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Setor de Ciências Exatas
Departamento de Matemática
CM005 - Álgebra Linear - Turma C (Engenharia Elétrica)
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Lista 2

✰ Conjuntos linearmente independentes e bases

1. Verifique se cada um dos conjuntos X abaixo é LI ou LD no espaço E indicado:

   (a) $X = \{(1,0,1,0,1,0),(0,1,0,1,0,1),(1,2,3,3,2,1),(-1,-2,-3,1,2,3),(-1,0,1,0,-1,0),$ $(1,1,1,1,1,1),(9,8,7,6,5,4)\}$ $(E = \mathbb{R}^6)$

   (b) $X = \{(0,1,2,0),(1,-1,0,1),(1,0,2,3)\}$ $(E = \mathbb{R}^4)$

   (c) $X = \{(1,0,-1,2,0),(-1,2,5,7,1),(1,1,1,2,3),(5,-6,-17,-17,-3)\}$ $(E = \mathbb{R}^5)$

   (d) $X = \{1,0,-1,2,0,1),(-1,2,5,7,1,0),(1,1,1,2,3,1),(5,-6,-17,-17,-3,0)\}$ $(E = \mathbb{R}^6)$

   (e) $X = \{(1,-2,-1,2,0),(-1,1,1,3,1),(1,-1,-1,-2,0),(0,3,0,3,0)\}$ $(E = \mathbb{R}^5)$

   (f) $X = \{13, 1+x-x^3, x^4-2x^3+2\}$ $(E = \mathscr{P}_4(\mathbb{R}))$

   (g) $X = \{3, -1+x, 2-x+4x^2, 2+x^3\}$ $(E = \mathscr{P}_3(\mathbb{R}))$

2. Encontre uma base e a dimensão do subespaço $S(X)$ gerado por X para cada um dos ítens do exercício anterior.

3. Ache uma base e a dimensão do espaço de soluções de cada um dos sistemas lineares abaixo em $\mathbb{R}^n$:

   (a) $\left\{\begin{array}{rcrcrcrcl} x_1 & - & x_2 & + & x_3 & - & 2x_4 & = & 0 \\ 2x_1 & + & 3x_2 & - & x_3 & + & 2x_4 & = & 0 \end{array}\right.$ $(n=4)$

   (b) $\left\{\begin{array}{rcrcrcrcrcl} -3x_1 & - & x_2 & & & + & x_4 & - & x_5 & = & 0 \\ x_1 & + & 3x_2 & + & x_3 & - & 4x_4 & + & 2x_5 & = & 0 \\ 4x_1 & + & 4x_2 & + & x_3 & - & 5x_4 & + & 2x_5 & = & 0 \end{array}\right.$ $(n=5)$

   (c) $2x_1 + 3x_2 - x_3 = 0$ $(n=3)$

   (d) $\left\{\begin{array}{rcrcl} x_1 & + & 6x_2 & = & 0 \\ 2x_1 & + & 3x_2 & = & 0 \end{array}\right.$ $(n=2)$

   (e) $\left\{\begin{array}{rcrcrcrcl} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 0 \\ 2x_1 & - & x_2 & + & x_3 & + & x_4 & = & 0 \\ -x_1 & + & 2x_2 & + & 4x_3 & - & x_4 & = & 0 \end{array}\right.$ $(n=4)$

   (f) $\left\{\begin{array}{rcrcrcrcrcrcl} -x_1 & + & x_2 & - & x_3 & + & x_4 & - & x_5 & + & x_6 & = & 0 \\ 2x_1 & + & 3x_2 & + & x_3 & + & 2x_4 & + & x_5 & - & x_6 & = & 0 \\ 3x_1 & + & x_2 & - & x_3 & + & x_4 & - & x_5 & + & 2x_6 & = & 0 \end{array}\right.$ $(n=6)$

(g) $\begin{cases} x_1 - x_2 - x_3 + 2x_4 = 0 \\ 2x_1 + 3x_3 - x_4 = 0 \end{cases}$ $(n = 4)$

4. Para cada um dos subconjuntos $X = \{u_1, \ldots, u_n\}$ abaixo, determine uma base $B \subset X$ e a dimensão do subespaço $S(X)$ gerado por X:

   (a) $E = \mathbb{R}^3$, $n = 2$, $u_1 = (1,2,-1)$, $u_2 = (2015, 4030, -2015)$

   (b) $E = \mathbb{R}^6$, $n = 4$, $u_1 = (0,1,0,2,0,1)$, $u_2 = (-1,1,-1,0,1,1)$, $u_3 = (-3,4,-3,2,3,4)$, $u_4 = (-1,4,-1,6,1,4)$

