2

Two-dimensional patterns, lattices and symmetry

2.1 Approaches to the study of crystal structures

In Chapter 1 we developed an understanding of simple crystal structures by first considering the ways in which atoms or ions could pack together and then introducing smaller atoms or ions into the interstices between the larger ones. This is a pragmatic approach as it not only provides us with an immediate and straightforward understanding of the atomic/ionic arrangements in some simple compounds, but also suggests the ways in which more complicated compounds can be built up.

However, it is not a systematic and rigorous approach, as all the possibilities of atomic arrangements in all crystal structures are not explored. The rigorous, and essentially mathematical, approach is to analyse and classify the geometrical characteristics of quite general two-dimensional patterns and then to extend the analysis to three dimensions to arrive at a completely general description of all the patterns to which atoms or molecules or groups of atoms or molecules might conform in the crystalline state.

These two distinct approaches—or strands of crystallographic thought—are apparent in the literature of the nineteenth and early twentieth centuries. In general, it was the metallurgists and chemists, such as Tammann* and Pope*, who were the pragmatists, and the theoreticians and geometers, such as Fedorov* and Schoenflies*, who were the analysts. It might be thought that the analytical is necessarily superior to the pragmatic approach because its generality and comprehensiveness provides a much more powerful starting point for progress to be made in the discovery and interpretation of the crystal structures of more and more complex substances. But this is not so. It was, after all, the simple models of sodium chloride and zinc blende of Pope (such as we also constructed in Chapter 1) that helped to provide the Braggs* with the necessary insight into crystal structures to enable them to make their great advances in the interpretation of X-ray diffraction photographs. In the same way, 40 years later, the discovery of the structure of DNA by Watson and Crick was based as much upon structural and chemical knowledge and intuition, together with model building, as upon formal crystallographic theory.

However, a more general appreciation of the different patterns into which atoms and molecules may be arranged is essential, because it leads to an understanding of the important concepts of symmetry, motifs and lattices. The topic need not be pursued rigorously—in fact it is unwise to do so because we might quickly 'lose sight of the wood for the trees!' The essential ideas can be appreciated in two dimensions, the subject of this chapter. The extension to three dimensions (Chapters 3 and 4) which relates to 'real crystal structures', should then present no conceptual difficulties.

2.2 Two-dimensional patterns and lattices

Consider the pattern of Fig. 2.1(a), which is made up of the letter **R** repeated indefinitely. What does **R** represent? Anything you like—a 'two-dimensional molecule', a cluster of atoms or whatever. Representing the 'molecule' as an **R**, an *asymmetric* shape, is in effect representing an *asymmetric* molecule. We shall discuss the different types or elements of symmetry in detail in Section 2.3 below, but for the moment our general everyday knowledge is enough. For example, consider the symmetry of the letters **RMS**. **R** is asymmetrical. **M** consists of two equal sides, each of which is a reflection or mirror image of the other, there is a **mirror line** of symmetry down the centre indicated by the letter m, thus $\underset{\leftarrow m}{\mathbf{M}}$. There is no mirror line in the **S**, but if it is rotated 180° about a point in its centre, an identical **S** appears; there is a **two-fold rotation axis** usually called a **diad axis** at the centre of the **S**. This is represented by a little lens-shape at the axis of rotation: **S**.

In Fig. 2.1(a) **R**, the repeating 'unit of pattern' is called the **motif**. These motifs may be considered to be situated at or near the intersections of an (imaginary) grid. The grid is called the **lattice** and the intersections are called **lattice points**.

Let us now draw this underlying lattice in Fig. 2.1(a). First we have to decide where to place each lattice point in relation to each motif: anywhere will do—above, below, to one side, in the 'middle' of the motif—the only requirement is that the *same* position with respect to the motif is chosen every time. We shall choose a position a little below the motif, as shown in Fig. 2.1(b). Now there are an infinite number of ways in which the lattice points may be 'joined up' (i.e. an infinite number of ways of drawing a lattice or grid of lines through lattice points). In practice, a grid is usually chosen which 'joins up' adjacent lattice points to give the lattice as shown in Fig. 2.1(c), and a unit cell of the lattice may also be outlined. Clearly, if we know (1) the size and shape of the

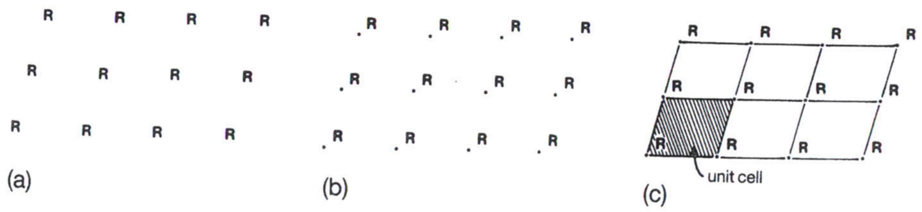

Fig. 2.1. (a) A pattern with the motif **R**, (b) with the lattice points indicated and (c) the lattice and a unit cell outlined.

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* Denotes biographical notes available in Appendix 3.

unit cell and (2) the motif which each lattice point represents, including its orientation with respect to the lattice point, we can draw the whole pattern or build up the whole structure indefinitely. The unit cell of the lattice and the motif therefore define the whole pattern or structure. This is very simple: but observe an importance consequence. Each motif is identical and, for an infinitely extended pattern, the environment (i.e. the spatial distribution of the surrounding motifs, and their orientation) around each motif is identical. This provides us with the definition of a lattice (which applies equally in two and three dimensions): *a lattice is an array of points in space in which the environment of each point is identical.* Again it should be stressed that by environment we mean the spatial distribution and orientation of the surrounding points.

Like all simple definitions (and indeed ideas), this definition of a lattice is often not fully appreciated; there is, to use a colloquial expression, 'more to it than meets the eye!' This is particularly the case when we come to three-dimensional lattices (Chapter 4), but, for the two-dimensional case, consider the patterns of points in Fig. 2.2 (which should be thought of as extending infinitely). Of these only (a) and (d) constitute a lattice; in (b) and (c) the points are certainly in a *regular* array, but the surroundings of each point are *not* all identical.

Figures 2.2(a) and (d) represent two two-dimensional lattice types, named **oblique** and **rectangular**, respectively, in view of the shapes of their unit cells. But what is the distinction between the oblique and rectangular lattices? Surely the rectangular lattice is just a special case of the oblique, i.e. with a 90° angle?

The distinction arises from different symmetries of the two lattices, and requires us to extend our everyday notions of symmetry and to classify a series of symmetry elements. This precise knowledge of symmetry can then be applied to both the motif and the lattice and will show that there are a limited number of patterns with different symmetries (only seventeen) and a limited number of two-dimensional lattices (only five).

