Advection)**
- Vertical passive mixing (method-of-images)

Mixing if $\sigma_z \approx h$, then $x = x_{MV}$

$$x_{MV} \approx \frac{h^2 U}{2E} = \frac{h^2 U}{2(0{,}07 u_* h)} \approx 70h$$

0,10U

→ independent of velocity
→ morphology dominates!

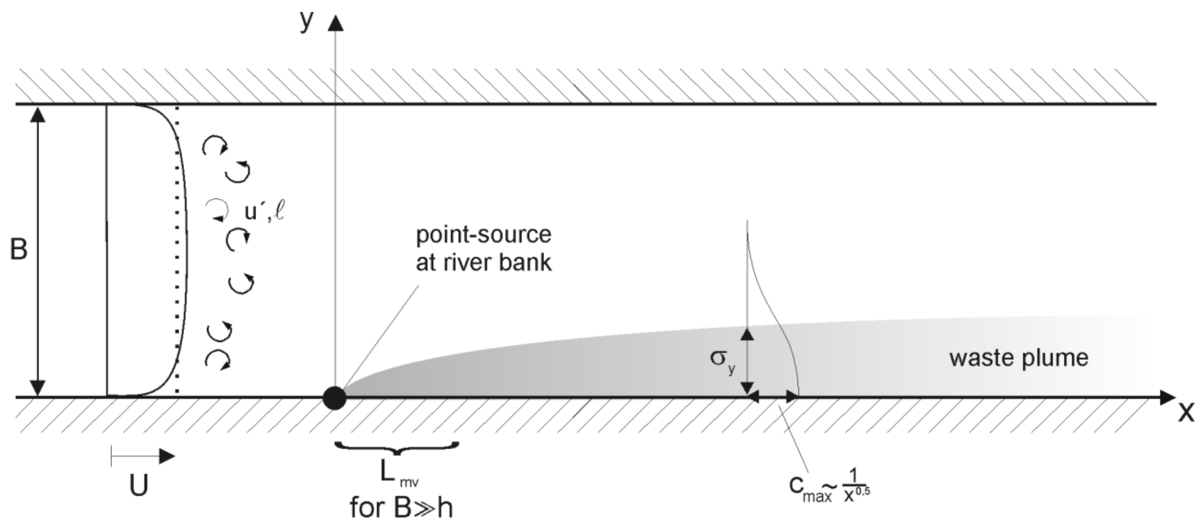
<u>Bsp. 1:</u> Rhein near Karlsruhe
B = 250m, h = 3m, U = 3m/s
x_{MV} = 70x3 = 210m ~fast!

<u>Bsp. 2:</u> Small river in Schwarzwald (Murg)
B = 5m, h = 0,5m
x_{MV} = 70x0,5 = 35m ~fast!

vertical mixing is fast process!

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Lateral mixing (Advection + Diffusion)



- Horizontal Diffusivity: $E_y = \alpha_y u_* h$ $\alpha_z = 0,5$ +/- 50%
  moderate heterogeneities
- Horizontal distribution: $\sigma_y = \sqrt{2E_y \frac{x}{U}}$
- Horizontal mixing (river bank)

Lateral mixing if $\sigma_y \approx B$, $x = x_{ML}$
→ independent of velocity
→ morphology dominates!

$$x_{ML} \approx \frac{B^2U}{2E} = \frac{B^2U}{2(0,5u_*h)} \approx 10\frac{B^2}{h}$$

0,10U

<u>Bsp. 1:</u> Rhein near Karlsruhe
$x_{ML} \approx 10x250^2/3$
$= 208.000$m $\approx$ 200km slow!

Bsp. 2: small river in Schwarzwald (Murg)
$x_{ML} \approx 10x5^2/0,5$
= 500m = 0,5km

lateral mixing is slow process!

Tobias Bleninger
Universidade Federal do Paraná (UFPR)
Departamento de Engenharia Ambiental(DEA)
www.tobias.br.ms

Referencias

- Lista de livros recomendados sobre Hidráulica Fluvial: www.iahr.net --> E-library --> Ressources --> River Engineering Textbook list
- Graf, W.H. (1998). Fluvial Hydraulics: Flow and Transport Processes in Channels of Simple Geometry. In collaboration with M.S. Altinakar, John Wiley and Sons, England, 681 pages [ISBN 0-471-97714-4] (capitulo de transporte de sedimentos online)
- Gerhard H. Jirka, Cornelia Lang. 2009. Einführung in die Gerinnehydraulik. Universitätsverlag Karlsruhe. ISBN: 978-3-86644-363-1. http://digbib.ubka.uni-karlsruhe.de/volltexte/1000011374
- Eduard Naudascher. 1992. Gerinne und Gerinnebauwerke. Springer-Verlag Wien New York. ISBN 3-21 1-82366-2
- Software e manuais do modelo HEC-RAS
- Materiais multimídias na hidráulica