UFPR - Universidade Federal do Paraná
Departamento de Matemática
CM043 - Cálculo III - Turma B - 2012/1
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Lista 1

☆ Sequências e séries de números reais

1. Decida se cada uma das sequências abaixo é convergente ou divergente, calculando o limite no caso convergente.

(1) $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$

(2) $1, \frac{1}{2}, 1, \frac{1}{4}, 1, \frac{1}{8}, 1, \frac{1}{16}, \dots$

(3) $\frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{3}{4}, \frac{1}{8}, -\frac{7}{8}, \dots$

(4) $a_n = \left(4 + \frac{1}{n}\right)^{\frac{1}{2}}$

(5) $a_n = \frac{\sqrt{n}+1}{n-1}$, $n \geq 2$

(6) $a_n = \frac{n^3+3n+1}{4n^3+2}$

(7) $a_n = \sqrt{n+1} - \sqrt{n}$

(8) $a_n = \frac{n+(-1)^n}{n-(-1)^n}$

(9) $a_n = \frac{2n}{n+1} - \frac{n+1}{2n}$

(10) $a_n = n(\sqrt{n^2+1} - n)$

(11) $a_n = \frac{\operatorname{sen} n}{n}$

(12) $a_n = \operatorname{sen} n$

(13) $a_n = \frac{2n+\operatorname{sen} n}{5n+1}$

(14) $a_n = \frac{(n+3)!-n!}{(n+4)!}$

(15) $a_n = \sqrt[n]{n^2+n}$

(16) $a_n = \frac{n\operatorname{sen}(n!)}{n^2+1}$

(17) $a_n = \frac{3^n}{2^n+10^n}$

(18) $a_n = \left(\frac{n+2}{n+1}\right)^n$

(19) $a_n = \frac{(n+1)^n}{n^{n+1}}$

(20) $a_n = na^n$, $a \in \mathbb{R}$

(21) $a_n = \frac{n!}{n^n}$

(22) $a_n = n - n^2 \operatorname{sen} \frac{1}{n}$

(23) $a_n = \sqrt[n]{a^n + b^n}$, $0 < a < b$

(24) $a_n = (-1)^n + \frac{(-1)^n}{n}$

(25) $a_n = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\dots\left(1 - \frac{1}{n}\right)$

(26) $a_n = \frac{\sqrt{n}+\operatorname{sen}(2n!-7)}{n+3\sqrt{n}}$

(27) $a_n = \frac{1}{n} \cdot \frac{1.3.5\dots(2n-1)}{2.4.6\dots(2n)}$

(28) $a_n = \sqrt[n]{n}$

(29) $a_n = \frac{n^\alpha}{e^n}$, $\alpha \in \mathbb{R}$

(30) $a_n = \frac{\ln n}{n^\alpha}$, $\alpha > 0$

(31) $a_n = \sqrt[n]{n!}$

(32) $a_n = \sqrt[n]{a}$, $a > 0$

(33) $a_n = \left(\frac{n-1}{n}\right)^n$

(34) $a_n = \left(\frac{n+1}{n}\right)^{n^2}$

(35) $a_n = \left(\frac{n+1}{n}\right)^{\sqrt{n}}$

(36) $a_n = \left(\frac{3n+5}{5n+11}\right)^n$

(37) $a_n = \left(\frac{3n+5}{5n+1}\right)^n \left(\frac{5}{3}\right)^n$

(38) $a_n = \left(1 + \frac{1}{n^2}\right)^n$

(39) $a_n = \operatorname{sen}\left(\frac{n\pi}{2}\right)$

(40) $a_n = \frac{n}{\sqrt[n]{n!}}$

(41) $a_n = \frac{1}{n}\sqrt[n]{(n+1)(n+2)\cdot\ldots\cdot(2n)}$

(42) $a_n = \sqrt[n]{\frac{(2n)!}{n!^2}}$

(43) $a_n = \frac{n^2-1}{n^5+(-1)^n n^2}$

(44) $a_n = \sqrt[n]{n^4+2012n^3-5}$

(45) $a_n = \left(1 + \frac{1}{n}\right)^{1/n}$

(46) $a_n = \frac{n!^2}{n^{2n}}$

(47) $a_n = \frac{5^n}{2^n+3^n+4^n}$

(48) $a_n = \frac{n+\sqrt{2n+3}}{\sqrt[4]{n}+\sqrt[7]{17n-8}}$

(49) $a_n = \frac{3n^3 - n^2 + 11n}{n^4 - 2n^3}$ (50) $a_n = \left(\frac{5n+7}{3n+8}\right)^{2n-4}$ (51) $a_n = \frac{(2n)!}{(n!)^2}$

2. Considere a sequência $a_1 = \sqrt{2}$, $a_2 = \sqrt{2\sqrt{2}}$, $a_3 = \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$

   (a) Verifique que a sequência é crescente e limitada superiormente por 2.