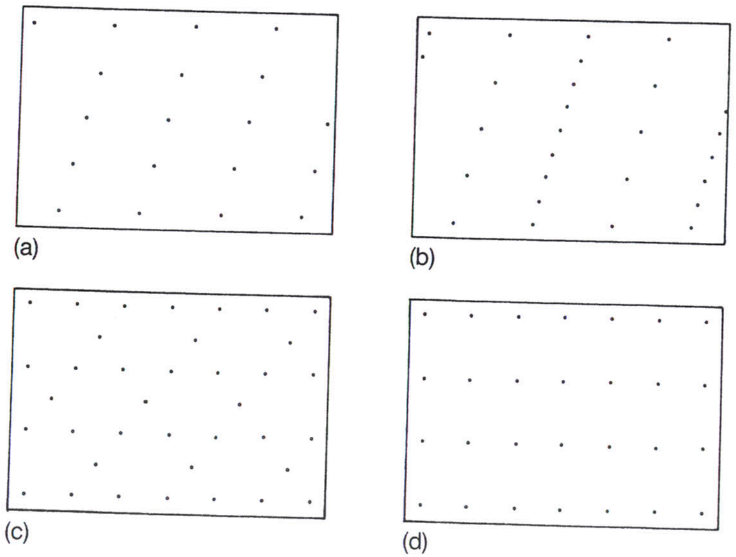
(a) (b) (c) (d)

Fig. 2.2. Patterns of points. Only (a) and (d) constitute lattices.

2.3 Two-dimensional symmetry elements

The clearest way of developing the concept of symmetry is to begin with an asymmetrical 'object'—say the **R** of Fig. 2.1—then to add successively mirror lines and axes of symmetry and to see how the **R** is repeated to form different patterns or groups. The different patterns or groups of **R**s which are produced correspond, of course, to objects or projections of molecules (i.e. 'two-dimensional molecules') with different symmetries which are not possessed by the **R** alone.

For example, consider Fig. 2.3(a). 'Right-' and 'left-'handed **R**s are reflected in the 'vertical' mirror line between them. This pair of **R**s has the same mirror symmetry as the letter **M**, or the projection of the *cis*-difluoroethene molecule. Now add another 'horizontal' mirror line as in Fig. 2.3(b). A group of four **R**s—two right and two left handed—is produced. This group has the same symmetry as the single letter **H** or the projection of the ethene molecule.

The **R** may also be repeated with a diad (two-fold rotation) axis, as in Fig.

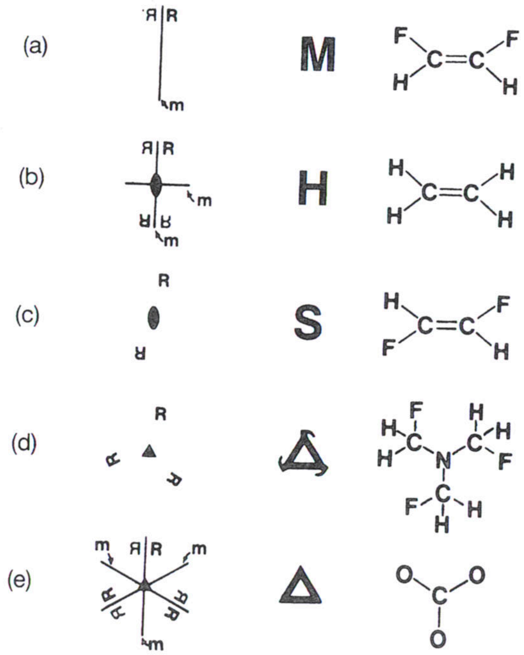

Fig. 2.3. Generation of motifs (a)–(e) with different symmetries (five out of the ten plane point groups) and examples of two-dimensional symbols and (right) molecules and ions. (a) *Cis*-difluoroethene, (b) ethene, (c) *trans*-difluoroethene, (d) tri(fluoroalkyl)ammonia molecule and (e) carbonate ion.

2.3(c). The two **R**s—both right handed—have the same symmetry as the letter **S** as we saw above, or the *trans*-difluoroethene molecule. Now look back to the group of **R**s in Fig. 2.3(b); notice that they are also related by a diad (two-fold rotation axis) at the intersection of the mirror lines: the action of reflecting the **R**s across two perpendicular mirror lines 'automatically' generates the two-fold symmetry as well. This effect, where the action of one symmetry element generates another, is quite general, as we shall see below.

Mirror lines and diad axes of symmetry are just two of the symmetry elements in two dimensions. In addition, there are three-fold, rotation or **triad** axes (represented by a little triangle ▲), four-fold rotation or **tetrad** axes (represented by a little square ■) and six-fold rotation or **hexad** axes (represented by a little hexagon ⬢). Figure 2.3(d) shows the **R** related by a triad (three-fold) axis. This paper windmill and the projection of the trifluoralkylammonia molecule also have this same symmetry. Now add a 'vertical' mirror line as in Fig. 2.3(e). Three more (left-handed) **R**s are generated, and at the same time, the **R**s are mirror related not just in the vertical line but also in two lines inclined at 60° as shown, another example of additional symmetry elements—in this case mirror lines— being automatically generated.

This procedure—of generating groups of **R**s which represent motifs with different symmetries—may be repeated with tetrad (four-fold) and hexad (six-fold) rotation axes of symmetry. Altogether there are ten such symmetries in two dimensions or **ten plane point groups***, so called because all the symmetry elements pass through one point. The ten plane point groups are labelled with 'shorthand' symbols which indicate the symmetry elements present; *m* for one mirror plane, *mm* or *mm*2 for two mirror planes (plus diad), 2 for a diad, 3 for a triad, 3*m* for a triad and three mirror planes and so on. The assiduous reader should complete Fig. 2.3 for himself or herself including, as well as the **R**s, common objects and molecules with the appropriate symmetries. In addition, it will become obvious why it is not possible to have five-, seven-, eight-, etc., fold rotation axes of symmetry. A five-fold 'pentad' axis, for example, would require five lattice points to be equally arranged about the axis, but such an arrangement of points could not be put together to form a lattice. Try it and see!

2.4 The five plane lattices

Having examined some of the types of symmetry which a two-dimensional motif can possess (Fig. 2.3), we can now determine how many two-dimensional or plane lattices there are. The important criterion is this: the lattice itself must possess at least the symmmetry of the motif; it may possess more symmetry elements but it cannot possess fewer. Or, put another way, the symmetry of the arrangement of lattice points around each lattice point must be the same as, or

greater than, the symmetry of the motif. A few examples will make this clear. Consider the rectangular lattice of Fig. 2.2(d). Through each lattice point and halfway between there are vertical and horizontal lines of symmetry intersecting diad axes. The motif of Fig. 2.3(b) also has this symmetry and therefore it follows that a pattern with such a motif will have a rectangular lattice. The motif of Fig. 2.3(a) has one mirror line, and a pattern with this motif will also have a rectangular lattice—in this case the symmetry of the lattice is greater than that of the motif. Note, on the other hand, that the motifs with one or two mirror lines of symmetry (Figs. 2.3(a) and (b)) cannot occur in a pattern with the oblique lattice because the oblique lattice does not itself have any mirror lines of symmetry.

