3a. Lista de Exercícios

✩ Integrais definidas

1. Calcule as integrais definidas abaixo:

(1) $\int_{-1}^{0} (2x - e^x)dx$

(2) $\int_{-2}^{2} (3x+1)^2 dx$

(3) $\int_{0}^{1} (2x+5)(3x+1)dx$

(4) $\int_{0}^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}d\theta$

(5) $\int_{0}^{2} \frac{1+\sqrt[3]{x}}{\sqrt{x}}dx$

(6) $\int_{0}^{2\pi} |\text{sen}\,\theta|d\theta$

(7) $\int_{0}^{\pi} x\,\text{sen}\,(nx), n \in \mathbb{N}$

(8) $\int_{0}^{\pi} x\cos(nx)dx, n \in \mathbb{N}$

(9) $\int_{-1}^{} 2xe^x dx$

(10) $\int_{-1}^{2} x^2 e^x dx$

(11) $\int_{0}^{\pi/2} \cos^2\theta d\theta$

(12) $\int_{0}^{\pi/2} \text{sen}^2\theta d\theta$

(13) $\int_{-3}^{3} (\text{sen}\,(x^5) - 7x^7\cos x - x + 1)dx$

(14) $\int_{-2}^{2} (x\cos(x^2+2x)+3x)dx$

(15) $\int_{0}^{2} xe^{x^2}dx$

(16) $\int_{0}^{\pi/4} \text{tg}^2\theta d\theta$

(17) $\int_{0}^{\pi/2} \text{sen}^4\theta d\theta$

(18) $\int_{0}^{\pi/2} \cos^4\theta d\theta$

(19) $\int_{0}^{\pi/4} \sec\theta d\theta$

(20) $\int_{0}^{1} x^2\sqrt{x+1}dx$

(21) $\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}}$

(22) $\int_{0}^{1} e^{\sqrt{x}}dx$

(23) $\int_{0}^{2\pi} \sqrt{1+\cos x}dx$

(24) $\int_{0}^{2} \frac{e^x}{\sqrt{1+e^x}}dx$

(25) $\int_{0}^{1} \sqrt{1+x^2}dx$

(26) $\int_{0}^{1/2} \frac{x}{\sqrt{1-x^4}}dx$

(27) $\int_{-1}^{1} x^3\,\text{sen}\,(x^2+1)dx$

(28) $\int_{-1}^{1} \frac{x^2}{4+x^6}dx$

(29) $\int_{0}^{1} \frac{x^3}{\sqrt{1+x^2}}dx$

(30) $\int_{1}^{2} \frac{1}{x(\ln x)^2}dx$

2. Encontre o volume de uma pirâmide cuja base é o quadrado de lado L e cuja altura é h.

3. Calcule o volume do sólido cuja base é a astróide de equação $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ e tal que as seções transversais por planos paralelos ao plano Oxz são quadrados.

4. Calcule $\lim_{n\to\infty} \frac{\pi}{n}\left(\text{sen}\,\frac{\pi}{n} + \text{sen}\,\frac{2\pi}{n} + ... + \text{sen}\,\frac{(n-1)\pi}{n}\right)$.

5. Calcule o comprimento do gráfico de $f(x) = \ln(\cos x)$, para $0 \leq x \leq \frac{\pi}{4}$.

6. Calcule o comprimento da astróide $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

7. Calcule a área da região interna ao laço formado pela curva $y^2 = x^2(x+3)$.

8. Calcule a área da região do plano limitada pela elipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

9. Determine o volume do sólido obtido pela rotação em torno do eixo Ox do conjunto

a) $A = \{(x,y) \in \mathbb{R}^2 : 0 \leq xy \leq 2,\ x^2 + y^2 \leq 5 \text{ e } x > 0\}$.

b) $A = \{(x,y) \in \mathbb{R}^2 : y \geq \sqrt{x} \text{ e } (x-1)^2 + y^2 \leq 1\}$.

c) $A = \{(x, y) \in \mathbb{R}^2 : 0 \le x \le 2 \text{ e } e^{-x} \le y \le e^x\}$.

d) $A = \{(x, y) \in \mathbb{R}^2 : x > 0, \ y \le 1 \text{ e } 1/x \le y \le 4/x^2\}$.

