

3a. Lista de Exercícios

☆ Integrais definidas e indefinidas

1. Calcule as integrais definidas abaixo:

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| (1) $\int_{-1}^0 (2x - e^x) dx$ | (2) $\int_{-2}^2 (3x + 1)^2 dx$ | (3) $\int_0^1 (2x + 5)(3x + 1) dx$ |
| (4) $\int_0^{\pi/4} \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta$ | (5) $\int_0^2 \frac{1 + \sqrt[3]{x}}{\sqrt{x}} dx$ | (6) $\int_0^{2\pi} \sin \theta d\theta$ |
| (7) $\int_0^{\pi} x \sin(nx), n \in \mathbb{N}$ | (8) $\int_0^{\pi} x \cos(nx) dx, n \in \mathbb{N}$ | (9) $\int_{-1}^2 x e^x dx$ |
| (10) $\int_{-1}^2 x^2 e^x dx$ | (11) $\int_0^{\pi/2} \cos^2 \theta d\theta$ | (12) $\int_0^{\pi/2} \sin^2 \theta d\theta$ |
| (13) $\int_{-3}^3 (\sin(x^5) - 7x^7 \cos x - x + 1) dx$ | (14) $\int_{-2}^2 (x \cos(x^2 + 2x) + 3x) dx$ | (15) $\int_0^2 x e^{x^2} dx$ |
| (16) $\int_0^{\pi/4} \operatorname{tg}^2 \theta d\theta$ | (17) $\int_0^{\pi/2} \sin^4 \theta d\theta$ | (18) $\int_0^{\pi/2} \cos^4 \theta d\theta$ |
| (19) $\int_0^{\pi/4} \sec \theta d\theta$ | (20) $\int_0^1 x^2 \sqrt{x+1} dx$ | (21) $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$ |
| (22) $\int_0^1 e^{\sqrt{x}} dx$ | (23) $\int_0^{2\pi} \sqrt{1 + \cos x} dx$ | (24) $\int_0^2 \frac{e^x}{\sqrt{1+e^x}} dx$ |
| (25) $\int_0^1 \sqrt{1+x^2} dx$ | (26) $\int_0^{1/2} \frac{x}{\sqrt{1-x^4}} dx$ | (27) $\int_{-1}^1 x^3 \sin(x^2 + 1) dx$ |
| (28) $\int_{-1}^1 \frac{x^2}{4+x^6} dx$ | (29) $\int_0^1 \frac{x^3}{\sqrt{1+x^2}} dx$ | (30) $\int_1^2 \frac{1}{x(\ln x)^2} dx$ |

2. Calcule as integrais indefinidas abaixo:

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| 1. $\int \frac{x^7 + x^2 + 1}{x^2} dx$ | 2. $\int e^{2x} dx$ | 3. $\int \cos 7x dx$ | 4. $\int \operatorname{tg}^2 x dx$ |
| 5. $\int \frac{7}{x-2} dx$ | 6. $\int \operatorname{tg}^3 x \sec^2 x dx$ | 7. $\int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}} dx$ | 8. $\int \operatorname{tg} x dx$ |
| 9. $\int \operatorname{tg}^3 x dx$ | 10. $\int \frac{x}{1+x^2} dx$ | 11. $\int \frac{x}{1+x^4} dx$ | 12. $\int \frac{x^2}{1+x^2} dx$ |
| 13. $\int x \sqrt{1-x^2} dx$ | 14. $\int \sec x dx$ | 15. $\int \frac{dx}{x \sqrt{1+\ln x}}$ | 16. $\int x^2 \sqrt[5]{x^3+1} dx$ |
| 17. $\int \frac{4x+8}{2x^2+8x+20} dx$ | 18. $\int \frac{\sqrt{\ln x}}{x} dx$ | 19. $\int \frac{dx}{(\operatorname{arcsen} x) \sqrt{1-x^2}}$ | 20. $\int \frac{e^x}{1+e^x} dx$ |
| 21. $\int \frac{\operatorname{sen} 2x}{1+\cos^2 x} dx$ | 22. $\int e^{x^3} x^2 dx$ | 23. $\int e^x \sqrt[3]{1+e^x} dx$ | 24. $\int \frac{\operatorname{sen} \sqrt{x}}{\sqrt{x}} dx$ |
| 25. $\int \frac{e^{\operatorname{arctg} x}}{1+x^2} dx$ | 26. $\int 2x(x+1)^{2010} dx$ | 27. $\int x \operatorname{sen} x dx$ | 28. $\int e^x \cos x dx$ |
| 29. $\int x^r \ln x dx, r \in \mathbb{R}$ | 30. $\int (\ln x)^2 dx$ | 31. $\int x e^{-x} dx$ | 32. $\int x \operatorname{arctg} x dx$ |

