

# *Supplementary material of: “Global convergence of a derivative-free inexact restoration filter algorithm for nonlinear programming”*

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## **Detailed results**

In the present manuscript we provide details on the results obtained in the original paper “Global convergence of a derivative-free inexact restoration filter algorithm for nonlinear programming”.

Table 1 shows the functional value and number of function evaluations for each compared solver: S-FIRD and F-FIRD (the proposed algorithms), IR (the inexact restoration algorithm with merit function) and DF0. Column Prob. is the number of the problem from Hock-Schittkowski collection,  $n$  is the number of variables,  $m_{\mathcal{I}}$  and  $m_{\mathcal{E}}$  are the number of inequality and equality constraints, respectively,  $f$  is the functional value and  $\#f$  is the number of function evaluations for each solver. Red entries indicate that the algorithm did not solve the problem, i.e., the point  $\bar{x}$  found by the algorithm did not satisfy

$$h(\bar{x}) \leq 10^{-8} \quad \text{or} \quad \frac{f_{\min} - f(\bar{x})}{\max\{1, f_{\min}, f(\bar{x})\}} \leq \tau, \quad (1)$$

with  $\tau = 10^{-4}$ , where  $h$  is an infeasibility measure and  $f_{\min}$  is the smallest function value found among all the algorithms under comparison. Entries with \* are associated with infeasible solutions and – means that the algorithm was unable to return any solution, due to an internal error or maximum CPU time reached.

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Table 1: Numerical experiments. Elements in red represent problems that were not considered solved, \* indicates that an infeasible solution was found and - indicates that some error occurred.

Prob.	$n$	$m_{\mathcal{I}}$	$m_{\mathcal{E}}$	S-FIRD		F-FIRD		IR		DFO	
				$f$	$\#f$	$f$	$\#f$	$f$	$\#f$	$f$	$\#f$
6	2	0	1	$9.9670e - 15$	52	$9.9670e - 15$	52	$7.8394e - 10$	370	$2.2009e - 23$	13
7	2	0	1	$-1.7321e + 00$	71	$-1.7321e + 00$	71	$-1.7321e + 00$	154	$-1.7321e + 00$	11
8	2	0	2	$-1.0000e + 00$	35	$-1.0000e + 00$	35	$-1.0000e + 00$	4	$-1.0000e + 00$	3
9	2	0	1	$-5.0000e - 01$	34	$-5.0000e - 01$	34	$-5.0000e - 01$	118	$-5.0000e - 01$	13
10	2	1	0	$-1.0000e + 00$	200	$-1.0000e + 00$	275	$-1.0000e + 00$	259	$-1.0000e + 00$	17
11	2	1	0	$-8.4985e + 00$	66	$-8.4985e + 00$	66	$-8.4985e + 00$	126	$-8.4985e + 00$	21
12	2	1	0	$-3.0000e + 01$	117	$-3.0000e + 01$	117	$-3.0000e + 01$	588	$-3.0000e + 01$	22
13	2	1	0	$9.9603e - 01$	117	$9.9603e - 01$	117	$1.0886e + 00$	46392	$9.9575e - 01$	7
14	2	1	1	$1.3935e + 00$	35	$1.3935e + 00$	35	$1.3935e + 00$	20	$1.3935e + 00$	9
15	2	2	0	$3.0650e + 00$	27	$3.0650e + 00$	27	$3.0650e + 00$	82	$3.6038e + 00$	19
16	2	2	0	$3.9821e + 00$	42	$3.9821e + 00$	42	$2.5000e - 01$	671	$2.5000e - 01$	38
17	2	2	0	$1.0000e + 00$	49	$1.0000e + 00$	49	$1.0000e + 00$	118	$1.0000e + 00$	16
18	2	2	0	$5.0000e + 00$	80	$5.0000e + 00$	80	$5.0000e + 00$	51355	$5.0000e + 00$	23
19	2	2	0	$-6.9618e + 03$	40	$-6.9618e + 03$	40	$-6.9618e + 03$	135	$-6.9618e + 03$	15
20	2	3	0	$3.8200e + 01$	32	$3.8200e + 01$	32	$4.0199e + 01$	88	$4.0199e + 01$	13
21	2	1	0	$-9.9960e + 01$	26	$-9.9960e + 01$	26	$-9.9960e + 01$	172	$-9.9960e + 01$	26
22	2	2	0	$1.0000e + 00$	30	$1.0000e + 00$	30	$1.0000e + 00$	35	$1.0000e + 00$	14
23	2	5	0	$2.0000e + 00$	35	$2.0000e + 00$	35	$2.0000e + 00$	37	$2.0000e + 00$	14
24	2	3	0	$-1.0000e + 00$	27	$-1.0000e + 00$	27	$-1.0000e + 00$	131	$-1.0000e + 00$	18
26	3	0	1	$1.0508e - 12$	171	$1.0508e - 12$	171	$1.7711e - 07$	10686	$2.5717e - 08$	50
27	3	0	1	$4.0000e + 00$	318	$4.0000e + 00$	290	$4.0003e + 00$	29184	$4.0000e + 00$	34
28	3	0	1	$3.0420e - 29$	52	$3.0420e - 29$	52	$1.5820e - 24$	524	$2.3064e - 18$	27
29	3	1	0	$-2.2627e + 01$	225	$-2.2627e + 01$	246	$-2.2627e + 01$	571	$-2.2627e + 01$	41
30	3	1	0	$1.0000e + 00$	87	$1.0000e + 00$	87	$1.0000e + 00$	788	$1.0000e + 00$	36
31	3	1	0	$6.0000e + 00$	82	$6.0000e + 00$	82	$6.0000e + 00$	536	$6.0000e + 00$	36
32	3	1	1	$1.0000e + 00$	55	$1.0000e + 00$	55	$1.0000e + 00$	84	$1.0000e + 00$	13
33	3	2	0	$-4.0000e + 00$	52	$-4.0000e + 00$	52	$-4.0000e + 00$	54	$-4.5858e + 00$	28
34	3	2	0	$-8.3402e - 01$	57	$-8.3402e - 01$	57	$-8.3403e - 01$	191	$-8.3403e - 01$	20