10. Calcule o volume do sólido obtido pela rotação em torno da reta $y = 3$ da região delimitada pelas parábolas $y = x^2$ e $y = 2 - x^2$.

11. Seja $A = \{(x, y) \in \mathbb{R}^2 : 0 \le x \le 1 \text{ e } \ln(x+1) + 2 \le y \le e^x + 4\}$. Determine o volume do sólido obtido pela rotação de A em torno da reta $y = 2$.

12. O disco $x^2 + y^2 \le a^2$ é girado em torno da reta $x = b$, com $b > a$, para gerar um sólido, com a forma de um pneu. Esse sólido é chamado **toro**. Calcule seu volume.

13. Calcule o volume de uma calota esférica de altura h, $h \le a$, de uma esfera de raio a.
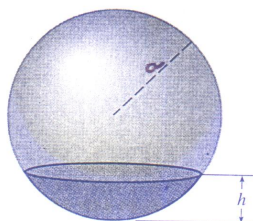
14. Determine o comprimento da curva $y = \cosh x$, $-3 \le x \le 4$.

15. Um anel esférico é o sólido que permanece após a perfuração de um buraco cilíndrico através do centro de uma esfera sólida. Se a esfera tem raio R e o anel esférico tem altura h, prove o fato notável de que o volume do anel depende de h, mas não de R.
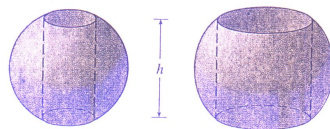
✩ Primitivas

16. Calcule as integrais indefinidas abaixo:

1. $\displaystyle \int \frac{x^7 + x^2 + 1}{x^2}\,dx$
2. $\displaystyle \int e^{2x}\,dx$
3. $\displaystyle \int \cos 7x\,dx$
4. $\displaystyle \int \operatorname{tg}^2 x\,dx$

5. $\displaystyle \int \frac{7}{x-2}\,dx$
6. $\displaystyle \int \operatorname{tg}^3 x \sec^2 x\,dx$
7. $\displaystyle \int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}}\,dx$
8. $\displaystyle \int \operatorname{tg} x\,dx$

9. $\displaystyle \int \operatorname{tg}^3 x\,dx$
10. $\displaystyle \int \frac{x}{1+x^2}\,dx$
11. $\displaystyle \int \frac{x}{1+x^4}\,dx$
12. $\displaystyle \int \frac{x^2}{1+x^2}\,dx$

13. $\displaystyle \int x\sqrt{1-x^2}\,dx$
14. $\displaystyle \int \sec x\,dx$
15. $\displaystyle \int \frac{dx}{x\sqrt{1+\ln x}}$
16. $\displaystyle \int x^2 \sqrt[5]{x^3+1}\,dx$

17. $\displaystyle \int \frac{4x+8}{2x^2+8x+20}\,dx$
18. $\displaystyle \int \frac{\sqrt{\ln x}}{x}\,dx$
19. $\displaystyle \int \frac{dx}{(\operatorname{arcsen} x)\sqrt{1-x^2}}$
20. $\displaystyle \int \frac{e^x}{1+e^x}\,dx$

21. $\displaystyle \int \frac{\operatorname{sen} 2x}{1+\cos^2 x}\,dx$
22. $\displaystyle \int e^{x^3} x^2\,dx$
23. $\displaystyle \int e^x \sqrt[3]{1+e^x}\,dx$
24. $\displaystyle \int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx$

21. $\displaystyle\int \frac{\operatorname{sen}2x}{1+\cos^2 x}\,dx$ 22. $\displaystyle\int e^{x^3}x^2\,dx$ 23. $\displaystyle\int e^x\sqrt[3]{1+e^x}\,dx$ 24. $\displaystyle\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx$