33. $\int \arcsen x \, dx$	34. $\int \sec^3 x \, dx$	35. $\int \cos^2 x \, dx$	36. $\int \sin^2 x \cos^3 x \, dx$
37. $\int \sin^2 x \cos^2 x \, dx$	38. $\int \frac{1 - \sen x}{\cos x} \, dx$	39. $\int \frac{3x^2 + 4x + 5}{(x-1)(x-2)(x-3)} \, dx$	40. $\int \frac{dx}{2x^2 + 8x + 20}$
41. $\int \frac{3x^2 + 4x + 5}{(x-1)^2(x-2)} \, dx$	42. $\int \frac{x^5 + x + 1}{x^3 - 8} \, dx$	43. $\int \frac{x^2}{\sqrt{1-x^2}} \, dx$	44. $\int x^2 \sqrt{1-x^2} \, dx$
45. $\int e^{\sqrt{x}} \, dx$	46. $\int \ln(x + \sqrt{1+x^2}) \, dx$	47. $\int \frac{dx}{\sqrt{5-2x+x^2}}$	48. $\int \sqrt{x} \ln x \, dx$
49. $\int \sen(\ln x) \, dx$	50. $\int \frac{x}{x^2 - 4} \, dx$	51. $\int \frac{3x^2 + 5x + 4}{x^3 + x^2 + x - 3} \, dx$	52. $\int \sqrt{a^2 + b^2 x^2} \, dx$
53. $\int \frac{dx}{\sqrt{a^2 + b^2 x^2}}$	54. $\int \sqrt{x^2 - 2x + 2} \, dx$	55. $\int \sqrt{3 - 2x - x^2} \, dx$	56. $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$
57. $\int \cos^3 x \, dx$	58. $\int \sen^5 x \, dx$	59. $\int \frac{\cos^5 x}{\sen^3 x} \, dx$	60. $\int \sen^3\left(\frac{x}{2}\right) \cos^5\left(\frac{x}{2}\right) \, dx$
61. $\int \frac{dx}{\sen^5 x \cos^3 x}$	62. $\int \sen^4 x \, dx$	63. $\int \sen^2 x \cos^5 x \, dx$	64. $\int \sen^2 x \cos^4 x \, dx$
65. $\int \cos^6(3x) \, dx$	66. $\int \frac{\cos^2 x}{\sen^6 x} \, dx$	67. $\int \frac{dx}{\sen^2 x \cos^4 x}$	68. $\int \sqrt{\frac{1-x}{1+x}} \, dx$
69. $\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}}$	70. $\int \frac{x+1}{x^2(x^2+4)^2} \, dx$	71. $\int \frac{\arctg x}{x^2} \, dx$	72. $\int \frac{x^2 dx}{\sqrt{2x-x^2}} \, dx$
73. $\int \frac{4x^2 - 3x + 3}{(x^2 - 2x + 2)(x+1)} \, dx$	74. $\int \frac{dx}{1+e^x}$	75. $\int \frac{\ln(x+1)}{x^2} \, dx$	76. $\int x^5 e^{-x^3} \, dx$
77. $\int \frac{x+1}{x^2(x^2+4)} \, dx$	78. $\int \arctg \sqrt{x} \, dx$	79. $\int \frac{2x+1}{x^2+2x+2} \, dx$	80. $\int \cos^3 x(1 + \sqrt{\sen x}) \, dx$