   (b) Calcule $\lim_{n\to\infty} a_n$.

3. Mostre que a sequência $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots$ converge para 2.

4. Calcule $\sqrt{2}^{\sqrt{2}^{\sqrt{2}\cdots}}$.

5. Neste exercício, estudaremos o crescimento das somas parciais da série harmônica.

   (a) Sabemos que $\ln x = \int_1^x \frac{dt}{t}$, para qualquer $x > 0$. Interpretando a integral como área abaixo do gráfico, mostre que

   $$1 + \frac{1}{2} + \ldots + \frac{1}{n} \le 1 + \ln n,$$

   para todo $n \ge 2$.

   (b) Mostre que $\ln 10 < \frac{12}{5}$ e conclua que para qualquer $m \in \mathbb{N}$,

   $$1 + \frac{1}{2} + \ldots + \frac{1}{10^m} \le 1 + \frac{12m}{5}.$$

   Isso mostra que, embora a série harmônica seja divergente, suas somas parciais crescem muito lentamente. (Por exemplo, $1 + \frac{1}{2} + \ldots + \frac{1}{10^{1000}} \le 2401$.)

   (c) Considere a sequência $x_n = 1 + \frac{1}{2} + \ldots + \frac{1}{n} - \ln n$, $n \ge 1$. Mostre que $\{x_n\}$ é uma sequência decrescente limitada inferiormente. O número

   $$\gamma \doteq \lim_{n\to\infty} 1 + \frac{1}{2} + \ldots + \frac{1}{n} - \ln n$$

   é chamado de *constante de Euler-Mascheroni* e vale aproximadamente 0.57721.

6. Decida se cada uma das séries abaixo é convergente e calcule sua soma quando possível:

   (1) $\sum_{n=0}^{\infty} \left(\frac{1}{10^n} + 2^n\right)$ (2) $\sum_{k=0}^{\infty} (-1)^k\, t^{\frac{k}{2}}$, $0 < t < 1$ (3) $\sum_{n=0}^{\infty} u^n(1+u^n)$, $|u| < 1$

   (4) $\sum_{n=0}^{\infty} x^n \cos\left(\frac{n\pi}{2}\right)$, $|x| < 1$ (5) $\sum_{n=0}^{\infty} \operatorname{sen}^{2n} x$, $|x| < \frac{\pi}{2}$ (6) $\sum_{n=1}^{\infty} \frac{1}{e^{n/2}}$

   (7) $\sum_{n=1}^{\infty} \cos\left(\frac{n\pi}{2}\right)$ (8) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$ (9) $\sum_{n=1}^{\infty} \frac{n}{\operatorname{sen} n}$

   (10) $\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{n+2012}}$ (11) $\sum_{n=1}^{\infty} \frac{2+\cos n}{n}$ (12) $\sum_{n=1}^{\infty} \operatorname{tg}\left(\frac{1}{n}\right)$

7. Verifique se cada uma das séries abaixo é convergente ou divergente, justificando sua resposta:

(1) $\sum_{n=3}^{\infty} \frac{1}{\sqrt{n^2-4}}$ (2) $\sum_{n=2}^{\infty} \frac{\arctan n}{n^2}$ (3) $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$ (4) $\sum_{n=1}^{\infty} \frac{2^n}{(n!)^{\lambda}}$, $\lambda > 0$

(5) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!^2}$ (6) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ (7) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n+2}}{\sqrt[4]{n^3+3}\sqrt[5]{n^3+5}}$ (8) $\sum_{n=2}^{\infty} \frac{1}{n^{\ln n}}$

(9) $\sum_{n=1}^{\infty} \left(1-\cos\frac{1}{n}\right)$ (10) $\sum_{n=2}^{\infty} \frac{1}{(\ln\ n)^n}$ (11) $\sum_{n=2}^{\infty} \frac{\ln\ n}{n^2}$ (12) $\sum_{n=2}^{\infty} \frac{\ln n}{n^p}$, $p > 0$