25. $\displaystyle\int \frac{e^{\operatorname{arctg}x}}{1+x^2}\,dx$ 26. $\displaystyle\int 2x(x+1)^{2010}\,dx$ 27. $\displaystyle\int x\operatorname{sen}x\,dx$ 28. $\displaystyle\int e^x\cos x\,dx$

29. $\displaystyle\int x^r\ln x\,dx, r\in\mathbb{R}$ 30. $\displaystyle\int (\ln x)^2\,dx$ 31. $\displaystyle\int xe^{-x}\,dx$ 32. $\displaystyle\int x\operatorname{arctg}x\,dx$

33. $\displaystyle\int \operatorname{arcsen}x\,dx$ 34. $\displaystyle\int \sec^3 x\,dx$ 35. $\displaystyle\int \cos^2 x\,dx$ 36. $\displaystyle\int \operatorname{sen}^2 x\cos^3 x\,dx$

37. $\displaystyle\int \operatorname{sen}^2 x\cos^2 x\,dx$ 38. $\displaystyle\int \frac{1-\operatorname{sen}x}{\cos x}\,dx$ 39. $\displaystyle\int \frac{3x^2+4x+5}{(x-1)(x-2)(x-3)}\,dx$ 40. $\displaystyle\int \frac{dx}{2x^2+8x+20}$

41. $\displaystyle\int \frac{3x^2+4x+5}{(x-1)^2(x-2)}\,dx$ 42. $\displaystyle\int \frac{x^5+x+1}{x^3-8}\,dx$ 43. $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$ 44. $\displaystyle\int x^2\sqrt{1-x^2}\,dx$

45. $\displaystyle\int e^{\sqrt{x}}\,dx$ 46. $\displaystyle\int \ln(x+\sqrt{1+x^2})\,dx$ 47. $\displaystyle\int \frac{dx}{\sqrt{5-2x+x^2}}$ 48. $\displaystyle\int \sqrt{x}\ln x\,dx$

49. $\displaystyle\int \operatorname{sen}(\ln x)\,dx$ 50. $\displaystyle\int \frac{x}{x^2-4}\,dx$ 51. $\displaystyle\int \frac{3x^2+5x+4}{x^3+x^2+x-3}\,dx$ 52. $\displaystyle\int \sqrt{a^2+b^2x^2}\,dx$

53. $\displaystyle\int \frac{dx}{\sqrt{a^2+b^2x^2}}$ 54. $\displaystyle\int \sqrt{x^2-2x+2}\,dx$ 55. $\displaystyle\int \sqrt{3-2x-x^2}\,dx$ 56. $\displaystyle\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

57. $\displaystyle\int \cos^3 x\,dx$ 58. $\displaystyle\int \operatorname{sen}^5 x\,dx$ 59. $\displaystyle\int \frac{\cos^5 x}{\operatorname{sen}^3 x}\,dx$ 60. $\displaystyle\int \operatorname{sen}^3\left(\frac{x}{2}\right)\cos^5\left(\frac{x}{2}\right)\,dx$

61. $\displaystyle\int \frac{dx}{\operatorname{sen}^5 x\cos^3 x}$ 62. $\displaystyle\int \operatorname{sen}^4 x\,dx$ 63. $\displaystyle\int \operatorname{sen}^2 x\cos^5 x\,dx$ 64. $\displaystyle\int \operatorname{sen}^2 x\cos^4 x\,dx$

65. $\displaystyle\int \cos^6(3x)\,dx$ 66. $\displaystyle\int \frac{\cos^2 x}{\operatorname{sen}^6 x}\,dx$ 67. $\displaystyle\int \frac{dx}{\operatorname{sen}^2 x\cos^4 x}$ 68. $\displaystyle\int \sqrt{\frac{1-x}{1+x}}\,dx$

69. $\displaystyle\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$ 70. $\displaystyle\int \frac{x+1}{x^2(x^2+4)^2}\,dx$ 71. $\displaystyle\int \frac{\operatorname{arctg}x}{x^2}\,dx$ 72. $\displaystyle\int \frac{x^2\,dx}{\sqrt{2x-x^2}}\,dx$