This procedure can be applied to all the other motifs. For example, a motif with tetrad (four-fold) symmetry applies to a pattern with a square lattice and a motif with triad (three-fold) symmetry (Figs 2.3(d) and (e)) and hexad (six-fold) symmetry will apply to a pattern with an hexagonal lattice. Altogether, five two-dimensional or plane lattices may be worked out, as shown in Fig. 2.4(a). They are described by the shapes of the unit cells which are drawn between lattice points—oblique *p*, rectangular *p*, rectangular *c* (which is distinguished from rectangular *p* by having an additional lattice point in the centre of the cell), square *p* and hexagonal *p*. Notice again that additional symmetry elements are generated 'in between' the lattice points as shown in Fig. 2.4(a) (right). For example, in the square lattice there is a tetrad at the centre of the cell, diads halfway along the edges and vertical, horizontal and diagonal mirror lines as well as the tetrads situated at the lattice points.

All two-dimensional patterns must be based upon one of these five plane lattices; no others are possible. This may seem very surprising—surely other shapes of unit cells are possible? The answer is 'yes', a large number of unit cell shapes are possible, but the pattern of lattice points which they describe will always be one of the five of Fig. 2.4(a). For example, the rectangular *c* lattice may also be described as a rhombic *p* or diamond *p* lattice, depending upon which unit cell is chosen to 'join up' the lattice points (Fig. 2.4(b)). These are just two alternative descriptions of the *same* arrangement of lattice points. So the choice of unit cell is arbitrary: *any* four lattice points can be joined up to form a unit cell. In practice we take a sensible course and mostly choose a unit cell that is as small as possible—or 'primitive' (symbol *p*)—which does not contain other lattice points within it. Sometimes a larger cell is more useful because the axes joining up the sides are at 90°. Examples are the rhombic or diamond lattice which is identical to the rectangular centred lattice described above and, to take an important three-dimensional case, the cubic cell (Fig. 1.6(c)) which is used to describe the ccp structure in preference to the primitive rhombohedral cell (Fig. 1.7(c)).

Now as there are ten point group symmetries which a motif can possess (five of which are shown in Fig 2.3), it may be thought that there are therefore only ten different types of two-dimensional patterns, distributed among the five plane lattices. However, there is a complication: the combination of a point group

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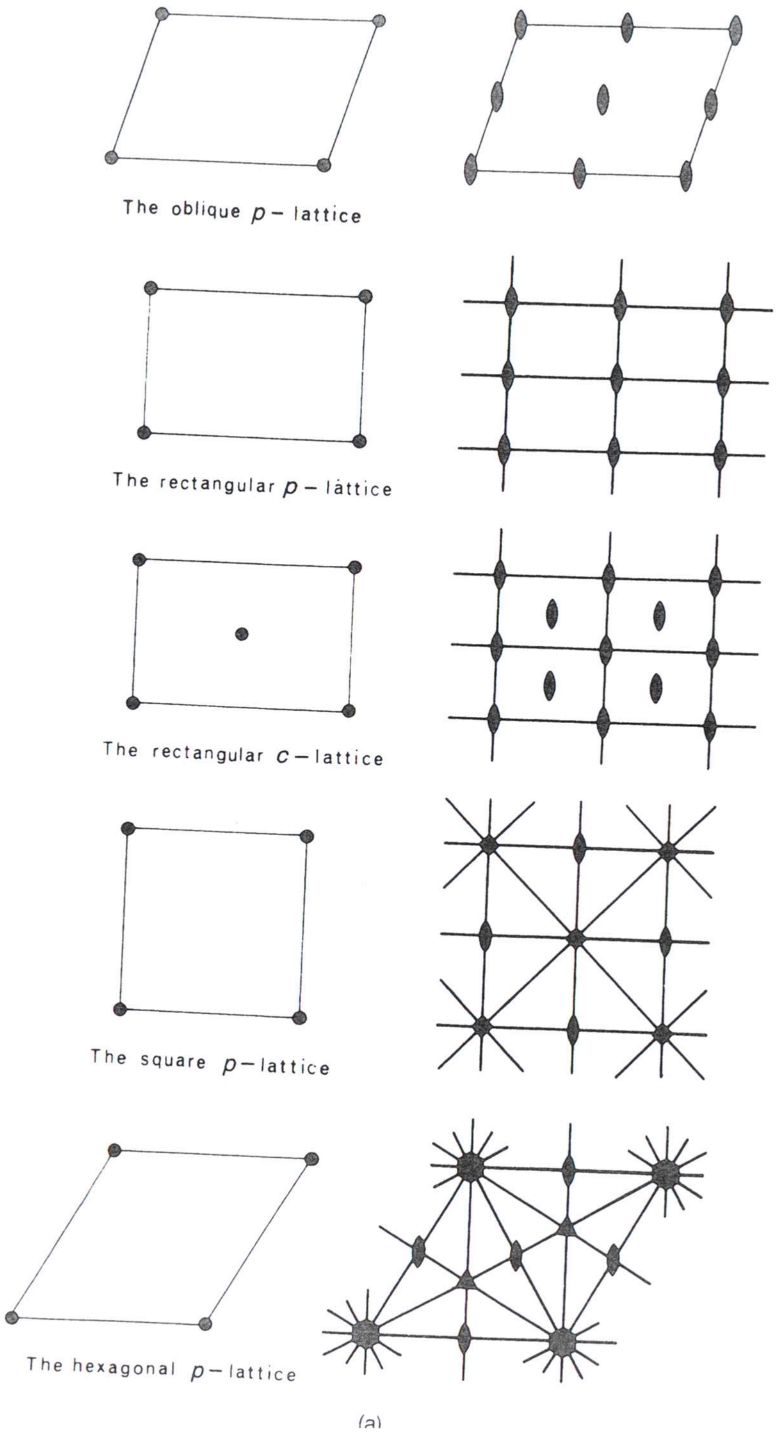
[*] This is only true for the case of repeating patterns. Separate objects in nature may have five-fold (pentagonal) symmetry (a starfish for example), or higher order axes of symmetry, but these do not occur in repeating patterns. The ten plane point groups which occur in plane patterns as discussed below are 1 (asymmetric), 2, 3, 4, 6, *m*, *mm*, 3*m*, 4*m*, and 6*m*.



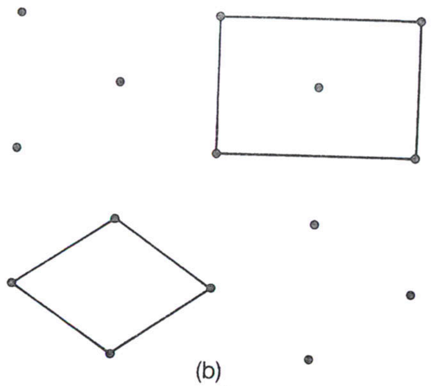


Fig. 2.4. (a) Unit cells of the five plane lattices (left), showing (right) the symmetry elements present (from *Essentials of Crystallography*, by D. McKie and C. McKie, Blackwell, 1986). (b) The rectangular c lattice, showing the alternative primitive (rhombic p or diamond p) unit cell.

symmetry with a lattice can give rise to an additional symmetry element called a **glide line**. Consider the two patterns in Fig. 2.5, both of which have a rectangular lattice. In Fig. 2.5(a) the motif has mirror symmetry as in Fig. 2.3(a); it consists of a pair of right- and left-handed **R**s. In Fig. 2.5(b) there is still a reflection—still pairs of right- and left-handed **R**s—but one set of **R**s has been translated, or glided half a lattice spacing. This symmetry is called a **reflection glide** or simply a **glide line of symmetry**. Notice that glide lines also arise automatically in the centre of the unit cell of Fig. 2.5(b) as do mirror lines in Fig. 2.5(a).