(13) $\sum_{n=2}^{\infty} \ln\left(1+\frac{1}{n^p}\right)$, $p > 0$ (14) $\sum_{n=2}^{\infty} \sqrt{n}\ln\left(\frac{n+1}{n}\right)$ (15) $\sum_{n=1}^{\infty} \frac{n!3^n}{n^n}$ (16) $\sum_{n=1}^{\infty} \frac{n!e^n}{n^n}$

(17) $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^n}$ (18) $\sum_{n=1}^{\infty} 3^n\left(\frac{n}{n+1}\right)^{n^2}$ (19) $\sum_{n=1}^{\infty} \frac{n^3}{(\ln 2)^n}$ (20) $\sum_{n=1}^{\infty} \frac{1}{(\arctan n)^n}$

(21) $\sum_{n=0}^{\infty} \frac{n+2}{(n+1)^3}$ (22) $\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)$ (23) $\sum_{n=1}^{\infty} \operatorname{sen}\left(\frac{1}{n}\right)$ (24) $\sum_{n=0}^{\infty} \frac{1+2^n}{1+3^n}$

(25) $\sum_{n=0}^{\infty} \frac{n}{(1+n^2)^p}$, $p > 0$ (26) $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt[17]{n}}$ (27) $\sum_{n=0}^{\infty} \left(\frac{2n+1}{3n+4}\right)^n$ (28) $\sum_{n=0}^{\infty} \frac{\operatorname{sen} 4n}{4^n}$

(29) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$, $p > 0$ (30) $\sum_{n=1}^{\infty} \ln(\cos(1/n))$ (31) $\sum_{n=0}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$ (32) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(33) $\sum_{n=1}^{\infty} \frac{1}{(n\ln n)^p}$, $p > 0$ (34) $\sum_{n=1}^{\infty} (\sqrt{1+n^2}-n)$ (35) $\sum_{n=1}^{\infty} \frac{\ln n}{n^p e^n}$, $p > 0$ (36) $\sum_{n=0}^{\infty} e^{-n}n!$

(37) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^p}$, $p > 0$ (38) $\sum_{n=1}^{\infty} \operatorname{sen}\left(\frac{1}{n\sqrt[4]{n^3+6}}\right)$ (39) $\sum_{n=1}^{\infty} \frac{\sqrt[8]{n^7+3n^3-2}}{\sqrt[6]{n^9+7n^2}}$ (40) $\sum_{n=0}^{\infty} \frac{n^2 2^n}{n!}$

(41) $\sum_{n=1}^{\infty} \frac{n^p}{e^{-an}}$, $a, p > 0$ (42) $\sum_{n=1}^{\infty} a^n n^p$, $a, p > 0$ (43) $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$ (44) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$

(45) $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$ (46) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{2012} e^{-n/3}$ (47) $\sum_{n=1}^{\infty} \frac{1+n+n^2}{\sqrt{1+n^2+n^6}}$ (48) $\sum_{n=1}^{\infty} \frac{n^n}{(n+1)^{n+1}}$

8. Classifique as séries abaixo absolutamente convergentes, condicionalmente convergentes ou divergentes:

(1) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ (2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\frac{3}{2}}}$ (3) $\sum_{n=1}^{\infty} (-1)^n \frac{2n^2+1}{n^3+3}$ (4) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln\ n}$

(5) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$ (6) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$ (7) $\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{\ln n}$ (8) $\sum_{n=1}^{\infty} (-1)^{2n+1} \frac{1}{\sqrt{n}}$

(9) $\sum_{n=1}^{\infty} (-1)^n \operatorname{sen}\frac{1}{n^p}$, $p > 0$ (10) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n^2}$ (11) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$ (12) $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{(n+1)^{n+1}}$

(13) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(1+n^2)^p}$, $p > 0$ (14) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(\ln n)^p}$, $p > 0$ (15) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1}$ (16) $\sum_{n=1}^{\infty} (-1)^n n! e^{-n}$

(17) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n^p}$, $p > 0$ (18) $\sum_{n=1}^{\infty} (-1)^n \frac{2\cdot 4\cdot \ldots (2n)}{3\cdot 5\cdot (2n+1)}$ (19) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt[3]{n^2+3n}}$ (20) $\sum_{n=1}^{\infty} (-1)^n \operatorname{sen}\left(\frac{1}{n}\right)$

(21) $\sum_{n=1}^{\infty} \frac{(-1)^n}{10^n n!}$ (22) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+7\sqrt{n+2}}$ (23) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{3/4}}$ (24) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{2\cdot 5\cdot \ldots \cdot (3n+2)}$

