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Departamento de Matemática
CM312 - Cálculo II - Engenharia Química - 2025/2
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Lista 1

✯ Técnicas de integração

1. Calcule as integrais definidas abaixo:

(1) $\int_{-1}^{0}(2x-e^x)\,dx$

(2) $\int_{-2}^{2}(3x+1)^2\,dx$

(3) $\int_{0}^{1}(2x+5)(3x+1)\,dx$

(4) $\int_{0}^{\pi/4}\frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

(5) $\int_{0}^{2}\frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx$

(6) $\int_{0}^{2\pi}|\operatorname{sen}\theta|\,d\theta$

(7) $\int_{0}^{\pi}x\operatorname{sen}(nx),\ n\in\mathbb{N}$

(8) $\int_{0}^{\pi}x\cos(nx)\,dx,\ n\in\mathbb{N}$

(9) $\int_{-1}^{2}xe^x\,dx$

(10) $\int_{-1}^{2}x^2e^x\,dx$

(11) $\int_{0}^{\pi/2}\cos^2\theta\,d\theta$

(12) $\int_{0}^{\pi/2}\operatorname{sen}^2\theta\,d\theta$

(13) $\int_{-3}^{3}(\operatorname{sen}(x^5)-x^7\cos x)\,dx$

(14) $\int_{-2}^{2}(x\cos(x^2+2x^4)+3x)\,dx$

(15) $\int_{0}^{2}xe^{x^2}\,dx$

(16) $\int_{0}^{\pi/4}\operatorname{tg}^2\theta\,d\theta$

(17) $\int_{0}^{\pi/2}\operatorname{sen}^4\theta\,d\theta$

(18) $\int_{0}^{\pi/2}\cos^4\theta\,d\theta$

(19) $\int_{0}^{\pi/4}\sec\theta\,d\theta$

(20) $\int_{0}^{1}x^2\sqrt{x+1}\,dx$

(21) $\int_{0}^{1/2}\frac{dx}{\sqrt{1-x^2}}$

(22) $\int_{0}^{1}e^{\sqrt{x}}\,dx$

(23) $\int_{0}^{2\pi}\sqrt{1+\cos x}\,dx$

(24) $\int_{0}^{2}\frac{e^x}{\sqrt{1+e^x}}\,dx$

(25) $\int_{0}^{1}\sqrt{1+x^2}\,dx$

(26) $\int_{0}^{1/2}\frac{x}{\sqrt{1-x^4}}\,dx$

(27) $\int_{-1}^{1}x^3\operatorname{sen}(x^2+1)\,dx$

(28) $\int_{-1}^{1}\frac{x^2}{4+x^6}\,dx$

(29) $\int_{0}^{1}\frac{x^3}{\sqrt{1+x^2}}\,dx$

2. Calcule as integrais definidas abaixo:

(1) $\int\frac{x^7+x^2+1}{x^2}\,dx$

(2) $\int e^{2x}\,dx$

(3) $\int\cos 7x\,dx$

(4) $\int\operatorname{tg}^2x\,dx$

(5) $\int\frac{7}{x-2}\,dx$

(6) $\int\operatorname{tg}^3x\sec^2x\,dx$

(7) $\int\frac{\operatorname{sen}^3x}{\sqrt{\cos x}}\,dx$

(8) $\int\operatorname{tg}x\,dx$

(9) $\int \operatorname{tg}^3 x\,dx$

(10) $\int \frac{x}{1+x^2}\,dx$

(11) $\int \frac{x}{1+x^4}\,dx$

(12) $\int \frac{x^2}{1+x^2}\,dx$

(13) $\int x\sqrt{1-x^2}\,dx$

(14) $\int \sec x\,dx$

(15) $\int \frac{dx}{x\sqrt{1+\ln x}}$

(16) $\int x^2\sqrt[5]{x^3+1}\,dx$

(17) $\int \frac{4x+8}{2x^2+8x+20}\,dx$

(18) $\int \frac{\sqrt{\ln x}}{x}\,dx$

(19) $\int \frac{dx}{(\arcsin x)\,\sqrt{1-x^2}}$

(20) $\int \frac{e^x}{1+e^x}\,dx$

(21) $\int \frac{\operatorname{sen} 2x}{1+\cos^2 x}\,dx$