The presence of the glide lines also has important consequences regarding the symmetry of the motif. In Fig. 2.5(a) the motif has mirror symmetry but in Fig. 2.5(b) it does not: the pair of right- and left-handed **R**s is asymmetric. It is the repetition of the translational symmetry elements—the glide lines—that determines the overall rectangular symmetry of the pattern.

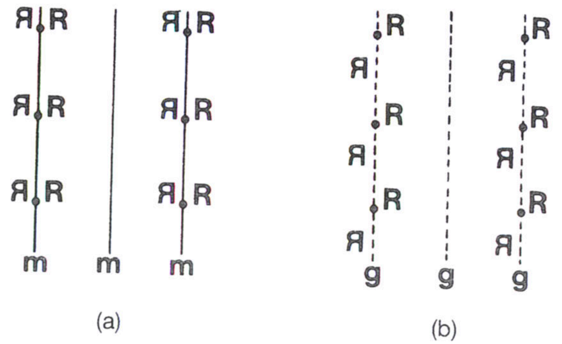


Fig. 2.5. Patterns with (a) reflection symmetry and (b) glide-reflection symmetry. The mirror lines (m) and glide lines (g) are indicated.

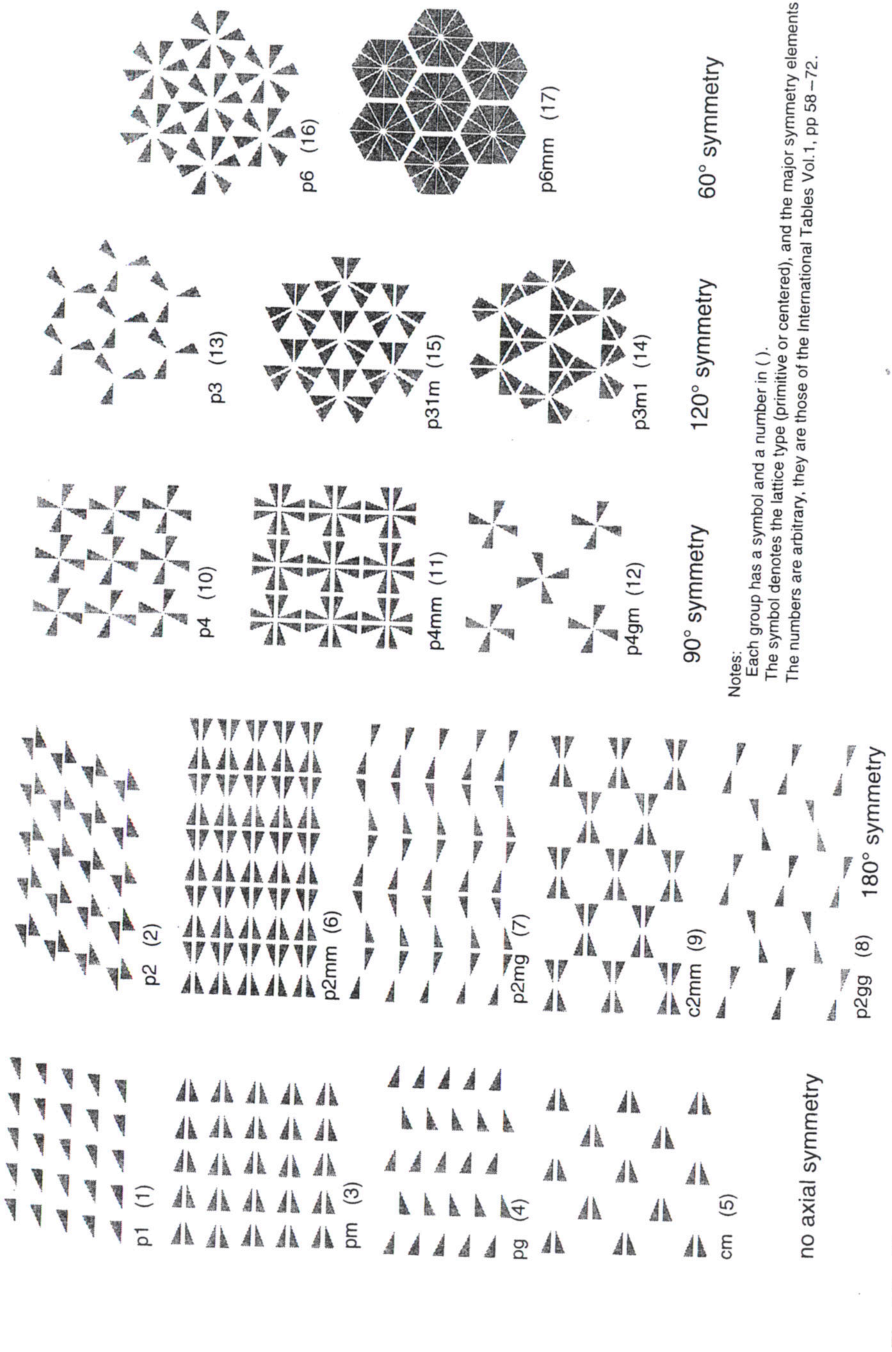


Fig. 2.6. The seventeen plane groups (from *Point and Plane Groups* by K. M. Crennell). The numbering 1–17 is that which is arbitrarily assigned in the International Tables. Note that the 'shorthand' symbols do not necessarily indicate all the symmetry elements which are present in the patterns.

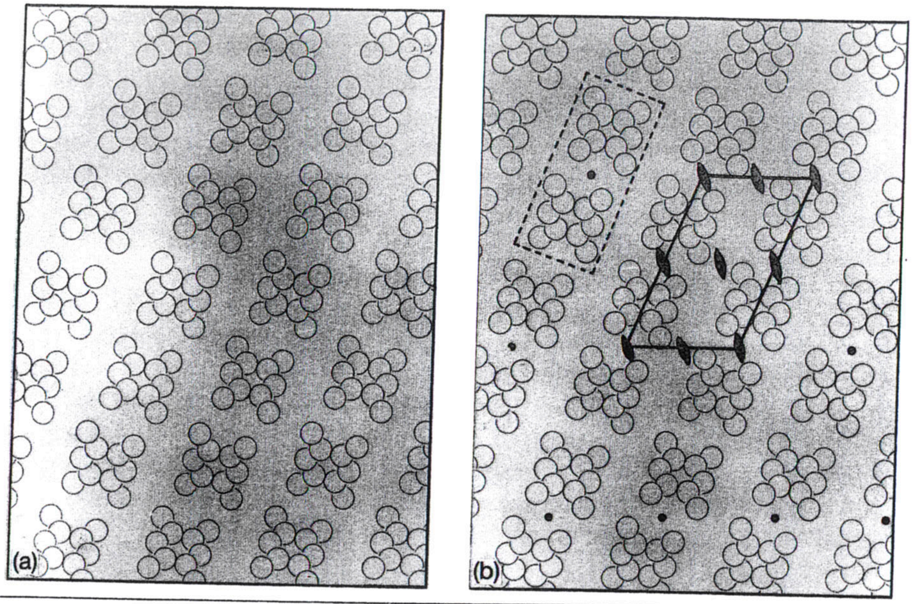


Fig. 2.7. Projection (a) of the structure of $C_6H_2(CH_3)_4$ (from *Contemporary Crystallography*, by M. J. Buerger, McGraw-Hill, 1970), with (b) the motif, lattice and symmetry elements indicated.