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Prob.	$n$	$m_{\mathcal{I}}$	$m_{\mathcal{E}}$	S-FIRD		F-FIRD		IR		DFO	
				$f$	$\#f$	$f$	$\#f$	$f$	$\#f$	$f$	$\#f$
35	3	1	0	$1.1111e-01$	107	$1.1111e-01$	107	$1.1111e-01$	475	$1.1111e-01$	32
36	3	1	0	$-3.3000e+03$	53	$-3.3000e+03$	53	$-3.3000e+03$	257	$-3.3000e+03$	29
37	3	2	0	$-3.4560e+03$	120	$-3.4560e+03$	120	$-3.4560e+03$	474	$-3.4560e+03$	75
39	4	0	2	$-1.0000e+00$	289	$-1.0000e+00$	307	$-1.0000e+00$	435	$-1.0000e+00$	25
40	4	0	3	$-2.5000e-01$	171	$-2.5000e-01$	147	$-2.5000e-01$	127	$-2.5000e-01$	16
41	4	0	1	$1.9259e+00$	117	$1.9259e+00$	117	$1.9259e+00$	456	$1.9259e+00$	37
42	4	0	2	$1.3858e+01$	298	$1.3858e+01$	507	$1.3858e+01$	536	$1.3858e+01$	15
43	4	3	0	$-4.4000e+01$	366	$-4.4000e+01$	366	$-4.4000e+01$	1506	$-4.4000e+01$	40
44	4	6	0	$-1.5000e+01$	75	$-1.5000e+01$	75	$-1.3000e+01$	264	$-1.5000e+01$	24
46	5	0	2	$1.0199e-07$	708	$1.6481e-08$	368	$5.0367e-07$	5880	$1.9017e-07$	83
47	5	0	3	$9.0668e-16$	127	$9.0668e-16$	127	$8.8559e-08$	422	$1.6635e-09$	52
48	5	0	2	$1.9946e-09$	111	$1.9946e-09$	111	$7.0719e-25$	847	$1.3348e-18$	32
49	5	0	2	$1.8761e-10$	288	$1.8761e-10$	288	$1.2096e-07$	20289	$3.4137e-06$	76
50	5	0	3	$1.9377e-17$	143	$1.9377e-17$	143	$1.2599e-27$	588	$1.3790e-07$	51
51	5	0	3	$2.2660e-17$	63	$2.2660e-17$	63	$2.0094e-27$	509	$2.7175e-18$	17
52	5	0	3	$5.3266e+00$	98	$5.3266e+00$	98	$5.3266e+00$	307	$5.3266e+00$	23
53	5	0	3	$4.0930e+00$	96	$4.0930e+00$	96	$4.0930e+00$	307	$4.0930e+00$	18
54	6	0	1	$-9.0327e-01$	11156	$-8.9591e-01^*$	12824*	$-1.5385e-01$	444	$-1.5399e-01$	23
55	6	0	6	$6.6667e+00$	115	$6.6667e+00$	115	$6.6667e+00$	18	$6.6667e+00$	9
56	7	0	4	$-3.4560e+00$	795	$-3.4560e+00$	736	$-8.3508e-03$	14487	$-3.4560e+00$	54
57	2	1	0	$2.8460e-02$	40	$2.8460e-02$	40	$3.0646e-02$	287	$2.8460e-02$	27
58	2	3	0	$3.1904e+00$	114	$3.1904e+00$	114	$3.1903e+00$	103	$3.1903e+00$	17
59	2	3	0	$-6.7546e+00$	63	$-6.7546e+00$	63	$-7.8042e+00$	754	$-6.7546e+00$	28
60	3	0	1	$3.2568e-02$	86	$3.2568e-02$	86	$3.2568e-02$	546	$3.2568e-02$	32
61	3	0	2	$-1.4365e+02$	75	$-1.4365e+02$	75	$-1.4365e+02$	180	$0.0000e+00^*$	1*
62	3	0	1	$-2.6273e+04$	72	$-2.6273e+04$	72	$-2.6273e+04$	811	$-2.6273e+04$	36
63	3	0	2	$9.6172e+02$	521	$9.6172e+02$	187	$9.6172e+02$	171	$9.6172e+02$	10
64	3	1	0	$6.2998e+03$	424	$6.2998e+03$	424	$6.2998e+03$	1438	$2.6604e+05^*$	1*
65	3	1	0	$9.5353e-01$	71	$9.5353e-01$	71	$9.5353e-01$	1463	$9.5353e-01$	39
66	3	2	0	$5.1816e-01$	218	$5.1816e-01$	337	$5.1816e-01$	382	$5.1816e-01$	26
67	3	14	0	$-1.1620e+03$	150	$-1.1620e+03$	150	$-1.1620e+03$	28674	$-1.1620e+03$	259
68	4	0	2	$-9.2043e-01$	257	$-9.2043e-01$	257	$-9.2042e-01$	5203	$-9.2042e-01$	117