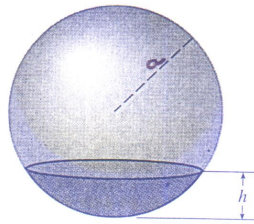
☆ Volumes, áreas e comprimentos

- Determine o volume de uma pirâmide cuja base é o quadrado de lado L e cuja altura é h .
- Calcule o volume do sólido cuja base é a astróide de equação $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ e tal que as seções transversais por planos paralelos ao plano Oxz são quadrados.
- Seja A a região do plano delimitada pelas desigualdades $y \geq 0$ e $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$. Calcule o volume do sólido de rotação obtido girando-se A em torno do eixo x .
- Calcule o comprimento do gráfico de $f(x) = \ln(\cos x)$, para $0 \leq x \leq \frac{\pi}{4}$.
- Calcule o comprimento da astróide $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.
- Calcule a área da região interna ao laço formado pela curva $y^2 = x^2(x+3)$.
- Calcule a área da região do plano limitada pela elipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
- Determine o volume do sólido obtido pela rotação do conjunto A em torno do eixo x :
 - $A = \{(x, y) \in \mathbb{R}^2 : 0 \leq xy \leq 2, x^2 + y^2 \leq 5 \text{ e } x > 0\}$
 - $A = \{(x, y) \in \mathbb{R}^2 : y \geq \sqrt{x} \text{ e } (x-1)^2 + y^2 \leq 1\}$.
 - $A = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2 \text{ e } e^{-x} \leq y \leq e^x\}$
 - $A = \{(x, y) \in \mathbb{R}^2 : x > 0, y \leq 1 \text{ e } 1/x \leq y \leq 4/x^2\}$

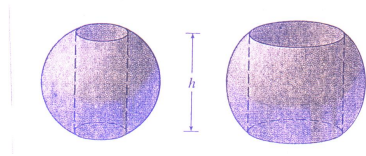
(e) $A = \{(x, y) \in \mathbb{R}^2 : y \geq 0, 2x^2 + y^2 \geq 1\}$

(f) $A = \{(x, y) \in \mathbb{R}^2 : y \geq 0, 1 \leq x^2 + y^2 \leq 4\}$

11. Calcule o volume do sólido obtido pela rotação em torno da reta $y = 3$ da região delimitada pelas parábolas $y = x^2$ e $y = 2 - x^2$.
12. Seja $A = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1 \text{ e } \ln(x + 1) + 2 \leq y \leq e^x + 4\}$. Determine o volume do sólido obtido pela rotação de A em torno da reta $y = 2$.
13. O disco $x^2 + y^2 \leq a^2$ é girado em torno da reta $x = b$, com $b > a$, para gerar um sólido, com a forma de um pneu. Esse sólido é chamado **toro**. Calcule seu volume.
14. Calcule o volume de uma calota esférica de altura h , $h \leq a$, de uma esfera de raio a .



15. Determine o comprimento da curva $y = \cosh x$, $-3 \leq x \leq 4$.
16. Um anel esférico é o sólido que permanece após a perfuração de um buraco cilíndrico através do centro de uma esfera sólida. Se a esfera tem raio R e o anel esférico tem altura h , prove o fato notável de que o volume do anel depende de h , mas não de R .



17. Calcule o volume do sólido obtido pela rotação de A em torno do eixo y :

(a) $A = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1 + (x^2/2), y \geq x^2 - 1\}$

(b) $A = \{(x, y) : x^2 \leq y \leq 4, x \geq 0\}$

(c) $A = \{(x, y) : 1 \leq x \leq e, 0 \leq y \leq \ln x\}$

(d) $A = \{(x, y) : 0 \leq x \leq 8, 0 \leq y \leq \sqrt[3]{x}\}$

(e) $A = \{(x, y) : 0 \leq x \leq 2, y \geq \sqrt{x-1}, 0 \leq y \leq x^2\}$

18. Calcule o volume da região delimitada pelas curvas abaixo:

(a) $y = x^2 - x - 6$ e $y = 0$

(b) $y = 20 - x^2$ e $y = x^2 - 12$

(c) $y = e^x - 1$, $y = x^2 - x$ e $x = 1$

(d) $x + y = 0$ e $x = y^2 + 3y$

(e) $y = \sqrt{x}$, $y = x^2$ e $x = 2$

★ Respostas

(1)