2.5 The seventeen plane groups

Glide lines give seven more two-dimensional patterns, giving seventeen in all—the seventeen **plane groups**. On a macroscopic scale the glide symmetry in a crystal would appear as simple mirror symmetry—the shift between the mirror-related parts of the motif would only by observable in an electron microscope which was able to resolve the individual mirror-related parts of the motif, i.e. distances of the order of 0.5–2 Å (50–200 pm).

The seventeen plane groups are shown in Fig. 2.6. They are labelled by 'shorthand' symbols which indicate the type of lattice (*p* for primitive, *c* for centred) and the symmetry elements present, *m* for mirror lines, *g* for glide lines, 4 for tetrads and so on.

It is essential to practice recognizing the motifs, symmetry elements and lattice types in two-dimensional patterns and therefore to find to which of the seventeen plane groups they belong. Any regular patterned object will do—wallpapers, fabric designs, or the examples at the end of this chapter. Figure 2.7 indicates the procedure you should follow. Cover up Fig. 2.7(b) and examine only Fig. 2.7(a); it is a projection of molecules of $C_6H_2(CH_3)_4$. You should recognize that the molecules or groups of atoms are *not* identical in this two-dimensional projection. The motif is a *pair* of such molecules and this is the 'unit of pattern' that is repeated. Now look for symmetry elements and (using a piece of tracing paper) indicate the positions of all of these on the pattern. Compare your pattern of symmetry elements with those shown in Fig. 2.7(b). If you did not obtain the same result you have not been looking carefully enough! Finally, insert the lattice points—one for each motif. Anywhere will do, but it is convenient to have them coincide with a symmetry element, as has been done in Fig. 2.7(b). The lattice is clearly oblique and the plane group is *p*2 (see Fig. 2.6).

The motifs of the seventeen patterns in Fig. 2.6 should be identified by circling them (lightly in pencil in case you make a mistake). You will find that, as a result of the presence of glide lines of symmetry, there are three plane groups (*pg*, *p*2*gg* and *p*4*gm*) in which the motif is asymmetric and one (*p*2*mg*) in which it has only one mirror-line of symmetry.

Another systematic way of identifying a plane pattern is to follow the 'flow diagram' shown in Fig. 2.8. The first step is to identify the highest order of rotation symmetry present, then to determine the presence or absence of reflection symmetry and so on through a series of 'yes' and 'no' answers, finally identifying one of the seventeen plane patterns whose plane group symbols are indicated 'in boxes', corresponding to those given in Fig. 2.6.

2.6 One-dimensional symmetry: border or frieze patterns

Identifying the number of one-dimensional patterns provides us with a good exercise in applying our more general knowledge of plane patterns. It is also a useful exercise in that it tells us about the different types of patterns that can be designed for the borders of wallpapers, edges of dress fabrics, friezes and cornices in buildings, and so on.

In plane patterns the symmetry operations and symmetry elements are (clearly) repeated in a plane; in one-dimensional patterns they can only be repeated in or along a line—i.e. the line or long direction of the border or frieze. This restriction immediately rules out all rotational symmetry elements with the exception of diads: two-fold symmetry alone can be repeated in a line: three-, four-, and six-fold symmetry elements require the repetition of a motif in directions other than the line of the border. For the same reason glide-reflection lines of symmetry, other than that along the line of the border, are ruled out. Mirror lines of symmetry are restricted to those along, and perpendicular to, the line of the border.

These restrictions result in seven one-dimensional groups, shown in Fig. 2.9. It is a good and satisfying exercise for you to derive these from first principles as outlined above. It is also useful to compare Fig. 2.9 with Fig. 2.6; the bracketed symbols in Fig. 2.9 indicate from which plane pattern the one-dimensional pattern may be derived. Notice that in one case two one-dimensional patterns—these with 'horizontal' and 'vertical' mirror planes—are derived from one plane pattern (*pm*). This is because the mirror lines in the plane group *pm* can be oriented either along, or perpendicular to, the line of the one-dimensional pattern.