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Prob.	$n$	$m_{\mathcal{I}}$	$m_{\mathcal{E}}$	S-FIRD		F-FIRD		IR		DFO	
				$f$	$\#f$	$f$	$\#f$	$f$	$\#f$	$f$	$\#f$
69	4	0	2	-9.5671e + 02	1861	-9.5671e + 02	1861	-9.5671e + 02	4125	-9.5671e + 02	192
70	4	1	0	8.2496e - 03	515	8.2496e - 03	515	2.6909e - 01	5571	7.5024e - 03	421
71	4	1	1	1.7014e + 01	569	1.7014e + 01	569	1.7014e + 01	4486	1.7014e + 01	39
72	4	2	0	7.2768e + 02	10344	7.2768e + 02	10344	7.2768e + 02	2724	5.0000e + 00*	1*
73	4	2	1	2.9894e + 01	58	2.9894e + 01	58	2.9894e + 01	211	2.9894e + 01	18
74	4	2	3	5.1265e + 03	215	5.1265e + 03	215	5.1265e + 03	392	0.0000e + 00*	1*
75	4	2	3	5.1744e + 03	134	5.1744e + 03	134	5.1744e + 03	139	0.0000e + 00*	1*
76	4	3	0	-4.6818e + 00	64	-4.6818e + 00	64	-4.6818e + 00	483	-4.6818e + 00	31
77	5	0	2	2.4151e - 01	134	2.4151e - 01	134	2.4151e - 01	691	2.4151e - 01	87
78	5	0	3	-2.9197e + 00	186	-2.9197e + 00	189	-2.9197e + 00	566	-2.9197e + 00	28
79	5	0	3	7.8777e - 02	97	7.8777e - 02	97	7.8777e - 02	310	7.8777e - 02	40
80	5	0	3	5.3950e - 02	159	5.3950e - 02	159	5.3950e - 02	644	5.3950e - 02	24
81	5	0	3	5.3950e - 02	141	5.3950e - 02	170	5.3950e - 02	774	5.3950e - 02	27
83	5	6	0	-3.0666e + 04	98	-3.0666e + 04	98	-3.0666e + 04	430	-3.0666e + 04	48
84	5	6	0	-5.2803e + 01	111	-5.2803e + 01	111	-5.2803e + 01	350	-5.2803e + 01	84
85	5	38	0	-1.9052e + 00	117	-1.9052e + 00	117	-	-	-1.9052e + 00	65
86	5	10	0	-3.2349e + 01	95	-3.2349e + 01	95	-3.2349e + 01	457	-3.2349e + 01	36
87	6	0	4	8.9276e + 03	580	8.9276e + 03	580	8.8535e + 03	860	4.2090e + 04*	1*
88	2	1	0	1.3627e + 00	255	1.3627e + 00	255	1.3627e + 00	267	1.3627e + 00	14
89	3	1	0	1.3627e + 00	384	1.3627e + 00	384	1.3627e + 00	623	1.3627e + 00	22
90	4	1	0	1.3627e + 00	439	1.3627e + 00	439	1.3687e + 00	825	1.3627e + 00	66
91	5	1	0	1.3627e + 00	489	1.3627e + 00	489	1.3710e + 00	1743	1.3627e + 00	71
92	6	1	0	1.3628e + 00	363	1.3628e + 00	363	1.3659e + 00	1271	1.3627e + 00	110
93	6	2	0	4.6966e + 00*	25*	4.6966e + 00*	25*	1.3508e + 02	98455	1.3508e + 02	95
95	6	4	0	1.5620e - 02	92	1.5620e - 02	92	1.5620e - 02	723	1.5620e - 02	30
96	6	4	0	1.5620e - 02	92	1.5620e - 02	92	1.5620e - 02	650	1.5620e - 02	30
97	6	4	0	2.1247e + 00*	31*	2.1247e + 00*	31*	4.0712e + 00	3763	3.1358e + 00	26
98	6	4	0	1.9334e + 00*	31*	1.9334e + 00*	31*	3.1358e + 00	1499	3.1358e + 00	25
99	7	0	2	-8.3108e + 08	157	-8.3108e + 08	157	-8.3087e + 08	4	-8.3108e + 08	98
100	7	4	0	6.8063e + 02	762	6.8063e + 02	1261	7.0920e + 02	524282	6.8063e + 02	229
101	7	6	0	1.8635e + 03	2804	1.8635e + 03	2804	1.8098e + 03	1510	1.8098e + 03	860
102	7	6	0	9.1883e + 02	3319	9.1883e + 02	3319	1.5144e + 03	17252	9.1188e + 02	396