(22) $\int e^{x^3}x^2\,dx$

(23) $\int e^x\sqrt[3]{1+e^x}\,dx$

(24) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx$

(25) $\int \frac{e^{\arctan x}}{1+x^2}\,dx$

(26) $\int 2x(x+1)^{2025}\,dx$

(27) $\int x\operatorname{sen} x\,dx$

(28) $\int e^x\cos x\,dx$

(29) $\int x^\alpha \ln x\,dx,\ \alpha\in\mathbb{R}$

(30) $\int (\ln x)^2\,dx$

(31) $\int xe^{-x}\,dx$

(32) $\int x\arctan x\,dx$

(33) $\int \arcsin x\,dx$

(34) $\int \sec^3 x\,dx$

(35) $\int \cos^2 x\,dx$

(36) $\int \operatorname{sen}^2 x\cos^3 x\,dx$

(37) $\int \operatorname{sen}^2 x\cos^2 x\,dx$

(38) $\int \frac{1-\operatorname{sen} x}{\cos x}\,dx$

(39) $\int \frac{dx}{x^2-3x+2}$

(40) $\int \frac{x}{x^2-3x+2}dx$

(41) $\int \frac{2x+1}{x^2-1}\,dx$

(42) $\int \frac{x+1}{x^2+1}\,dx$

(43) $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

(44) $\int x^2\sqrt{1-x^2}\,dx$

(45) $\int e^{\sqrt{x}}\,dx$

(46) $\int \ln(x+\sqrt{1+x^2})\,dx$

(47) $\int \frac{dx}{\sqrt{5-2x+x^2}}$

(48) $\int \sqrt{x}\ln x\,dx$

(49) $\int \operatorname{sen}(\ln x)\,dx$

(50) $\int \frac{x}{x^2-4}\,dx$

(51) $\int \frac{x^2}{x^2-2x-3}\,dx$

(52) $\int \sqrt{a^2+b^2x^2}\,dx,\ a,b>0$

(53) $\int \sqrt{x^2-2x+2}\,dx$

(54) $\int \sqrt{3-2x-x^2}\,dx$

(55) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

(56) $\int \cos^3 x\,dx$

(57) $\int \operatorname{sen}^5 x\,dx$

(58) $\int \frac{\cos^5 x}{\operatorname{sen}^3 x}\,dx$

(59) $\int \operatorname{sen}^3 x\cos^5 x\,dx$

(60) $\int \operatorname{sen}^4 x\,dx$

(61) $\int \operatorname{sen}^2 x\cos^5 x\,dx$

(62) $\int \operatorname{sen}^2 x\cos^4 x\,dx$

(63) $\int \sqrt{\frac{1-x}{1+x}}\,dx$

(64) $\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$

(65) $\int \frac{\arctan x}{x^2}\,dx$

(66) $\displaystyle\int \frac{x^2}{\sqrt{2x-x^2}}\,dx$

(67) $\displaystyle\int \frac{dx}{1+e^x}$

(68) $\displaystyle\int \frac{\ln(x+1)}{x^2}\,dx$

(69) $\displaystyle\int x^5 e^{-x^3}\,dx$

(70) $\displaystyle\int \arctan\sqrt{x}\,dx$

(71) $\displaystyle\int \frac{2x+1}{x^2+2x+2}\,dx$

(72) $\displaystyle\int \cos^3 x(1+\sqrt{\operatorname{sen} x})\,dx$

☆ Funções reais de duas e três variáveis

3. Ache e esboce o domínio das funções:

(1) $f(x,y)=\sqrt{x-y}$

(2) $f(x,y)=\arctan\dfrac{y}{x}$

(3) $f(x,y)=\dfrac{1}{\sqrt{x^2+y^2-1}}$

(4) $f(x,y)=\dfrac{x}{y^x}$

(5) $f(x,y)=\operatorname{tg}(x-y)$

(6) $f(x,y)=\ln(xy^2-x^3)$

(7) $f(x,y)=\ln(16-4x^2-y^2)$

4. Esboce uma família de curvas de nível de:

(1) $f(x,y)=\dfrac{x+y}{x-y}$