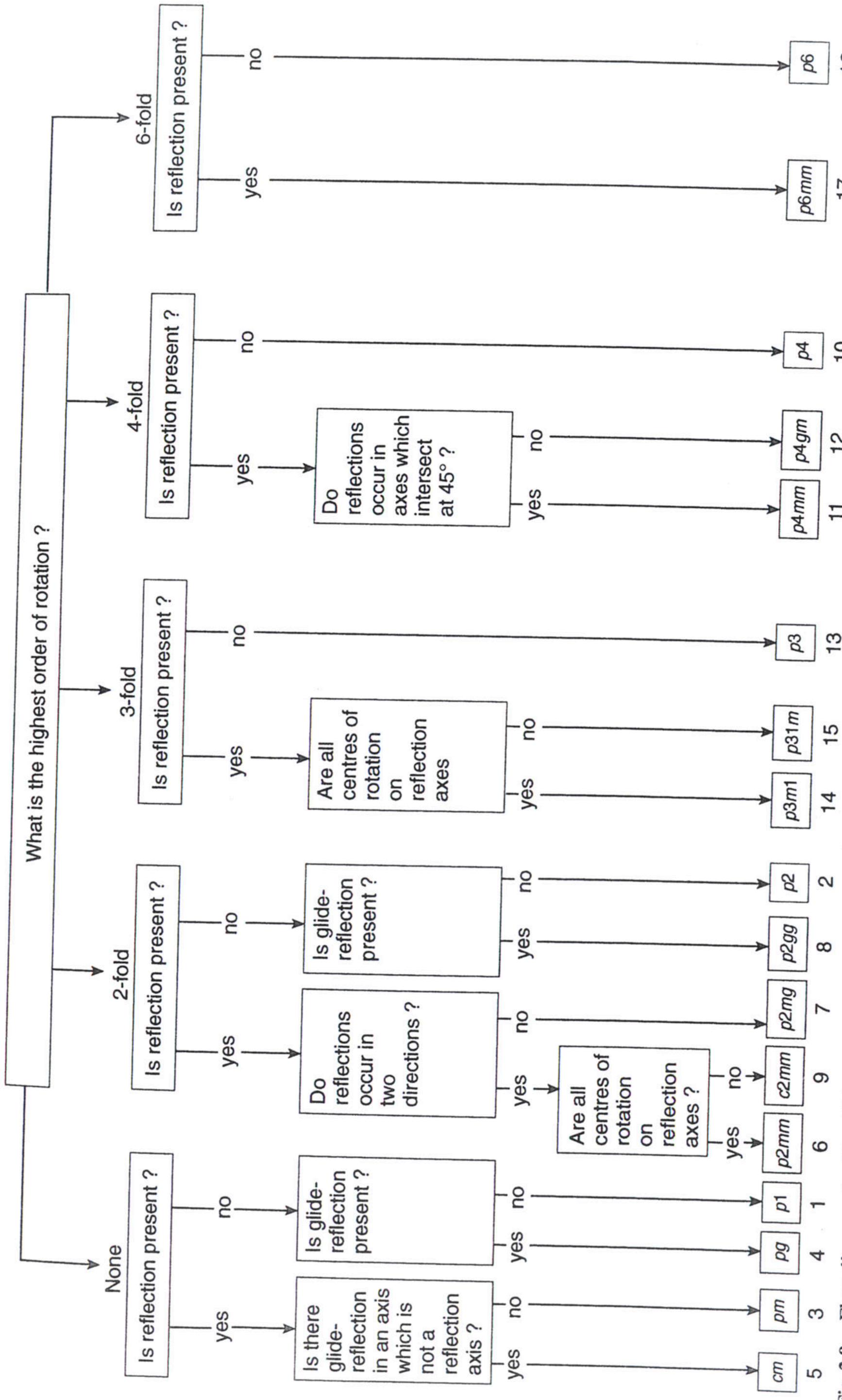


Fig. 2.8. Flow diagram for identifying one of the seventeen plane patterns (redrawn from *The Geometry of Regular Repeating Patterns* by M. A. Hann and G. M. Thomson, the Textile Institute, Manchester, 1992). The numbering is that which is arbitrarily assigned in the International Tables (see Fig 2.6).

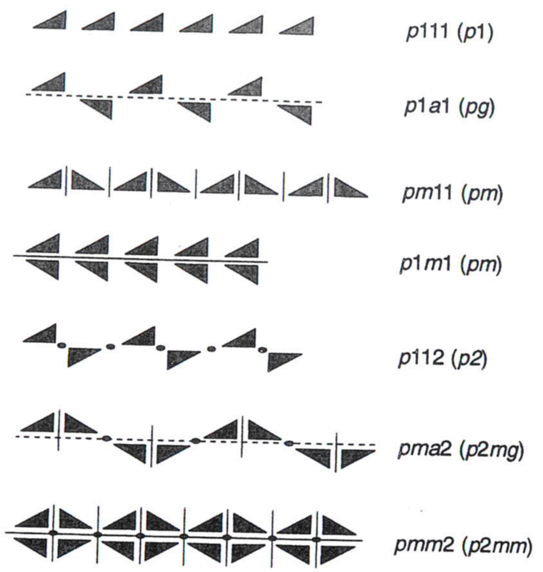


Fig. 2.9. The seven one-dimensional groups or seven classes of border or frieze patterns with their symmetry symbols (bracketed), the plane pattern symbols from which they may be derived. Solid lines indicate mirror lines, dotted lines indicate glide lines and symbols indicate diads (redrawn from *The Geometry of Regular Repeating Patterns*, loc. cit.).

Figure 2.15 (see Exercise 2.6) also shows examples of some of the border patterns. You can practice recognizing such patterns either by overlaying the pattern with a piece of tracing paper, and indicating the positions of the diads, mirror and glide lines as described above for plane patterns or by following the flow diagram (Fig. 2.10).

2.7 Symmetry, patterns and cultures

We have a rich inheritance of plane and border patterns in printed and woven textiles, wallpapers, bricks and tiles which have been designed and made by countless craftsmen and artisans in the past 'without benefit of crystallography'. The question we may now ask is: 'Have all the seventeen plane groups and seven one-dimensional groups been utilized in pattern design or are some patterns and some symmetries more evident than others? If so, is there any relationship between the preponderance or absence of certain types of symmetry elements in patterns and the civilization or culture which produced them?'

Questions such as these have exercised the minds of archaeologists, anthropologists and historians of art and design. They are, to be sure, questions more of cultural than crystallographic significance, but patterns play such a large part in our everyday experience that a crystallographer can hardly fail to be absorbed by them, just as he or she is absorbed by the three-dimensional patterns of crystals.

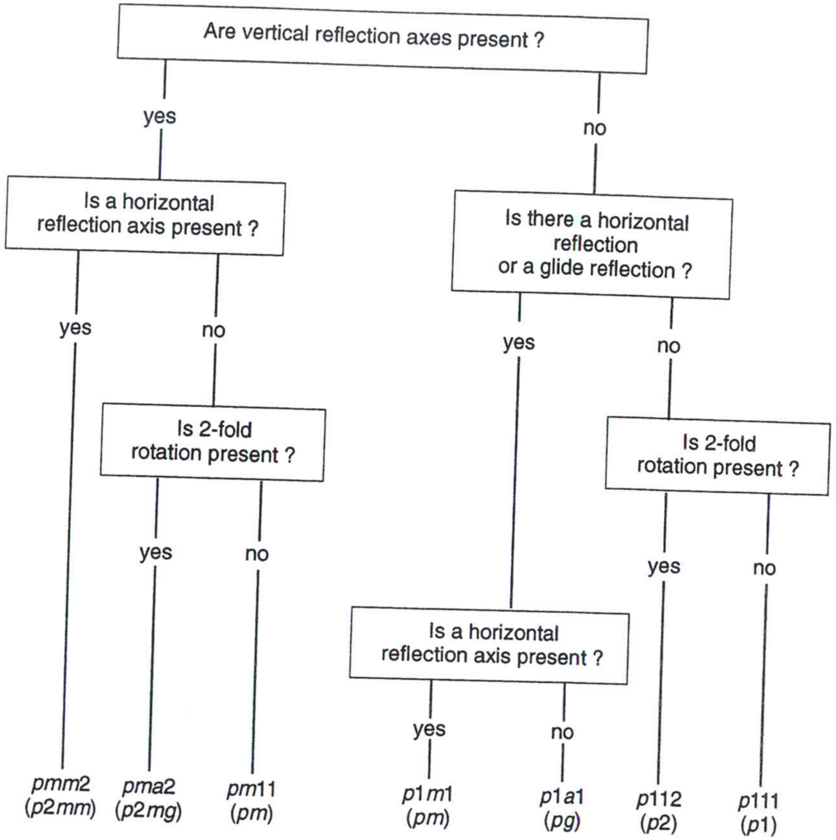


Fig. 2.10. Flow diagram for identifying one of the seven border patterns (from *The Geometry of Regular Repeating Patterns*, loc. cit.).

The study of plane and one-dimensional patterns (and indeed three-dimensional (space) patterns) is complicated by the question of colour—'real' colours in the case of plane and one-dimensional patterns, or colours representing some property, such as electron spin direction or magnetic moment, in space patterns (Chapter 4). Colour changes may also be analysed in terms of symmetry elements in which colours are alternated in a systematic way. Clearly, the greater the number of colours, the greater the complexity. The simplest cases to consider are two-colour (e.g. black and white) patterns. Figure 2.11 shows the generation of plane motifs through the operation of what are called counterchange or colour symmetry elements, which are distinguished from ordinary (rotation) axes and mirror lines of symmetry by a prime superscript. For example, the operation of a $2'$ axis is a twice repeated rotation of an asymmetric object by 180° plus a colour change at each rotation; the operation of an m' mirror line is a reflection plus colour change. Figure 2.11 shows the operation of all the two-dimensional counterchange point symmetry elements.