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Prob.	$n$	$m_{\mathcal{I}}$	$m_{\mathcal{E}}$	S-FIRD		F-FIRD		IR		DFO	
				$f$	$\#f$	$f$	$\#f$	$f$	$\#f$	$f$	$\#f$
103	7	6	0	$5.4367e+02$	727	$5.4367e+02$	727	-	-	$5.4367e+02$	322
104	8	6	0	$3.9512e+00$	572	$3.9512e+00$	578	$3.9512e+00$	8301	$3.9512e+00$	168
105	8	1	0	$1.1384e+03$	8009	$1.1384e+03$	8009	$1.1384e+03$	26279	$1.1390e+03$	8000
106	8	6	0	$7.0492e+03$	5855	$7.0492e+03$	8203	$7.0492e+03$	63675	$7.0492e+03$	91
107	9	0	6	$5.0550e+03$	151	$5.0550e+03$	151	$5.0550e+03$	265	$5.0550e+03$	25
108	9	13	0	$-8.6603e-01$	758	$-8.6603e-01$	758	-	-	$-8.6603e-01$	73
109	9	4	6	$5.3621e+03$	488	$5.3621e+03$	488	$5.6072e+03$	61526	$0.0000e+00^*$	1*
111	10	0	3	$-1.6320e+17^*$	204*	$-1.6320e+17^*$	204*	$-1.5978e+35^*$	2098*	$-4.7720e+01$	427
112	10	0	3	$-4.7739e+01$	342	$-4.7739e+01$	342	$-4.7761e+01$	4764	$-4.7761e+01$	121
113	10	8	0	$2.4306e+01$	169	$2.4306e+01$	169	$2.4306e+01$	3403	$2.4306e+01$	355
114	10	8	3	$-1.7688e+03$	1774	$-1.7688e+03$	1774	$-1.7656e+03$	154924	$-1.7688e+03$	261
116	13	15	0	$9.3491e+01^*$	458*	$9.3491e+01^*$	458*	$9.7588e+01$	36564	$4.5000e+02^*$	1*
117	15	5	0	$3.2349e+01$	537	$3.2349e+01$	537	$3.2349e+01$	10605	$3.2349e+01$	2370
118	15	29	0	$6.6482e+02$	198	$6.6482e+02$	198	$6.6482e+02$	2010	$6.6482e+02$	191
119	16	0	8	$2.4490e+02$	563	$2.4490e+02$	563	$2.4490e+02$	1201	$2.4490e+02$	111
215	2	1	0	$0.0000e+00$	26	$0.0000e+00$	26	$-5.3831e-09$	99	$0.0000e+00$	15
216	2	0	1	$9.9938e-01$	47	$9.9938e-01$	47	$9.9938e-01$	160	$9.9938e-01$	12
217	2	1	1	$-8.0000e-01$	47	$-8.0000e-01$	47	$-8.0000e-01$	32	$-8.0000e-01$	9
218	2	1	0	$0.0000e+00$	98	$0.0000e+00$	105	$4.9784e-03$	356	$0.0000e+00$	28
219	4	0	2	$-1.0000e+00$	373	$-1.0000e+00$	373	$-1.0000e+00$	415	$-1.0000e+00$	25
220	2	1	0	$1.0000e+00$	83	$1.0000e+00$	83	$3.0248e+01$	171	$1.0000e+00$	245
221	2	1	0	$-1.0000e+00$	37	$-1.0000e+00$	37	$-9.5768e-01$	52404	$-1.0000e+00$	9
222	2	1	0	$-1.5000e+00$	39	$-1.5000e+00$	39	$-1.5000e+00$	161	$-1.5000e+00$	12
223	2	2	0	$-8.3403e-01$	43	$-8.3402e-01$	33	$-8.3403e-01$	144	$-8.3403e-01$	15
224	2	4	0	$-3.0400e+02$	28	$-3.0400e+02$	28	$-3.0400e+02$	222	$-3.0400e+02$	26
225	2	5	0	$2.0000e+00$	31	$2.0000e+00$	31	$2.0000e+00$	37	$2.0000e+00$	14
226	2	2	0	$-5.0000e-01$	70	$-5.0000e-01$	70	$-5.0000e-01$	153	$-5.0000e-01$	13
227	2	2	0	$1.0000e+00$	43	$1.0000e+00$	43	$1.0000e+00$	58	$1.0000e+00$	13
228	2	2	0	$-3.0000e+00$	27	$-3.0000e+00$	27	$-3.0000e+00$	401	$-3.0000e+00$	26
230	2	2	0	$3.7500e-01$	35	$3.7500e-01$	35	$3.7500e-01$	80	$3.7500e-01$	16
231	2	2	0	$3.4450e-04$	8330	$3.4450e-04$	8330	$2.7605e-06$	24277	$1.1487e-07$	78
232	2	3	0	$-1.0000e+00$	27	$-1.0000e+00$	27	$-1.0000e+00$	126	$-1.0000e+00$	15