73. $\displaystyle\int \frac{4x^2-3x+3}{(x^2-2x+2)(x+1)}\,dx$ 74. $\displaystyle\int \frac{dx}{1+e^x}$ 75. $\displaystyle\int \frac{\ln(x+1)}{x^2}\,dx$ 76. $\displaystyle\int x^5 e^{-x^3}\,dx$

77. $\displaystyle\int \frac{x+1}{x^2(x^2+4)}\,dx$ 78. $\displaystyle\int \operatorname{arctg}\sqrt{x}\,dx$ 79. $\displaystyle\int \frac{2x+1}{x^2+2x+2}\,dx$ 80. $\displaystyle\int \cos^3 x(1+\sqrt{\operatorname{sen}x})\,dx$

✩ Funções definidas por integrais

17. Calcule $g'(x)$ onde

    (a) $g(x)=\displaystyle\int_{\cos x}^{\operatorname{sen}x} e^{t^2}\,dt$ (b) $g(x)=\displaystyle\int_{\sqrt{x}}^{2\sqrt{x}} \operatorname{sen}(t^2)\,dt$ (c) $g(x)=\displaystyle\int_{\operatorname{sen}x}^{x^3} \frac{dt}{1+t^4}$

18. Esboce o gráfico das funções abaixo:

    (a) $f(x)=\displaystyle\int_0^x e^{-t^2}\,dt$ (b) $f(x)=\displaystyle\int_0^x \frac{\operatorname{sen}t}{t}\,dt$

19. Calcule $\displaystyle\int_0^{\pi/2} \frac{\operatorname{sen}x\cos x}{x+1}\,dx$ em termos de $A=\displaystyle\int_0^{\pi} \frac{\cos x}{(x+2)^2}\,dx$.

20. Seja f uma função contínua em um intervalo I contendo a origem e seja

$$y=y(x)=\int_0^x \operatorname{sen}(x-t)f(t)\,dt$$

    Prove que $y''+y=f(x)$ e $y(0)=y'(0)=0$, para todo $x\in I$.

21. Seja $F(x) = \int_0^x \sqrt{1 + t^3}\, dt$. Calcule $\int_0^2 xF(x)\,dx$ em termos de $F(2)$.

22. Calcule $\displaystyle\lim_{x \to 0} \frac{\int_0^{x^2} \cos(t^2)\, dt}{\int_0^x e^{-t^2}\, dt}$.

23. Mostre que $f(x) = \int_0^{1/x} \frac{1}{t^2 + 1}\, dt + \int_0^x \frac{1}{t^2 + 1}\, dt$ é constante em $(0, \infty)$. Qual o valor dessa constante?

24. Seja $f(x) = \int_0^x \frac{1}{\sqrt{1 + t^4}}\, dt$, $x \in \mathbb{R}$.

    (a) Mostre que f é crescente e ímpar.

    (b) Mostre que $f(x) \le f(1) + 1 - \dfrac{1}{x}$, $\forall x \ge 1$. (Sugestão: Integre $0 \le \frac{1}{\sqrt{1+t^4}} \le \frac{1}{t^2}$ de 1 a x.)

    (c) Mostre que $\displaystyle\lim_{x \to \infty} f(x)$ existe e é um número real positivo.

    (d) Esboce o gráfico de $f(x)$, localizando seu ponto de inflexão.

25. Seja $f(x) = \int_0^x e^{\frac{x^2 - t^2}{2}}\, dt$. Mostre que $f'(x) - xf(x) = 1$, para todo $x \in \mathbb{R}$.

26. Seja $F : [1, +\infty[\to \mathbb{R}$ dada por $F(x) = \int_1^x \sqrt{t^3 - 1}\, dt$.

    (a) Calcule o comprimento do gráfico de F entre $x = 1$ e $x = 4$.