   (c) $E = \mathbb{R}^4$, $n = 4$, $u_1 = (0,1,2,3)$, $u_2 = (-1,1,2,1)$, $u_3 = (1,1,1,1)$, $u_4 = (2,1,1,3)$

   (d) $E = \mathbb{R}^5$, $n = 5$, $u_1 = (1,2,0,1,2)$, $u_2 = (0,0,-1,0,1)$, $u_3 = (-3,1,0,0,0)$, $u_4 = (0,0,0,1,-1)$, $u_5 = (1,1,1,1,1)$

   (e) $E = \mathbb{R}^3$, $n = 6$, $u_1 = (0,-1,0)$, $u_2 = (1,3,4)$, $u_3 = (1,2,4)$, $u_4 = (-1,-4,-4)$, $u_5 = (5,12,20)$, $u_6 = (4,8,16)$

5. Complete cada um dos subconjuntos $X = \{u_1, \ldots, u_n\}$ linearmente independentes abaixo de forma a obter uma base de E:

   (a) $E = \mathbb{R}^2$, $n = 1$, $u_1 = (1,3)$

   (b) $E = \mathbb{R}^3$, $n = 1$, $u_1 = (0,1,1)$

   (c) $E = \mathbb{R}^3$, $n = 1$, $u_1 = (2015, 2016, 2017)$

   (d) $E = \mathbb{R}^3$, $n = 2$, $u_1 = (1,2,4)$, $u_2 = (0,1,4)$

   (e) $E = \mathbb{R}^3$, $n = 2$, $u_1 = (0,1,2)$, $u_3 = (-1,0,-1)$

   (f) $E = \mathbb{R}^7$, $n = 3$, $u_1 = (1,2,0,=1,0,-2,0)$, $u_2 = (3,0,1,2,1,0,2015)$, $u_3 = (-1,1,0,1,0,3,0)$

   (g) $E = \mathscr{P}^3(\mathbb{R})$, $n = 2$, $u_1 = -3$, $u_2 = 1 - x^2$

   (h) $E = \mathscr{P}^8(\mathbb{R})$, $n = 3$, $u_1 = 1 - x + x^2$, $u_2 = 1 + x - x^2$, $u_3 = x^4 - 2x^3$

6. Obtenha uma base e a dimensão do subespaço soma $F_1 + F_2$, onde $F_1 = S(X_1)$, $F_2 = S(X_2)$, nos casos abaixo:

   (a) $X_1 = \{(1,2,1,0), (0,1,2,0)\}$, $X_2 = \{(1,1,0,1), (1,1,-1,0)\}$

   (b) $X_1 = \{(0,1,0,1), (0,0,0,1), (0,2,0,-2)\}$, $X_2 = \{(1,1,0,0), (0,1,0,0)\}$

   (c) $X_1 = \{(-1,0,0,1,1), (1,2,1,0,0)\}$, $X_2 = \{(1,0,1,0,1), (1,2,3,4,1), (0,0,0,0,1), (-1,0,-2,0,1)\}$

   (d) $X_1 = \{(1,2,0), (1,1,1)\}$, $X_2 = \{(0,1,-1), (0,-1,-2)\}$

   (e) $X_1 = \{(0,0,1,0,-1,0), (1,2,1,0,-1,3), (-1,0,-1,0,0,-1), (1,2,-1,0,1,1)\}$,
   $X_2 = \{(1,0,1,1,2,1), (0,2,1,0,-2,2)\}$

7. Obtenha um sistema linear cujo espaço de soluções seja o subespaço gerado pelo conjunto X:

   (a) $X = \{(1,1,0,1,-1), (0,1,1,2,0)\}$

   (b) $X = \{(1,2,1), (0,-1,0), (2,2,2)\}$

   (c) $X = \{(0,0,-1,2), (1,3,1,-1), (0,1,-2,1)\}$

   (d) $X = \{(-1,-2,0,-1,0), (0,2,0,-1,2), (1,2,1,1,1), (0,3,-1,0,1)\}$

(e) $X = \{(-2, 1, 0, 1, 2), (1, 1, -1, 1, -1)\}$

(f) $X = \{(0, -1, 0, 0, 1), (0, 1, 1, 1, 0), (-2, 1, -1, 0, 1), (-1, 1, -1, 0, 0)\}$

(g) $X = \{(0, 1, 0, 1, 0, 0), (-1, 1, 2, 0, 1, 1), (0, 0, 0, 1, 0, -2)\}$

8. Dados $m < n$, mostre que qualquer subespaço de dimensão m contido em $\mathbb{R}^n$ é o espaço solução de um sistema linear com $n - m$ equações independentes.