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Prob.	$n$	$m_{\mathcal{I}}$	$m_{\mathcal{E}}$	S-FIRD		F-FIRD		IR		DFO	
				$f$	$\#f$	$f$	$\#f$	$f$	$\#f$	$f$	$\#f$
233	2	1	0	$3.6789e-04$	972	$3.6789e-04$	972	$3.8501e-05$	45591	$8.4346e-07$	67
234	2	1	0	$-8.0000e-01$	90	$-8.0000e-01$	90	$-8.0000e-01$	3325	$-8.0000e-01$	19
235	3	0	1	$4.0000e-02$	232	$4.0000e-02$	218	$4.0000e-02$	9874	$4.0000e-02$	30
236	2	2	0	$-5.8903e+01$	64	$-5.8903e+01$	64	$-8.1975e+00$	333	$-8.1975e+00$	31
237	2	3	0	$-5.8903e+01$	48	$-5.8903e+01$	48	$-5.8900e+01$	18569	$-5.8903e+01$	27
238	2	3	0	$-5.8903e+01$	57	$-5.8903e+01$	57	$-8.1975e+00$	629	$-8.1975e+00$	31
239	2	1	0	$-5.8903e+01$	64	$-5.8903e+01$	64	$-8.1975e+00$	349	$-8.1975e+00$	31
248	3	1	1	$-8.0000e-01$	183	$-8.0000e-01$	242	$-8.0000e-01$	321	$-8.0000e-01$	23
249	3	1	0	$1.0000e+00$	87	$1.0000e+00$	87	$1.0096e+00$	254	$1.0000e+00$	36
250	3	2	0	$-3.3000e+03$	53	$-3.3000e+03$	53	$-3.3000e+03$	338	$-3.3000e+03$	29
251	3	1	0	$-3.4560e+03$	120	$-3.4560e+03$	120	$-3.4560e+03$	446	$-3.4560e+03$	74
252	3	0	1	$4.0000e-02$	159	$4.0000e-02$	159	$4.0664e-02$	16083	$4.0000e-02$	135
253	3	1	0	$6.9282e+01$	63	$6.9282e+01$	63	$6.9282e+01$	341	$6.9282e+01$	35
254	3	0	2	$-1.7321e+00$	99	$-1.7321e+00$	99	$-1.7282e+00$	64	$-1.7321e+00$	13
262	4	3	1	$-1.0000e+01$	90	$-1.0000e+01$	90	$-1.0000e+01$	135	$-1.0000e+01$	22
263	4	2	2	$-1.0000e+00$	168	$-1.0000e+00$	168	$-9.7943e-01$	60	$-1.0000e+00$	25
264	4	3	0	$-4.3999e+01$	222	$-4.4000e+01$	180	$-4.1914e+01^*$	2086383*	$-4.4000e+01$	41
265	4	0	2	$1.9036e+00$	83	$1.9036e+00$	83	$1.9036e+00$	160	$9.7475e-01$	13
268	5	5	0	$1.3560e+00$	528	$1.3560e+00$	528	$1.9889e-01$	332781	$2.3590e-12$	138
269	5	0	3	$4.0930e+00$	96	$4.0930e+00$	96	$4.0930e+00$	307	$4.0930e+00$	18
270	5	1	0	$1.1466e-16$	83	$1.1466e-16$	83	$2.6695e-11$	453	$0.0000e+00$	81
277	4	4	0	$5.0762e+00$	105	$5.0762e+00$	105	$5.0762e+00$	503	$5.0762e+00$	56
278	6	6	0	$7.8385e+00$	267	$7.8385e+00$	247	$7.8385e+00$	75539	$7.8385e+00$	121
279	8	8	0	$1.0606e+01$	329	$1.0606e+01$	329	-	-	$1.0606e+01$	147
280	10	10	0	$1.3375e+01$	335	$1.3375e+01$	335	$1.3375e+01$	142946	$1.3375e+01$	275
284	15	10	0	$-1.8400e+03$	495	$-1.8400e+03$	3297	-	-	$-1.8400e+03$	434
285	15	10	0	$-8.2520e+03$	295	$-8.2520e+03$	4502	-	-	$-8.2520e+03$	278
315	2	3	0	$-8.0000e-01$	41	$-8.0000e-01$	41	$-7.9853e-01$	159	$-8.0000e-01$	13
316	2	0	1	$3.3431e+02$	36	$3.3431e+02$	36	$3.3431e+02$	156	$8.0000e+02^*$	1*
317	2	0	1	$3.7247e+02$	149	$3.7247e+02$	149	$3.7247e+02$	151	$8.0000e+02^*$	1*
318	2	0	1	$4.1275e+02$	118	$4.1275e+02$	118	$4.1275e+02$	151	$8.0000e+02^*$	1*
319	2	0	1	$4.5240e+02$	327	$4.5240e+02$	327	$4.5240e+02$	201	$8.0000e+02^*$	1*