    (b) Calcule $\displaystyle\lim_{x \to 2} \frac{F(x^3) - F(8)}{\operatorname{sen}(x - 2)}$

☆ **Respostas**

(1)

(1) $e^{-1} - 2$; (2) 52; (3) 31/2; (4) $1 + \pi/4$; (5) $2\sqrt{3} + \frac{6}{5}\sqrt[6]{32}$; (6) 4; (7) 0 se $n = 0$ e $(-1)^{n+1}\pi/n$ se $n > 0$; (8) 0 se n é par e $-2/n^2$ se n é ímpar; (9) $e^2 + 2/e$; (10) $e^2 - 1/e$; (11) $\pi/4$; (12) $\pi/4$; (13) 6; (14) 0; (15) $(e^4 - 1)/2$; (16) $1 - \pi/4$; (17) $3\pi/8$; (18) $3\pi/8$; (19) $\ln(1 + \sqrt{2})$; (20) $16/105$; (21) $\pi/6$; (22) 2; (23) $4\sqrt{2}$; (24) $2(\sqrt{1 + e^2} - \sqrt{2})$; (25) $\frac{\ln(\sqrt{2}+1) + \sqrt{2}}{2}$; (26) $\frac{\operatorname{arcsen}(1/4)}{2}$; (27) 0; (28) $\frac{\operatorname{arctg}(1/2)}{3}$; (29) $\frac{2 - \sqrt{2}}{3}$; (30) $\frac{1}{\ln 2} - \frac{1}{\ln 3}$.

(2) $\frac{l^2 h}{3}$; **(3)** $\frac{128}{105}a^3$; **(4)** 2; **(5)** $\ln(1 + \sqrt{2})$; **(6)** $6a$; **(7)** $\frac{24}{5}\sqrt{3}$; **(8)** πab;

(9) (a) $\frac{5\sqrt{5} - 2}{3}\pi$; (b) $\frac{\pi}{6}$; (c) $\frac{\pi}{2}(e^2 - e^{-2})^2$; (d) $\frac{5\pi}{6}$. **(10)** $\frac{32}{3}\pi$; **(11)** $\pi\left(\frac{e^2}{2} + 4e - 2(\ln 2)^2 + 4\ln 2 - \frac{3}{2}\right)$

(12) $(2\pi b)(\pi a^2)$; **(13)** $\pi h^2(a - \frac{h}{3})$; **(14)** $\sinh 4 + \sinh 3$.

(16)