(25) $\sum_{n=1}^{\infty} (-1)^n n \operatorname{tg}(1/n)$ (26) $\sum_{n=1}^{\infty} \frac{(-2)^n n!}{e^{n^2}}$ (27) $\sum_{n=0}^{\infty} (-1)^n \frac{\operatorname{sen}(7n)}{9+5^n}$ (28) $\sum_{n=0}^{\infty} \frac{5^n}{3^n+4^n}$

(29) $\sum_{n=0}^{\infty} \frac{3^n}{4^n+5^n}$ (30) $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$ (31) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+4}$ (32) $\sum_{n=1}^{\infty} (-1)^n \frac{\cos(n!)}{\sqrt[3]{n^4+\operatorname{sen} n}}$

☆ Séries de potências

9. Determine o intervalo máximo de convergência de cada uma das séries de potências abaixo:

(1) $\sum_{n=1}^{\infty} \frac{n}{4^n} x^n$ (2) $\sum_{n=1}^{\infty} n!\, x^n$ (3) $\sum_{n=1}^{\infty} \frac{x^n}{n^3+1}$ (4) $\sum_{n=1}^{\infty} \frac{(3n)!}{(2n)!} x^n$

(5) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{n3^n}$ (6) $\sum_{n=1}^{\infty} \frac{(x+1)^n}{(n+1)\ln^2(n+1)}$ (7) $\sum_{n=1}^{\infty} \frac{10^n}{(2n)!}(x-7)^n$ (8) $\sum_{n=1}^{\infty} \frac{\ln n}{e^n}(x-e)^n$

(9) $\sum_{n=1}^{\infty} \frac{n!}{n^n}(x+3)^n$ (10) $\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{(2n+1)\sqrt{n+1}}$ (11) $\sum_{n=0}^{\infty} \frac{n^2}{4^n}(x-4)^{2n}$ (12) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}\, x^n$

(13) $\sum_{n=1}^{\infty} 2^n x^{n^2}$ (14) $\sum_{n=1}^{\infty} \frac{3^n}{n4^n} x^n$ (15) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} x^n$ (16) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} x^{2n}$

(17) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} x^{n^2}$ (18) $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$ (19) $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^{3n}$ (20) $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^{n!}$

(21) $\sum_{n=1}^{\infty} \frac{x^n}{(2+(-1)^n)^n}$ (22) $\sum_{n=1}^{\infty} \frac{(x+1)^n}{a^n+b^n}$, $b>a>0$ (23) $\sum_{n=1}^{\infty} \left(\frac{2^n+3}{3^n+2}\right) x^n$ (24) $\sum_{n=1}^{\infty} \left(\frac{3n+2}{5n+7}\right)^n x^n$

(25) $\sum_{n=1}^{\infty} (\operatorname{sen} n) x^n$ (26) $\sum_{n=0}^{\infty} \frac{1}{(3+(-1)^n)^n} x^n$ (27) $\sum_{n=1}^{\infty} \frac{n!}{1\cdot 3\cdot \ldots \cdot (2n-1)} x^n$ (28) $\sum_{n=1}^{\infty} \frac{n^2}{2\cdot 4\cdot \ldots \cdot (2n)} x^n$

10. Suponha que $\sum_{n=0}^{\infty} a_n x^n$ converge quando $x=-4$ e diverge quando $x=6$. O que pode ser dito sobre a convergência ou divergência das séries a seguir?

(a) $\sum_{n=0}^{\infty} a_n$ (b) $\sum_{n=0}^{\infty} a_n 8^n$ (c) $\sum_{n=0}^{\infty} a_n(-3)^n$ (d) $\sum_{n=0}^{\infty} (-1)^n a_n 9^n$

11. Usando derivação e integração termo a termo, calcular as somas das séries de potências:

(1) $\sum_{n=1}^{\infty} \frac{x^n}{n}$ (2) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ (3) $\sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}$ (4) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}$

(5) $\sum_{n=0}^{\infty} (n+1)x^n$ (6) $\sum_{n=1}^{\infty} nx^n$ (7) $\sum_{n=1}^{\infty} nx^{2n-1}$ (8) $\sum_{n=2}^{\infty} \frac{x^n}{n(n-1)}$

(9) $\sum_{n=1}^{\infty} n^2 x^{n-1}$ (10) $\sum_{n=1}^{\infty} n^3 x^n$ (11) $\sum_{n=0}^{\infty} (n+4)x^n$ (12) $\sum_{n=1}^{\infty} \frac{x^{4n}}{4n}$