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Prob.	$n$	$m_{\mathcal{I}}$	$m_{\mathcal{E}}$	S-FIRD		F-FIRD		IR		DFO	
				$f$	$\#f$	$f$	$\#f$	$f$	$\#f$	$f$	$\#f$
320	2	0	1	4.8553e + 02	347	4.8553e + 02	347	4.8553e + 02	128	8.0000e + 02*	1*
321	2	0	1	4.9611e + 02	217	4.9611e + 02	217	4.9611e + 02	240	8.0000e + 02*	1*
322	2	0	1	4.9996e + 02	330	4.9996e + 02	351	4.9996e + 02	219	4.9996e + 02	18
323	2	2	0	3.7989e + 00	272	3.7989e + 00	272	3.7989e + 00	184	3.7989e + 00	14
324	2	2	0	5.0000e + 00	80	5.0000e + 00	80	5.0000e + 00	51244	5.0000e + 00	23
325	2	2	1	3.7915e + 00	32	3.7915e + 00	32	7.4641e + 00	42	9.0000e + 00	2
326	2	2	0	-7.9808e + 01	36	-7.9808e + 01	36	-7.9808e + 01	101	-7.9808e + 01	13
327	2	1	0	2.8460e - 02	40	2.8460e - 02	40	3.0646e - 02	287	2.8460e - 02	26
329	2	3	0	-6.9618e + 03	45	-6.9618e + 03	45	-6.9618e + 03	120	-6.9618e + 03	14
330	2	1	0	1.6206e + 00	65	1.6206e + 00	65	1.6206e + 00	245	-	-
331	2	1	0	4.2584e + 00	49	4.2584e + 00	49	4.2584e + 00	283	4.2584e + 00	24
335	3	0	2	-4.4721e - 03	346	-4.4721e - 03	335	-2.7812e - 01*	2118771*	-4.4721e - 03	19
336	3	0	2	-3.3790e - 01	431	-3.3790e - 01	526	-3.3790e - 01	253	-3.3790e - 01	17
337	3	1	0	6.0000e + 00	82	6.0000e + 00	82	6.0000e + 00	519	6.0000e + 00	36
338	3	0	2	-5.2262e + 08*	184*	-5.2262e + 08*	184*	-1.0993e + 01	6140	-7.2057e + 00	15
339	3	1	0	3.3617e + 00	111	3.3617e + 00	111	3.3617e + 00	1150	-	-
340	3	1	0	-5.4000e - 02	92	-5.4000e - 02	92	-5.4000e - 02	389	-5.4000e - 02	37
341	3	1	0	-2.2627e + 01	175	-2.2627e + 01	175	-2.2627e + 01	658	-2.2627e + 01	35
342	3	1	0	-2.2627e + 01	256	-2.2627e + 01	256	-2.2627e + 01	658	-2.2627e + 01	35
343	3	2	0	-5.6848e + 00	892	-5.6848e + 00	892	-5.6848e + 00	157	-5.6848e + 00	40
344	3	0	1	3.2568e - 02	86	3.2568e - 02	86	3.2568e - 02	635	3.2568e - 02	32
345	3	0	1	3.2568e - 02	53	3.2568e - 02	53	3.2595e - 02	103325	3.2568e - 02	39
346	3	2	0	-5.6848e + 00	892	-5.6848e + 00	892	-5.6848e + 00	157	-5.6848e + 00	40
347	3	1	0	1.7375e + 04	71	1.7375e + 04	71	1.7375e + 04	287	1.7375e + 04	18
353	4	2	1	-3.9934e + 01	62	-3.9934e + 01	62	-3.9934e + 01	500	-3.9934e + 01	13
354	4	1	0	1.1378e - 01	176	1.1378e - 01	176	1.2756e - 01	415	1.1378e - 01	101
355	4	0	1	6.9675e + 01	489	6.9675e + 01	489	6.9675e + 01	36407	6.9675e + 01	51
359	5	14	0	-5.2803e + 06	93	-5.2803e + 06	93	-3.2333e + 06	4	-5.2803e + 06	133
360	5	2	0	-5.2803e - 01	111	-5.2803e - 01	111	-2.5414e - 01	512	-5.2803e - 01	82
361	5	6	0	-7.7641e + 05	107	-7.7641e + 05	107	-7.7641e + 05	6	-7.7641e + 05	66
367	7	3	2	-3.3971e + 01	312	-3.3971e + 01	312	-3.7413e + 01	1161	-3.7413e + 01	70
372	9	12	0	1.9511e + 04	650	1.9511e + 04	650	1.1001e + 05	1964	1.0453e + 05	382