(1) $\frac{x^6}{6} + x - \frac{1}{x} + C$

(2) $\frac{e^{2x}}{2} + C$

(3) $\frac{1}{7}\operatorname{sen}7x + C$

(4) $\operatorname{tg}x - x + C$

(5) $7\ln|x-2| + C$

(6) $\frac{1}{4}\operatorname{tg}^4 x + C$

(7) $2\sqrt{\cos x}(\frac{1}{5}\cos^2 x - 1) + C$

(8) $-\ln|\cos x| + C$

(9) $\frac{1}{2}\operatorname{tg}^2 x + \ln|\cos x| + C$

(10) $\frac{1}{2}\ln(1+x^2) + C$

(11) $\frac{1}{2}\operatorname{arctg}x^2 + C$

(12) $x - \operatorname{arctg}x + C$

(13) $-\frac{1}{3}\sqrt{(1-x^2)^3} + C$

(14) $\ln|\sec x + \operatorname{tg}x| + C$

(15) $2\sqrt{1 + \ln x} + C$

(16) $\frac{5}{18}\sqrt[5]{(x^3+1)^6} + C$

(17) $\ln(2x^2 + 8x + 20) + C$

(18) $\frac{2}{3}\sqrt{(\ln x)^3} + C$

(19) $\ln|\operatorname{arcsen}x| + C$

(20) $\ln(1+e^x) + C$

(21) $-\ln(1 + \cos^2 x) + C$

(22) $\frac{1}{3}e^{x^3} + C$

(23) $\frac{3}{4}\sqrt[3]{(1+e^x)^4} + C$

(24) $-2\cos\sqrt{x} + C$

(25) $e^{\operatorname{arctg}x} + C$

(26) $2(x+1)^{2011}(\frac{x+1}{2012} - \frac{1}{2011}) + C$

(27) $-x\cos x + \operatorname{sen}x + C$

(28) $\frac{1}{2}e^x(\operatorname{sen}x + \cos x) + C$

(29) $\begin{cases} \frac{x^{r+1}}{r+1}\ln x - \frac{x^{r+1}}{(r+1)^2} + C, \text{ se } r \neq -1 \\ \frac{1}{2}(\ln x)^2 + C, \text{ se } r = -1 \end{cases}$

(30) $x(\ln x)^2 - 2(x\ln x - x) + C$

(31) $(-x-1)e^{-x} + C$

(32) $\frac{x^2}{2}\operatorname{arctg}x - \frac{x}{2} + \frac{1}{2}\operatorname{arctg}x + C$

(33) $x\operatorname{arcsen}x + \sqrt{1-x^2} + C$

(34) $\frac{1}{2}\sec x\operatorname{tg}x + \frac{1}{2}\ln\sec x + \operatorname{tg}x| + C$

(35) $\frac{1}{2}(x + \operatorname{sen}x\cos x) + C$

(36) $\frac{1}{3}\operatorname{sen}^3 x - \frac{1}{5}\operatorname{sen}^5 x + C$

(37) $\frac{1}{8}(x - \frac{1}{4}\operatorname{sen}4x) + C$

(38) $\ln|1 + \operatorname{sen}x| + C$

(39) $6\ln|x-1| - 25\ln|x-2| + 22\ln|x-3| + C$

(40) $\frac{\sqrt{6}}{12}\operatorname{arctg}(\frac{x+2}{\sqrt{6}}) + C$

(41) $-22\ln|x-1| + \frac{12}{x-1} + 25\ln|x-2| + C$

(42) $\frac{x^3}{3} + \frac{35}{12}\ln|x-2| + \frac{61}{24}\ln(1 + (\frac{x+1}{\sqrt{3}})^2) + \frac{\sqrt{3}}{12}\operatorname{arctg}(\frac{x+1}{\sqrt{3}}) + C$

(43) $\frac{1}{2}\operatorname{arcsen}x - \frac{1}{2}x\sqrt{1-x^2} + C$

(44) $\frac{x}{8}(2x^2-1)\sqrt{1-x^2} + \frac{1}{8}\operatorname{arcsen}x + C$

(45) $2(\sqrt{x} - 1)e^{\sqrt{x}} + C$

(46) $x\ln(x + \sqrt{1+x^2}) - \sqrt{1+x^2} + C$

(47) $\ln|\sqrt{5-2x+x^2} + x - 1| + C$

(48) $\frac{2}{3}x\sqrt{x}(\ln x - \frac{2}{3}) + C$

(49) $\frac{x}{2}(\operatorname{sen}(\ln x) - \cos(\ln x)) + C$

(50) $\frac{1}{2}\ln|x^2-4| + C$

(51) $2\ln|x-1| + \frac{1}{2}\ln(x^2+2x+3) + \frac{1}{\sqrt{2}}\operatorname{arctg}(\frac{x+1}{\sqrt{2}}) + C$

(52) $x\sqrt{a^2+b^2x^2} + \frac{a^2}{2b}\ln(\frac{bx}{a} + \frac{\sqrt{a^2+b^2x^2}}{a}) + C$

(53) $\frac{1}{b}\ln(\frac{bx}{a} + \frac{\sqrt{a^2+b^2x^2}}{a}) + C$

(54) $\frac{x-1}{2}\sqrt{x^2-2x+2} + \frac{1}{2}\ln(x-1+\sqrt{x^2-2x+2}) + C$

(55) $\frac{x+1}{2}\sqrt{3-2x-x^2} + 2\operatorname{arcsen}(\frac{x+1}{2}) + C$

(56) $\frac{1}{\sqrt{2}}\operatorname{arctg}(\frac{x\sqrt{2}}{\sqrt{1-x^2}}) + C$