(2) $f(x,y)=x-\sqrt{1-y^2}$

(3) $f(x,y)=\dfrac{x^2}{x^2-y^2}$

(4) $f(x,y)=\dfrac{2xy^2}{x^2+y^4}$

5. Esboce os gráficos de:

(1) $f(x,y)=1-x-y$

(2) $f(x,y)=\dfrac{x}{x^2+1}$

(3) $f(x,y)=\sqrt{x^2+9y^2}$

(4) $f(x,y)=\dfrac{1}{4x^2+9y^2}$

(5) $f(x,y)=(x-y)^2$

(6) $f(x,y)=x^2+y^2+2y+3$

(7) $f(x,y)=\dfrac{1}{(x^2+2y^2)^2}$

(8) $f(x,y)=\ln(9x^2+y^2)$

(9) $f(x,y)=2-\sqrt[4]{x^2+4y^2}$

(10) $f(x,y)=\sqrt{x^2+y^2-9}$

(11) $f(x,y)=\sqrt{x^2+y^2+1}$

☆ Limites e continuidade

6. Calcule os seguintes limites, caso existam. Se não existirem, explique por quê:

(1) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{xy}{x^2+y^2}$

(2) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2y\cos(x^2+y^2)}{x^2+y^2}$

(3) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^3+y^3}{x^2+y^2}$

(4) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2y}{2x^4+x^2y+y^2}$

(5) $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2+3xy+4y^2}{3x^2+5y^2}$

(6) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$

(7) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^3-y}$

(8) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4\operatorname{sen}(x^2+y^2)}{x^4+y^2}$

(9) $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x+y)^3}{x^2+y^2}$

(10) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{x^2+y^2}\operatorname{sen}\left(\dfrac{xy}{\sqrt{x^2+y^2}}\right)$

(11) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y+y^4+x^4}{x^3y-xy^3}$

(12) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+\operatorname{sen}(x^2+y^2)}{y^4+\operatorname{sen}(x^2+y^2)}$

(13) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\operatorname{sen}(x^2+y^2)}{x^2+y^2}$

(14) $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$

(15) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^4}{x^2+y^8}$

(16) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2\operatorname{sen}^2 y}{x^2+2y^2}$

7. Determine o conjunto dos pontos de continuidade das funções abaixo:

(a) $f(x,y)=\dfrac{\operatorname{sen}(xy)}{e^x-y^2}$ (b) $f(x,y)=\dfrac{\sqrt{x-y^3}}{1-x^2-y^2}$

(c) $f(x,y)=\arctan(x+\sqrt{1/y})$ (d) $f(x,y)=\arcsin(x^2+y^2)$

(e) $f(x,y)=\begin{cases} \dfrac{x^2y^3}{2x^2+y^3} & \text{, se } (x,y)\neq(0,0) \\ 1 & \text{, se } (x,y)=(0,0) \end{cases}$

(f) $f(x,y)=\begin{cases} \dfrac{(x^2-y^2)(x-1)^2}{(x^2+y^2)((x-1)^2+(y-1)^2)} & \text{, se } (x,y)\neq(0,0) \text{ e } (x,y)\neq(1,1) \\ 1 & \text{, se } (x,y)=(0,0) \text{ ou } (x,y)=(1,1) \end{cases}$