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Prob.	$n$	$m_{\mathcal{I}}$	$m_{\mathcal{E}}$	S-FIRD		F-FIRD		IR		DFO	
				$f$	$\#f$	$f$	$\#f$	$f$	$\#f$	$f$	$\#f$
373	9	0	6	$1.3390e+04$	332	$1.3390e+04$	332	$1.3390e+04$	3200	$7.5289e+05^*$	$1^*$
374	10	35	0	$2.3326e-01$	1469	$2.3326e-01$	1921	$3.0438e-01$	262	$2.3326e-01$	144
375	10	0	9	$-6.4477e+08^*$	$256^*$	$-6.4477e+08^*$	$256^*$	$-1.5624e+01$	20161	$-1.5161e+01$	17
376	10	14	1	$-4.4301e+03$	1039	$-4.4301e+03$	1039	-	-	$-4.4301e+03$	192
377	10	0	3	$-7.9499e+02$	193	$-7.9499e+02$	193	$-7.9499e+02$	677	$-7.9499e+02$	33
378	10	0	3	$-4.7760e+01$	26072	$-4.7739e+01$	2548	$-4.7752e+01$	1307311	$-4.7760e+01$	360
380	12	3	0	$3.1687e+05$	1904	$3.1687e+05$	1904	-	-	$3.1683e+05$	10462
381	13	3	1	$1.0149e+00$	204	$1.0149e+00$	204	$1.0149e+00$	2629	$1.0149e+00$	70
382	13	3	1	$1.0383e+00$	420	$1.0383e+00$	399	$1.0383e+00$	6273	-	-
383	14	0	1	$1.9747e+01^*$	$122^*$	$1.9747e+01^*$	$122^*$	-	-	$7.2870e+00$	242
384	15	10	0	$-8.3103e+03$	4864	$-8.3103e+03$	6375	-	-	$-8.3103e+03$	303
385	15	10	0	$-8.3153e+03$	2226	$-8.3153e+03$	4538	-	-	$-8.3153e+03$	488
386	15	11	0	$-8.1644e+03$	4508	$-8.1644e+03$	8107	$-8.1416e+03$	1222	$-8.1644e+03$	389
387	15	11	0	$-8.2501e+03$	5177	$-8.2501e+03$	100170	$-8.2491e+03$	3064	$-8.2501e+03$	405
388	15	15	0	$-5.8211e+03$	5315	$-5.8211e+03$	26499	$-5.0809e+03$	86477	$-5.8211e+03$	508
389	15	15	0	$-5.8097e+03$	2948	$-5.8097e+03$	4437	$-5.4083e+03$	78056	$-5.8097e+03$	546
394	20	0	1	$1.9167e+00$	1083	$1.9167e+00$	1083	$1.9167e+00$	37025	$1.9167e+00$	1296
395	50	0	1	$1.9167e+00$	2460	$1.9167e+00$	2460	$1.9167e+00$	132811	-	-



## Performance and data profiles

Another tests have been performed by modifying the value of  $\tau$  in the second condition of (1). Figures 1-4 show data and performance profiles for the 4 compared algorithms in the set of 206 test problems from the Hock-Schittkowski collection, for  $\tau \in \{10^{-1}, 10^{-2}, 10^{-4}, 10^{-8}\}$ , respectively.

