UFPR - Universidade Federal do Paraná
Setor de Ciências Exatas
Departamento de Matemática
CM312 - Cálculo II - Turma Honors
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CM312 H

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GRR____

INFORMAÇÕES IMPORTANTES

1. A prova tem 3 questões.
2. Você pode usar todos os resultados demonstrados em aula.
3. Justifique suas respostas.
4. Boa prova!

Questão 1 Calcule as seguintes integrais definidas (cada item vale (**15 pontos**)):

① $\displaystyle\int_{-\pi/4}^{\pi/4} (\underbrace{\text{sen}\left(x^3 + x\cos x\right)}_{\text{ÍMPAR}} + \sec^2 x)\, dx = \underbrace{\int_{-\pi/4}^{\pi/4} \text{sen}(x^3 + x\cos x)\, dx}_{= 0} + \int_{-\pi/4}^{\pi/4} \sec^2 x\, dx$

$$= \text{tg}\, x \Big|_{-\pi/4}^{\pi/4} = 1 - (-1) = 2.$$

② $\displaystyle\int_0^1 \frac{dx}{1+e^x} = \int_0^1 \frac{(1+e^x) - e^x}{1+e^x}\, dx$

$$= \int_0^1 \left(1 - \frac{e^x}{1+e^x}\right) dx = 1 - \int_0^1 \frac{e^x}{1+e^x}\, dx$$

$u = e^x + 1$
$du = e^x dx$

$$= 1 - \int_2^{e+1} \frac{du}{u}$$

$$= 1 - \Big[\ln|u|\Big]_2^{e+1}$$

$$= 1 - \left(\ln(e+1) - \ln 2\right)$$

$$= 1 - \ln(1+e) + \ln 2.$$

Questão 2 Calcule as seguintes integrais indefinidas (cada item vale (**20 pontos**)):

① $\int \frac{x+1}{x^2-9}dx$

$$\frac{x+1}{x^2-9} = \frac{x+1}{(x-3)(x+3)} = \frac{2/3}{x-3} + \frac{1/3}{x+3} \therefore$$

$$\int \frac{x+1}{x^2-9}\,dx = \frac{2}{3}\int \frac{dx}{x-3} + \frac{1}{3}\int \frac{dx}{x+3} = \frac{2}{3}\ln|x-3| + \frac{1}{3}\ln|x+3| + C$$

$$= \ln\left|(x-3)^{2/3}(x+3)^{1/3}\right| + C$$

② $\int \frac{x^3}{\sqrt{1-x^2}}dx$ $= \int \frac{\operatorname{sen}^3\theta}{\cos\theta}\cdot\cos\theta\, d\theta$

$x = \operatorname{sen}\theta$
$dx = \cos\theta\, d\theta$

$$= \int \operatorname{sen}^2\theta \cdot \operatorname{sen}\theta\, d\theta$$

$$= \int (1-\cos^2\theta)\operatorname{sen}\theta\, d\theta$$

$u = \cos\theta$
$du = -\operatorname{sen}\theta\, d\theta$

$$= -\int (1-u^2)\,du = -u + \frac{u^3}{3} + C$$

$$= -\cos\theta + \frac{\cos^3\theta}{3} + C$$

$$= -\sqrt{1-x^2} + \frac{\left(\sqrt{1-x^2}\right)^3}{3} + C$$

③ $\displaystyle\int \frac{\operatorname{sen}^5 x}{\sqrt{\cos x}}\,dx = \int \frac{\operatorname{sen}^4 x}{\sqrt{\cos x}} \cdot \operatorname{sen} x\, dx$

$$= \int \frac{(1-\cos^2 x)^2}{\sqrt{\cos x}} \cdot \operatorname{sen} x\, dx$$

$u = \cos x$

$du = -\operatorname{sen} x\, dx$

$$= -\int \frac{(1-u^2)^2}{\sqrt{u}}\, du = -\int \frac{1 - 2u^2 + u^4}{\sqrt{u}}\, du$$

$$= -\int \left(u^{-1/2} - 2u^{3/2} + u^{7/2}\right) du = -\frac{u^{1/2}}{1/2} + 2\,\frac{u^{5/2}}{5/2} - \frac{u^{9/2}}{9/2} + C$$

$$= -2u^{1/2} + \frac{4}{5}u^{5/2} - \frac{2}{9}u^{9/2} + C$$

$$= -2\sqrt{\cos x} + \frac{4}{5}\left(\sqrt{\cos x}\right)^5 - \frac{2}{9}\left(\sqrt{\cos x}\right)^9 + C$$

Questão 3 (15 pontos) Mostre que a integral

$$\int_0^\infty \frac{\sqrt[4]{x^7+\cos(x^2)+2}}{\sqrt[3]{2x^9+\ln(1+x^2)+1}}\,dx$$

converge absolutamente.

SEJAM $f(x) = \dfrac{\sqrt[4]{x^7+\cos(x^2)+2}}{\sqrt[3]{2x^9+\ln(1+x^2)+1}}$ E $g(x) = \dfrac{1}{x^{5/4}}$, $x>0$.

TEMOS QUE

$$\left|\frac{f(x)}{g(x)}\right| = \frac{\sqrt[4]{x^5}\cdot\sqrt[4]{x^7+\cos(x^2)+2}}{\sqrt[3]{2x^9+\ln(1+x^2)+1}}$$

$$= \frac{\sqrt[4]{\cancel{x^{12}}\left(1+\frac{\cos(x^2)}{x^7}+\frac{2}{x^7}\right)}}{\sqrt[3]{\cancel{x^9}\left(2+\frac{\ln(1+x^2)}{x^9}+\frac{1}{x^9}\right)}}$$

$$= \frac{\sqrt[4]{1+\frac{\cos(x^2)}{x^7}+\frac{2}{x^7}}}{\sqrt[3]{2+\frac{\ln(1+x^2)}{x^9}+\frac{1}{x^9}}} \xrightarrow{x\to+\infty} \frac{1}{\sqrt[3]{2}}$$

COMO $\displaystyle\int_1^\infty g(x)\,dx = \int_1^\infty \frac{dx}{x^{5/4}}$ CONVERGE (POIS $5/4 > 1$),

TEMOS QUE $\displaystyle\int_1^\infty f(x)\,dx$ CONVERGE ABSOLUTAMENTE.

COMO f É CONTÍNUA EM $x=0$ $\left(f(0) = \dfrac{\sqrt[4]{3}}{1}\right)$,

SEGUE QUE $\displaystyle\int_0^\infty f(x)\,dx$ CONVERGE ABSOLUTAMENTE.