(1) $e^{-1} - 2$; (2) 52; (3) $31/2$; (4) $4\pi ab^2/3$; (5) $2\sqrt{3} + \frac{6}{5}\sqrt[6]{32}$; (6) 4; (7) 0 se $n = 0$ e $(-1)^{n+1}\pi/n$ se $n > 0$;
 (8) 0 se n é par e $-2/n^2$ se n é ímpar; (9) $e^2 + 2/e$; (10) $e^2 - 1/e$; (11) $\pi/4$; (12) $\pi/4$; (13) 6; (14) 0; (15)
 $(e^4 - 1)/2$; (16) $1 - \pi/4$; (17) $3\pi/8$; (18) $3\pi/8$; (19) $\ln(1 + \sqrt{2})$; (20) $16/105$; (21) $\pi/6$; (22) 2; (23) $4\sqrt{2}$; (24)
 $2(\sqrt{1 + e^2} - \sqrt{2})$; (25) $\frac{\ln(\sqrt{2}+1)+\sqrt{2}}{2}$; (26) $\frac{\arcsen(1/4)}{2}$; (27) 0; (28) $\frac{\text{arctg}(1/2)}{3}$; (29) $\frac{2-\sqrt{2}}{3}$; (30) $\frac{1}{\ln 2} - \frac{1}{\ln 3}$.

(2)

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| (1) $\frac{x^6}{6} + x - \frac{1}{x} + C$ | (2) $\frac{e^{2x}}{2} + C$ | (3) $\frac{1}{7}\text{sen } 7x + C$ |
| (4) $\text{tg } x - x + C$ | (5) $7\ln x - 2 + C$ | (6) $\frac{1}{4}\text{tg}^4 x + C$ |
| (7) $2\sqrt{\cos x}(\frac{1}{5}\cos^2 x - 1) + C$ | (8) $-\ln \cos x + C$ | (9) $\frac{1}{2}\text{tg}^2 x + \ln \cos x + C$ |
| (10) $\frac{1}{2}\ln(1 + x^2) + C$ | (11) $\frac{1}{2}\text{arctg } x^2 + C$ | (12) $x - \text{arctg } x + C$ |
| (13) $-\frac{1}{3}\sqrt{(1 - x^2)^3} + C$ | (14) $\ln \sec x + \text{tg } x + C$ | (15) $2\sqrt{1 + \ln x} + C$ |
| (16) $\frac{5}{18}\sqrt[5]{(x^3 + 1)^6} + C$ | (17) $\ln(2x^2 + 8x + 20) + C$ | (18) $\frac{2}{3}\sqrt{(\ln x)^3} + C$ |
| (19) $\ln \arcsen x + C$ | (20) $\ln(1 + e^x) + C$ | (21) $-\ln(1 + \cos^2 x) + C$ |
| (22) $\frac{1}{3}e^{x^3} + C$ | (23) $\frac{3}{4}\sqrt[3]{(1 + e^x)^4} + C$ | (24) $-2\cos\sqrt{x} + C$ |
| (25) $e^{\text{arctg } x} + C$ | (26) $2(x + 1)^{2011}(\frac{x+1}{2012} - \frac{1}{2011}) + C$ | (27) $-x\cos x + \text{sen } x + C$ |
| (28) $\frac{1}{2}e^x(\text{sen } x + \cos x) + C$ | (29) $\begin{cases} \frac{x^{r+1}}{r+1}\ln x - \frac{x^{r+1}}{(r+1)^2} + C, \text{ se } r \neq -1 \\ \frac{1}{2}(\ln x)^2 + C, \text{ se } r = -1 \end{cases}$ | (30) $x(\ln x)^2 - 2(x\ln x - x) + C$ |