12. Use as séries do exercício anterior para calcular:

(a) $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ (b) $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{5^n(n-1)}$

13. Determine as expansões em séries de potências em torno de $x_0 = 0$ das seguintes funções e os valores de x para os quais essas expansões são válidas:

(a) $\frac{1}{(1+x)^2}$ (b) $\frac{1}{(1+x)^3}$ (c) $\frac{2x}{1+x^4}$ (d) $\ln(1+x)$ (e) $\ln\left(\frac{1}{1+3x^2}\right)$

14. Verifique que

(a) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $x \in \mathbb{R}$ (b) $\operatorname{sen} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, $x \in \mathbb{R}$

(c) $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, $x \in \mathbb{R}$ (d) $\ln\left(\frac{1+x}{1-x}\right) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$, $|x|<1$

(e) $\arctan\, x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$, $|x| \le 1$ (f) $\sum_{n=0}^{\infty} \frac{(n+1)(n+2)(n+3)}{6} x^n = \frac{1}{(1-x)^4}$, $x \ne 1$

15. Utilizando as séries do exercício anterior, obtenha um valor aproximado de:

   (a) e, com erro inferior a 10^{-5}.
   (b) $\operatorname{sen} 1$, com erro inferior a 10^{-5} e a 10^{-7}.
   (c) $\ln 2$ e $\ln 3$, com erro inferior a 10^{-5}.
   (d) $\arctan(1/2)$ e $\arctan(1/3)$, com erro inferior a 10^{-5}.
   (e) $\pi/4$, com erro inferior a 10^{-5}, usando que $\pi/4 = \arctan(1/2) + \arctan(1/3)$, (esta igualdade segue da identidade $\operatorname{tg}(x+y) = \frac{\operatorname{tg}(x)+\operatorname{tg}(y)}{1-\operatorname{tg}(x)\operatorname{tg}(y)}$

16. Calcule $\frac{d^{320}\arctan}{dx^{320}}(0)$ e $\frac{d^{321}\arctan}{dx^{321}}(0)$

17. Desenvolva em série de potências de x as seguintes funções, indicando os intervalos de convergência:

   (a) $f(x) = x^2 e^x$ (b) $f(x) = \cos\sqrt{x}$ (c) $f(x) = \operatorname{sen}(x^2)$

   (d) $f(x) = \cos^2 x$ (e) $f(x) = \int_0^x \frac{\operatorname{sen} t}{t} dt$ (f) $f(x) = \int_0^x e^{-t^2} dt$

   (g) $f(x) = \int_0^x \frac{\ln(1+t)}{t} dt$ (h) $f(x) = \int_0^x \operatorname{sen}(t^2)\, dt$ (i) $f(x) = \frac{e^{x^2}-1}{x}$

18. Utilizando a expansão em série de potências das funções envolvidas, calcule os seguintes limites:

   (a) $\lim_{x\to 0} \frac{\operatorname{sen} x}{x}$ (b) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$ (c) $\lim_{x\to 0} \frac{x-\operatorname{sen} x}{x^3}$ (d) $\lim_{x\to 0} \frac{\ln(1+x)}{x}$

   (e) $\lim_{x\to 0} \frac{\arctan x}{x}$ (f) $\lim_{x\to 0} \frac{e^x-1}{x}$ (g) $\lim_{x\to 0} \frac{e^{x^3}-1}{x^2}$ (h) $\lim_{x\to 0} \frac{\ln(1+x)-\ln(1-x)}{x}$

☆ **Respostas**

(1)

- (2), (3), (12), (24), (39) são divergentes;
- (5), (7), (11), (14), (16), (17), (19), (21), (22), (25), (26), (27), (29), (30), (36), (43), (46), (49) convergem para zero;
- (1), (8), (15), (28), (32), (35), (38), (44), (45) convergem para 1;
- (31), (34), (47), (48), (50), (51) divergem para $+\infty$;
- (4) converge para 2; (6) converge para 1/4; (9) converge para 3/2; (10) converge para 1/2; (13) converge para 2/5; (18) converge para e; (20) diverge se $a < 0$, diverge para $+\infty$ se $a \geq 1$ e converge para zero se $0 \leq a < 1$; (23) converge para b; (33) converge para $1/e$; (37) converge para $e^{22/15}$; (40) converge para e; (41) converge para $4/e$; (42) converge para 4.