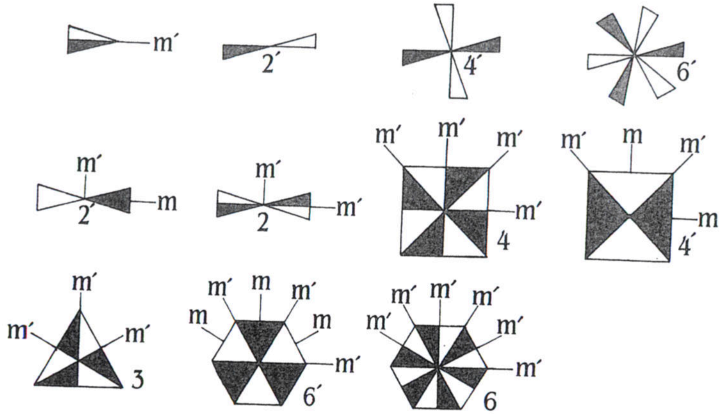


Fig. 2.11. The operation of two-colour (black/white) counterchange symmetry elements (denoted by prime superscripts).

Accounting for two-colour symmetry gives rise to a total of forty-six (rather than seventeen) plane patterns and seventeen (rather than seven) one-dimensional patterns.

Probably the most influential and pioneering study of patterns was *The Grammar of Ornament* by Owen Jones, first published in 1856*. Owen Jones attempted to categorize both plane and border patterns in terms of the different cultures that produced them, and although the symmetry aspects of patterns are touched on in the most fragmentary way, there is no doubt that the superb illustrations and encyclopaedic character of the book provided later writers with material which could be classified and analysed in crystallographic terms. Perhaps the best known of these was M. C. Escher (1898–1971) who drew inspiration for his drawings of tessellated figures from visits to the Alhambra in the 1930s, and also presumably from Owen Jones' chapter on 'Moresque Ornament' in which he describes the Alhambra as 'the very summit of Moorish art, as the Parthenon is of Greek art'. Escher's patterns encompass all the seventeen plane groups, eleven of which are represented in the Alhambra.

More recent work has identified clear preponderances of certain plane symmetry groups, and the absences of others†. For example, nearly 50% of traditional Javanese batik (wax-resist textile) patterns belong to plane group $4mm$ (Fig. 2.6), others, such as $p3$, $p3m1$, $p31m$ and $p6$ are wholly absent. In Jacquard-woven French silks of the last decade of the nineteenth century, nearly 80% of the patterns belong to plane group pg. In Japanese textile designs of the Edo period *all* plane groups are represented, with a marked preponderance for groups $p2mm$ and $c2mm$. What these differences mean, or tell us about the cultures which gave rise to them, is, as the saying goes, 'another question'.

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* Owen Jones. *The Grammar of Ornament*, Day & Sons Ltd., London, reprinted by Studio Editions, London (1986).

† M. A. Hann. *Symmetry of Regular Repeating Patterns: Case studies from various cultural settings*. ournal of the Textile Institute (1992), Vol. 83, pp. 579–580.

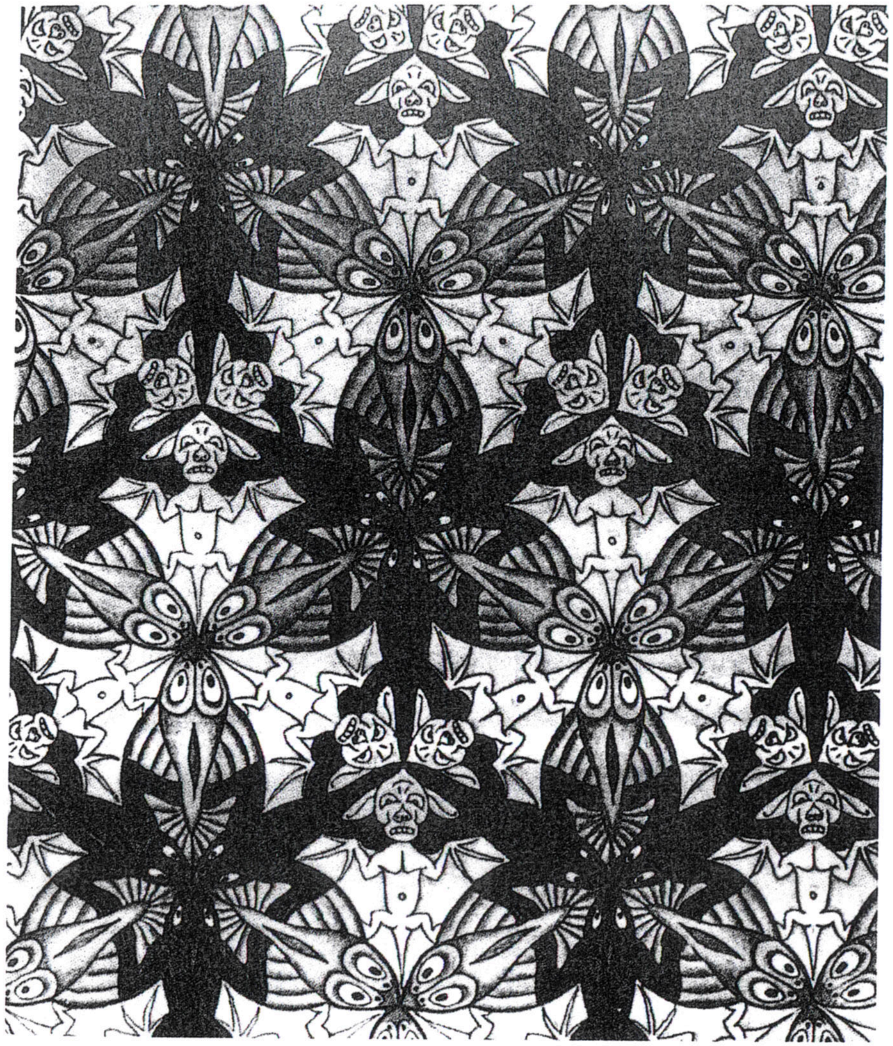

Fig. 2.12. A plane pattern (from *Symmetry Aspects of M. C. Escher's Periodic Drawings*, 2nd edn, by C. H. MacGillavry. Published for the International Union of Crystallography by Bohn, Scheltema and Holkema, Utrecht, 1976).

Exercises

2.1 Lay tracing paper over the plane patterns in Fig. 2.6. Outline a unit cell in each case and indicate the positions of all the symmetry elements within the unit cell. Notice in particular the differences in the distribution of the triad axes and mirror lines in the plane groups $p31m$ and $p3m1$.

2.2 Figure 2.12 is a design by M. C. Escher. Using a tracing paper overlay, indicate the positions of all the symmetry elements. With the help of the flow diagram (Fig. 2.8), determine the plane lattice type.