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| (31) $(-x - 1)e^{-x} + C$ | (32) $\frac{x^2}{2}\text{arctg } x - \frac{x}{2} + \frac{1}{2}\text{arctg } x + C$ |
| (33) $x\arcsen x + \sqrt{1 - x^2} + C$ | (34) $\frac{1}{2}\sec x \text{tg } x + \frac{1}{2}\ln \sec x + \text{tg } x + C$ |
| (35) $\frac{1}{2}(x + \text{sen } x \cos x) + C$ | (36) $\frac{1}{3}\text{sen}^3 x - \frac{1}{5}\text{sen}^5 x + C$ |
| (37) $\frac{1}{8}(x - \frac{1}{4}\text{sen } 4x) + C$ | (38) $\ln 1 + \text{sen } x + C$ |
| (39) $6\ln x - 1 - 25\ln x - 2 + 22\ln x - 3 + C$ | (40) $\frac{\sqrt{6}}{12}\text{arctg}(\frac{x+2}{\sqrt{6}}) + C$ |
| (41) $-22\ln x - 1 + \frac{12}{x-1} + 25\ln x - 2 + C$ | (44) $\frac{x}{8}(2x^2 - 1)\sqrt{1 - x^2} + \frac{1}{8}\arcsen x + C$ |
| (42) $\frac{x^3}{3} + \frac{35}{12}\ln x - 2 + \frac{61}{24}\ln(1 + (\frac{x+1}{\sqrt{3}})^2) + \frac{\sqrt{3}}{12}\text{arctg}(\frac{x+1}{\sqrt{3}}) + C$ | (46) $x\ln(x + \sqrt{1 + x^2}) - \sqrt{1 + x^2} + C$ |
| (43) $\frac{1}{2}\arcsen x - \frac{1}{2}x\sqrt{1 - x^2} + C$ | (48) $\frac{2}{3}x\sqrt{x}(\ln x - \frac{2}{3}) + C$ |
| (45) $2(\sqrt{x} - 1)e^{\sqrt{x}} + C$ | (50) $\frac{1}{2}\ln x^2 - 4 + C$ |
| (47) $\ln \sqrt{5 - 2x + x^2} + x - 1 + C$ | (54) $\frac{x-1}{2}\sqrt{x^2 - 2x + 2} + \frac{1}{2}\ln(x - 1 + \sqrt{x^2 - 2x + 2}) + C$ |
| (49) $\frac{x}{2}(\text{sen}(\ln x) - \cos(\ln x)) + C$ | (57) $\text{sen } x - \frac{1}{3}\text{sen}^3 x + C$ |
| (51) $2\ln x - 1 + \frac{1}{2}\ln(x^2 + 2x + 3) + \frac{1}{\sqrt{2}}\text{arctg}(\frac{x+1}{\sqrt{2}}) + C$ | (59) $\frac{1}{2}\text{sen}^2 x - \frac{1}{2\text{sen}^2 x} - 2\ln \text{sen } x + C$ |
| (52) $x\sqrt{a^2 + b^2x^2} + \frac{a^2}{2b}\ln(\frac{bx}{a} + \frac{\sqrt{a^2 + b^2x^2}}{a}) + C$ | 61) $\frac{1}{2}\text{tg}^2 x + 3\ln \text{tg } x - \frac{3}{2\text{tg}^2 x} - \frac{1}{4\text{tg}^4 x} + C$ |
| (53) $\frac{1}{b}\ln(\frac{bx}{a} + \frac{\sqrt{a^2 + b^2x^2}}{a}) + C$ | (64) $\frac{x}{16} - \frac{1}{64}\text{sen}(4x) + \frac{1}{48}\text{sen}^3(2x) + C$ |
| (55) $\frac{x+1}{2}\sqrt{3 - 2x - x^2} + 2\arcsen(\frac{x+1}{2}) + C$ | (67) $\text{tg } x + \frac{1}{3}\text{tg}^3 x - 2\cotg(2x) + C$ |
| (56) $\frac{1}{\sqrt{2}}\text{arctg}(\frac{x\sqrt{2}}{\sqrt{1-x^2}}) + C$ | (69) $2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln \sqrt[6]{x} - 1 + C$ |
| (58) $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$ | (72) $\frac{3}{2}\arcsen(x - 1) - (\frac{x+3}{2})\sqrt{2x - x^2} + C$ |
| (60) $\frac{1}{4}\cos^8(\frac{x}{2}) - \frac{1}{3}\cos^6(\frac{x}{2}) + C$ | |
| (62) $\frac{3}{8}x - \frac{1}{4}\text{sen}(2x) + \frac{1}{32}\text{sen}(4x) + C$ | |
| (63) $\frac{1}{3}\text{sen}^3 x - \frac{2}{5}\text{sen}^5 x + \frac{1}{7}\text{sen}^7 x + C$ | |
| (65) $\frac{5}{16}x + \frac{1}{12}\text{sen}(6x) + \frac{1}{64}\text{sen}(12x) - \frac{1}{144}\text{sen}^3(6x) + C$ | |
| (66) $-\frac{1}{3}\cotg^3 x - \frac{1}{5}\cotg^5 x + C$ | |
| (68) $\arcsen x + \sqrt{1 - x^2} + C$ | |
| (70) $\frac{1}{16}\ln x - \frac{1}{16x} - \frac{1}{32}\ln(x^2 + 4) - \frac{3}{64}\text{arctg } \frac{x}{2} + \frac{4-x}{32(x^2+4)} + C$ | |
| (71) $\frac{-\text{arctg } x}{x} + \ln x - \ln\sqrt{1 + x^2} + C$ | |
| (73) $2\ln x + 1 + \ln(x^2 - 2x + 2) + 3\text{arctg}(x - 1) + C$ | |