(2) $\lim_{n\to\infty} a_n = 2$; **(4)** A sequência converge para 2;

(6) (1) diverge; (2) converge para $\frac{1}{1+\sqrt{t}}$; (3) converge para $\frac{1}{1-u} + \frac{1}{1-u^2}$; (4) converge para $\frac{1}{1+x^2}$; (5) converge para $\sec^2 x$; (6) converge para $\frac{1}{\sqrt{e}-1}$; as demais são todas divergentes.

(7) (1), (5), (6), (14), (15), (16), (18), (19), (22), (23), (26), (34), (36),(39), (45), (47) são divergentes; (12), (13), (25), (29), (33) convergem se e somente se $p > 1$; (35) converge para qualquer valor de

$p > 0$; (42) converge se e só se $0 \le a < 1$; (37) diverge para qualquer $p > 0$; as demais são todas convergentes.

(8) (2), (6), (10), (12), (18), (21), (23), (24), (26), (27), (29), (32) são absolutamente convergentes; (1), (3), (4), (5), (8), (11), (15), (19), (20), (31) são condicionalmente convergentes; (5), (9), (17) são absolutamente convergentes para $p > 1$ e condicionalmente convergentes se $0 < p \le 1$; (14) é condicionalmente convergente para qualquer valor de $p > 0$; as demais são divergentes.

(9) (1) $(-4,4)$; (2) $\{0\}$; (3) $[-1,1]$; (4) $\{0\}$; (5) $(2,8]$; (6) $[-2,0)$; (7) $(-\infty,+\infty)$; (8) $(0,2e)$; (9) $(-3-e,-3+e)$; (10) $[2,4]$; (11) $[3,5]$; (12) $(-1,1]$; (13) $(-1,1)$; (14) $[-4/3,4/3)$; (15) $(-1/4,1/4)$; (16) $(-1/2,1/2)$; (17) $(-1,1)$; (18) $(-e,e)$; (19) $(-\sqrt[3]{e},\sqrt[3]{e})$; (20) $[-1,1]$; (21) $(-1,1)$; (22) $(-1-b,-1+b)$; (23) $(-3/2,3/2)$; (24) $(-5/3,5/3)$; (25) $(-1,1)$; (26) $(-2,2)$; (27) $\{0\}$; (28) $(-\infty,+\infty)$.

(10) (a), (c) convergem e (b), (d) divergem.

(11) (1) $-\ln(1-x)$; (2) $\ln(1+x)$; (3) $\ln\left(\sqrt{\frac{1+x}{1-x}}\right)$; (4) $\arctan x$; (5) $\frac{1}{(1-x)^2}$; (6) $\frac{x}{(1-x)^2}$; (7) $\frac{x}{(1-x^2)^2}$; (8) $(1+x)\ln(1+x)-x$; (9) $\frac{1+x}{(1-x)^3}$; (10) $\frac{x+4x^2+x^3}{(1-x)^4}$; (11) $\frac{4-3x}{(1-x)^2}$; (12) $\frac{-1}{4}\ln(1-x^4)$.

(12) (a) $\ln 2$; (b) $\frac{3}{128}$; (c) $\frac{6}{5}\ln\frac{6}{5}-\frac{1}{5}$.

(13) (a) $\sum_{n=0}^{\infty}(-1)^{n+1}(n+1)x^n$, $|x|<1$; (b) $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(n+1)(n+2)}{1}x^n$, $|x|<1$; (c) $\sum_{n=0}^{\infty}2(-1)^n x^{4n+1}$, $|x|<1$; (d) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^n$, $|x|<1$; (e) $\sum_{n=1}^{\infty}\frac{(-1)^n 3^n}{n}x^{2n}$, $|x|<1$.

(17) (a) $\sum_{n=0}^{\infty}\frac{x^{n+2}}{n!}$, $x\in\mathbb{R}$; (b) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(2n)!}$, $x\in\mathbb{R}$; (c) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+2}}{(2n+1)!}$, $x\in\mathbb{R}$; (d) $1+\sum_{n=1}^{\infty}\frac{(-1)^n 2^{2n-1}}{(2n)!}x^{2n}$, $x\in\mathbb{R}$; (e) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2(2n)!}x^{2n+1}$, $x\in\mathbb{R}$; (f) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)n!}x^{2n+1}$, $x\in\mathbb{R}$; (g) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}x^n$, $|x|<1$; (h) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!(4n+3)}x^{4n+3}$, $x\in\mathbb{R}$; (i) $\sum_{n=1}^{\infty}\frac{x^{2n-1}}{n!}$, $x\in\mathbb{R}$.