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Setor de Ciências Exatas
Departamento de Matemática
CM005 - Álgebra Linear - Turma C (Engenharia Elétrica)
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Lista 5

☆ Produto interno

1. Determine o ângulo entre u e v e a projeção de u na direção de v:

   (a) $u = (2,1,3)$ e $v = (6,3,9)$
   (b) $u = (2,-3)$ e $v = (3,2)$
   (c) $u = (4,1)$ e $v = (3,2)$
   (d) $u = (-2,3,1)$ e $v = (1,2,4)$
   (e) $u = (3,4)$ e $v = (1,0)$
   (f) $u = (2,4,3)$ e $v = (1,1,1)$

2. Determine o vetor de F mais próximo de u nos seguintes casos:

   (a) $F = S((1,2))$, $u = (0,1)$
   (b) $F = S((1,0,0),(0,0,1))$, $u = (-1,2,0)$
   (c) $F = \{(x_1,x_2,x_3) \in \mathbb{R}^3 : 2x_1 + x_2 - x_3 = 0\}$, $u = (0,0,1)$
   (d) $F = \{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 : x_1 - x_3 - x_4 = 0\}$, $u = (1,1,1,1)$
   (e) $F = \{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 : x_1 + x_2 - x_3 - x_4 = 0\}$, $u = (0,1,0,-1)$
   (f) $F = \{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 0,\ x_2 - 2x_3 = 0\}$, $u = (0,1,0,0)$

3. Use o processo de ortonormalização de Gram-Schmidt para obter bases ortonormais a partir das bases abaixo. A seguir, determine as coordenadas do vetor w em relação à base ortonormal obtida:

   (a) $\{(1,6),(-1,2)\}$, $w = (1,0)$
   (b) $\{(1,2,-2),(4,3,2),(1,2,1)\}$, $w = (2,1,-2)$
   (c) $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$, $w = (0,0,1,-1)$

4. Determine bases ortonormais para $\ker A$, $\ker A^t$, $\operatorname{Im} A$ e $\operatorname{Im} A^t$ para as matrizes A abaixo:

   (a) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ (b) $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$ (c) $A = \begin{pmatrix} -1 & 1 & 1 & 2 \\ 1 & 0 & 0 & 2 \\ 1 & 1 & 1 & 6 \end{pmatrix}$

   (d) $A = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ (e) $A = \begin{pmatrix} -1 & 1 \\ 1 & -2 \\ 1 & 0 \\ 0 & 0 \\ 1 & -1 \end{pmatrix}$ (f) $A = \begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ -1 & 0 & 0 & 2 & 0 \end{pmatrix}$

5. Para cada um dos subespaços F abaixo, determine uma base ortonormal para o complemento ortogonal $F^{\perp}$ de F:

   (a) $F = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 = x_3\}$

   (b) $F = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_2 = 0\}$

   (c) $F = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 - 2x_2 + 4x_3 = 0\}$

   (d) $F = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_3 = 0\}$

   (e) $F = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 = x_2 = 0\}$

   (f) $F = S((1,0,1,0), (0,1,0,1))$

   (g) $F = S((1,2,0,0), (0,1,0,-1), (3,11,0,-5))$

   (h) $F = S((1,1,1,1,1), (0,0,0,0,1))$

   (i) $F = S((1,-1,2,0,1), (0,1,0,0,-1))$

   (j) $F = S((1,-1,1,-1,1), (0,2,0,-1,2))$

6. Encontre matrizes ortogonais U tais que $U^{-1}AU$ seja diagonal:

   (a) $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ (b) $A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$ (c) $A = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{pmatrix}$

   (d) $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & -2 & 3 \end{pmatrix}$ (e) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (f) $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$