$$(74) x - \ln(1 + e^x) + C$$

$$(75) -\frac{\ln(x+1)}{x} + \ln|x| - \ln(x+1) + C$$

$$(76) -\frac{1}{3}(x^3 + 1)e^{-x^3} + C$$

$$(77) \frac{1}{4}\ln|x| - \frac{1}{4x} - \frac{1}{8}\ln(x^2 + 4) - \frac{1}{16}\operatorname{arctg}\left(\frac{x}{2}\right) + C$$

$$(78) (x+1)\operatorname{arctg}\sqrt{x} - \sqrt{x}$$

$$(79) \ln(x^2 + 2x + 2) - \operatorname{arctg}(x+1) + C$$

$$(80) \operatorname{sen} x + 2\sqrt{\operatorname{sen} x} - \frac{\operatorname{sen}^3 x}{3} - \frac{2\sqrt{\operatorname{sen}^5 x}}{5} + C$$

(2) $\frac{l^2 h}{3}$; **(3)** $\frac{128}{105}a^3$; **(4)** 2; **(5)** $\ln(1 + \sqrt{2})$; **(6)** $6a$; **(7)** $\frac{24}{5}\sqrt{3}$; **(8)** πab ;

(9) (a) $\frac{5\sqrt{5}-2}{3}\pi$; (b) $\frac{\pi}{6}$; (c) $\frac{\pi}{2}(e^2 - e^{-2})^2$; (d) $\frac{5\pi}{6}$; (e) $2\pi\sqrt{2}/3$; (f) $28\pi/3$; **(10)** $\frac{32}{3}\pi$; **(11)** $\pi\left(\frac{e^2}{2} + 4e - 2(\ln 2)^2 + 4\ln 2 - \frac{3}{2}\right)$

(12) $(2\pi b)(\pi a^2)$; **(13)** $\pi h^2(a - \frac{h}{3})$; **(14)** $\sinh 4 + \sinh 3$.

(17) (a) $7\pi/2$; (b) 8π ; (c) $\pi(e^2 + 1)/2$; (d) $768\pi/7$; (e) $88\pi/15$;

(18) $125/6$