(57) $\operatorname{sen}x - \frac{1}{3}\operatorname{sen}^3 x + C$

(58) $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

(59) $\frac{1}{2}\operatorname{sen}^2 x - \frac{1}{2\operatorname{sen}^2 x} - 2\ln|\operatorname{sen}x| + C$

(60) $\frac{1}{4}\cos^8(\frac{x}{2}) - \frac{1}{3}\cos^6(\frac{x}{2}) + C$

61) $\frac{1}{2}\operatorname{tg}^2 x + 3\ln|\operatorname{tg}x| - \frac{3}{2\operatorname{tg}^2 x} - \frac{1}{4\operatorname{tg}^4 x} + C$

(62) $\frac{3}{8}x - \frac{1}{4}\operatorname{sen}(2x) + \frac{1}{32}\operatorname{sen}(4x) + C$

(63) $\frac{1}{3}\operatorname{sen}^3 x - \frac{2}{5}\operatorname{sen}^5 x + \frac{1}{7}\operatorname{sen}^7 x + C$

(64) $\frac{x}{16} - \frac{1}{64}\operatorname{sen}(4x) + \frac{1}{48}\operatorname{sen}^3(2x) + C$

(65) $\frac{5}{16}x + \frac{1}{12}\operatorname{sen}(6x) + \frac{1}{64}\operatorname{sen}(12x) - \frac{1}{144}\operatorname{sen}^3(6x) + C$

(66) $-\frac{1}{3}\operatorname{cotg}^3 x - \frac{1}{5}\operatorname{cotg}^5 x + C$

(67) $\operatorname{tg}x + \frac{1}{3}\operatorname{tg}^3 x - 2\operatorname{cotg}(2x) + C$

(68) $\operatorname{arcsen}x + \sqrt{1-x^2} + C$

(69) $2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + C$

(70) $\frac{1}{16}\ln|x| - \frac{1}{16x} - \frac{1}{32}\ln(x^2+4) - \frac{3}{64}\operatorname{arctg}\frac{x}{2} + \frac{4-x}{32(x^2+4)} + C$

(71) $\frac{-\operatorname{arctg}x}{x} + \ln|x| - \ln\sqrt{1+x^2} + C$

(72) $\frac{3}{2}\operatorname{arcsen}(x-1) - \left(\frac{x+3}{2}\right)\sqrt{2x-x^2} + C$

(73) $2\ln|x+1| + \ln(x^2-2x+2) + 3\operatorname{arctg}(x-1) + C$

(74) $x - \ln(1+e^x) + C$

(75) $-\frac{\ln(x+1)}{x} + \ln|x| - \ln(x+1) + C$

(76) $-\frac{1}{3}(x^3+1)e^{-x^3} + C$

(77) $\frac{1}{4}\ln|x| - \frac{1}{4x} - \frac{1}{8}\ln(x^2+4) - \frac{1}{16}\operatorname{arctg}\left(\frac{x}{2}\right) + C$

(78) $(x+1)\operatorname{arctg}\sqrt{x} - \sqrt{x}$

(79) $\ln(x^2+2x+2) - \operatorname{arctg}(x+1) + C$

(80) $\operatorname{sen}x + 2\sqrt{\operatorname{sen}x} - \frac{\operatorname{sen}^3 x}{3} - \frac{2\sqrt{\operatorname{sen}^5 x}}{5} + C$

(17) (a) $g'(x) = e^{\operatorname{sen}^2 x}\cos x + e^{\cos^2 x}\operatorname{sen}x$; (b) $g'(x) = \frac{2\operatorname{sen}4x - \operatorname{sen}x}{2\sqrt{x}}$; (c) $g'(x) = \frac{3x^2}{1+x^{12}} - \frac{\cos x}{1+\operatorname{sen}^4 x}$; **(22)** 0; **(23)** $\pi/2$; **(26)** (a) 62/5; (b) $12\sqrt{511}$.