GABARITO

CM312 — P1

Questão 1 Calcule as seguintes integrais definidas (cada item vale **(15 pontos)**):

① $\int_0^2 xe^{x^2}\,dx = \left.\frac{e^{x^2}}{2}\right|_0^2 = \frac{1}{2}\left(e^4 - 1\right).$

② $\int_{-\pi/4}^{\pi/4} \left(\underbrace{\operatorname{sen}\left(x^3 + x\cos x\right)}_{\text{ÍMPAR}} + \sec^2 x\right) dx = \int_{-\pi/4}^{\pi/4} \underbrace{\sec^2 x}_{\text{PAR}}\,dx$

$= 2\int_0^{\pi/4} \sec^2 x\,dx$

$= 2 \left. \operatorname{tg} x \right|_0^{\pi/4}$

$= 2(1-0) = 2.$

Questão 2 Calcule as seguintes integrais indefinidas (cada item vale (**20 pontos**)):

① $\int \frac{x+1}{x^2-9}dx$

$$\frac{x+1}{x^2-9} = \frac{2/3}{x-3} + \frac{1/3}{x+3}$$

$$\therefore \int \frac{x+1}{x^2-9}\,dx = \frac{2}{3}\ln|x-3| + \frac{1}{3}\ln|x+3| + C$$

$$= \ln\left|(x-3)^{2/3}(x+3)^{1/3}\right| + C$$

② $\int \frac{x^3}{\sqrt{1-x^2}}dx$ $= \int \frac{\operatorname{sen}^3\theta}{\cos\theta}\cdot\cos\theta\, d\theta$

$x = \operatorname{sen}\theta$
$dx = \cos\theta\, d\theta$

$$= \int \operatorname{sen}^3\theta\, d\theta$$

$$= \int (1-\cos^2\theta)\operatorname{sen}\theta\, d\theta$$

$u = \cos\theta$
$du = -\operatorname{sen}\theta\, d\theta$

$$= -\int (1-u^2)\,du$$

$$= -u + \frac{u^3}{3} + C$$

$$= -\cos\theta + \frac{\cos^3\theta}{3} + C$$

$$= -\sqrt{1-x^2} + \frac{\left(\sqrt{1-x^2}\right)^3}{3} + C.$$

③ $\int \frac{\text{sen}^3 x}{\sqrt{\cos x}} dx = \int \frac{\text{sen}^2 x}{\sqrt{\cos x}} \cdot \text{sen}\, x\, dx$

$$= \int \frac{1-\cos^2 x}{\sqrt{\cos x}} \cdot \text{sen}\, x\, dx$$

$u = \cos x$

$du = -\text{sen}\, x\, dx$

$$= -\int \left(\frac{1}{\sqrt{u}} - \frac{u^2}{\sqrt{u}} \right) du$$

$$= -2u^{1/2} + \frac{u^{5/2}}{5/2} + C$$

$$= -2\sqrt{u} + \frac{2}{5}(\sqrt{u})^5 + C$$

$$= -2\sqrt{\cos x} + \frac{2}{5}(\sqrt{\cos x})^5 + C.$$

Questão 3 **(15 pontos)** Determine e esboce o maior domínio possível para a função

$$f(x,y) = \frac{x+y}{\sqrt{1-\operatorname{sen}(x^2+y^2)}}$$

DEVEMOS TER QUE

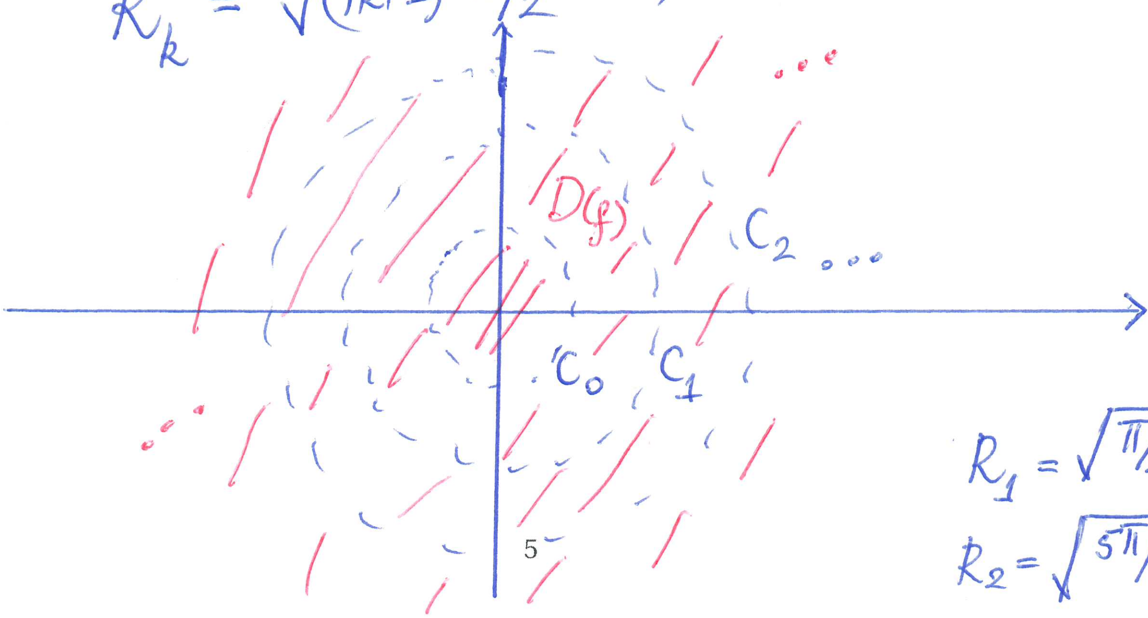
$$1 - \operatorname{sen}(x^2+y^2) > 0 \text{ SE } (x,y) \in D(f).$$

LOGO, COMO $x^2+y^2 \geq 0$, $(x,y) \in \mathbb{R}^2$, SEGUE QUE

$x^2+y^2 \neq \frac{\pi}{2}, \frac{\pi}{2}+2\pi, \frac{\pi}{2}+4\pi, \ldots$, OU SEJA,

$x^2+y^2 \neq (4k+1)\frac{\pi}{2}$, $k \in \mathbb{Z}, k \geq 0$. ASSIM, O DOMÍNIO DE f CONSISTE DO PLANO $\mathbb{R}^2$ SUBTRAÍDO DAS CIRCUNFERÊNCIAS C_k DE CENTRO NA ORIGEM E RAIO

$$R_k \doteq \sqrt{(4k+1)\pi/2}, \quad k \in \mathbb{Z}, k \geq 0.$$



$R_1 = \sqrt{\pi/2}$

$R_2 = \sqrt{5\pi/2}$

$$D(f) = \{(x,y) \in \mathbb{R}^2 \,/\, x^2+y^2 \neq (4k+1)\tfrac{\pi}{2},\ k \in \mathbb{Z}, k \geq 0\}.$$