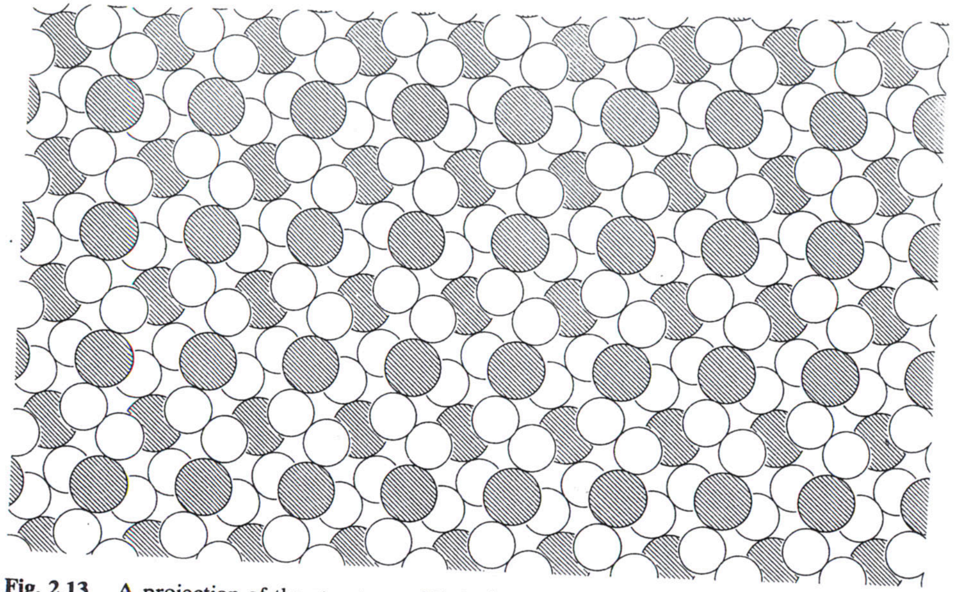

Fig. 2.13. A projection of the structure of FeS_2 (from *Contemporary Crystallography* by M. J. Buerger, McGraw-Hill, New York, 1970).

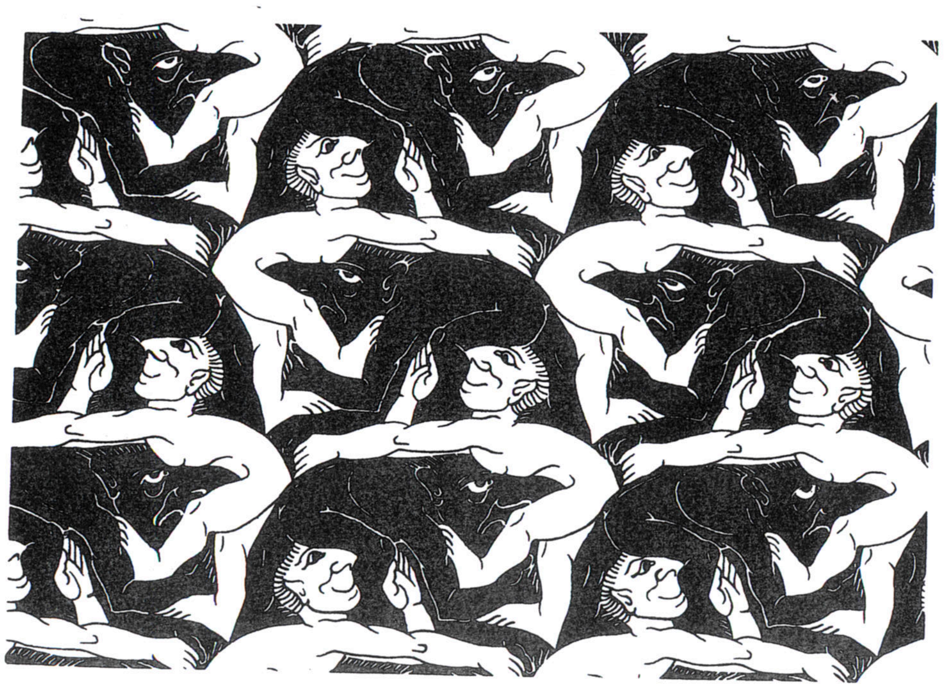

Fig. 2.14. A plane pattern (from C. H. MacGillavry, loc. cit.).

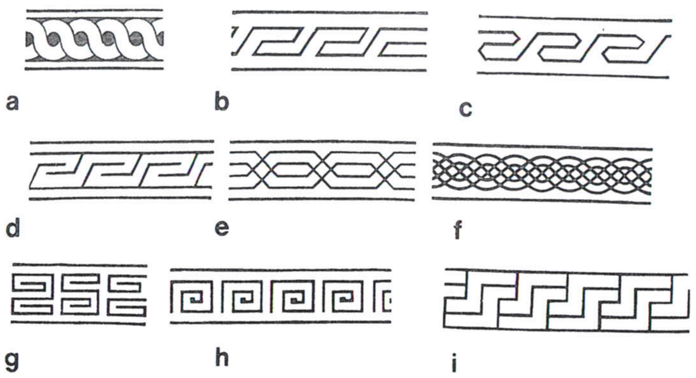

Fig. 2.15. Examples of border or frieze patterns (from *The Grammar of Ornament* by Owen Jones, Day & Son, London 1856, reprinted by Studio Editions, London, 1986). a, b, Greek; c, d, Arabian; e, Moresque; f, Celtic; g, h, Chinese; i, Mexican.

2.3 Figure 2.13 is a projection of the structure of FeS_2 (shaded atoms Fe, unshaded atoms S). Using a tracing paper overlay, indicate the positions of the symmetry elements, outline a unit cell and, with the help of the flow diagram in Fig. 2.8, determine the plane pattern type.

2.4 Figure 2.14 is a design by M. C. Escher. Can you see that the two sets of men are related by glide lines of symmetry? Draw in the positions of these glide lines, and determine the plane lattice type.

2.5 Determine (with reference to Fig. 2.11) the counterchange (black–white) point group symmetry of a chessboard.

2.6 Figure 2.15 shows examples of border or frieze patterns from *The Grammar of Ornament* by Owen Jones. Using a tracing paper overlay, indicate the positions of the symmetry elements and, with the help of the flow diagram (Fig. 2.10), determine the one-dimensional lattice types.

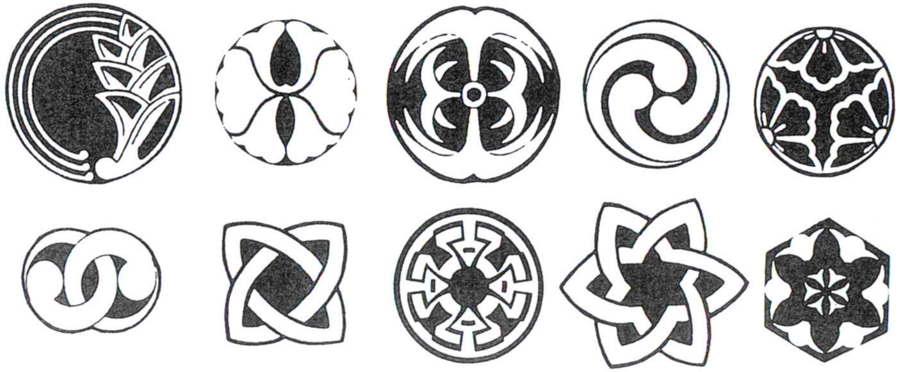

Fig. 2.16. Examples of motifs, representing the ten plane point groups (from *The Geometry of Regular Repeating Patterns*, loc. cit.).