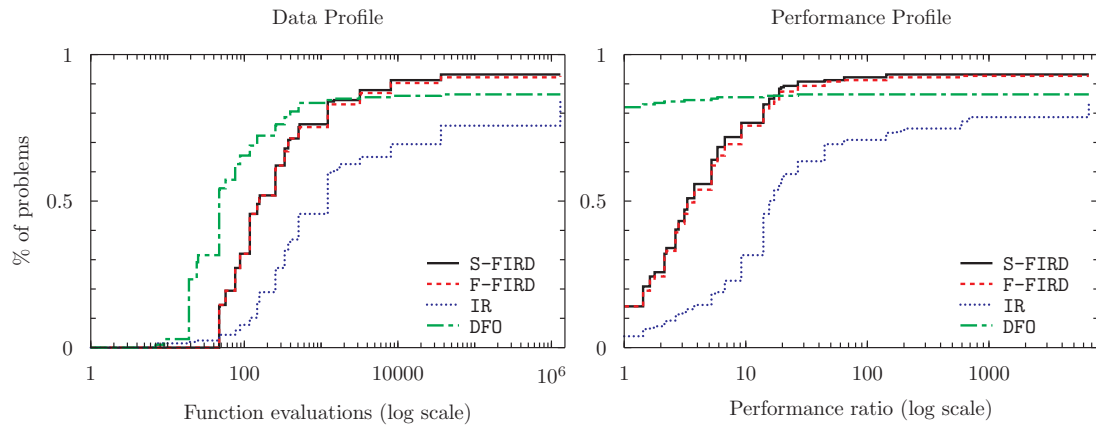


Figure 1: Data and performance profiles obtained when  $\tau = 10^{-1}$  is selected in (1).

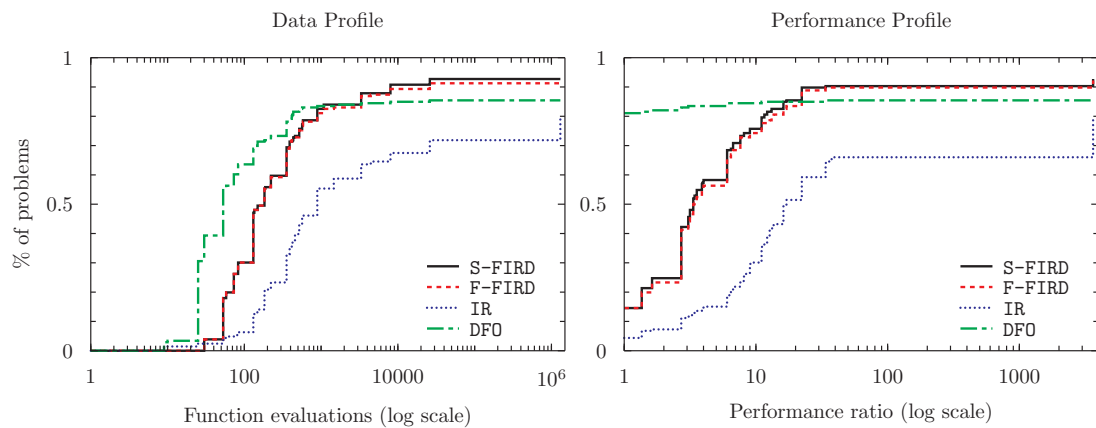


Figure 2: Data and performance profiles obtained when  $\tau = 10^{-2}$  is selected in (1).

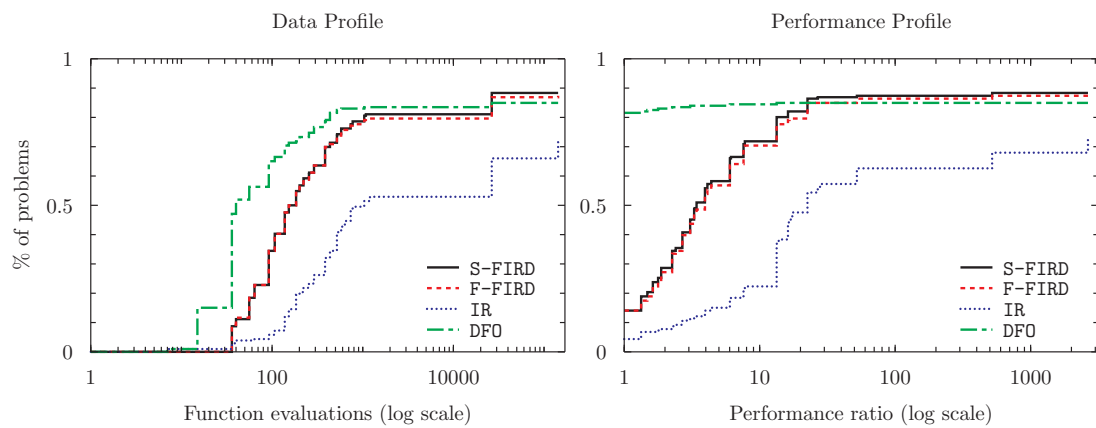


Figure 3: Data and performance profiles obtained when  $\tau = 10^{-4}$  is selected in (1). This case was considered for constructing Table 1.

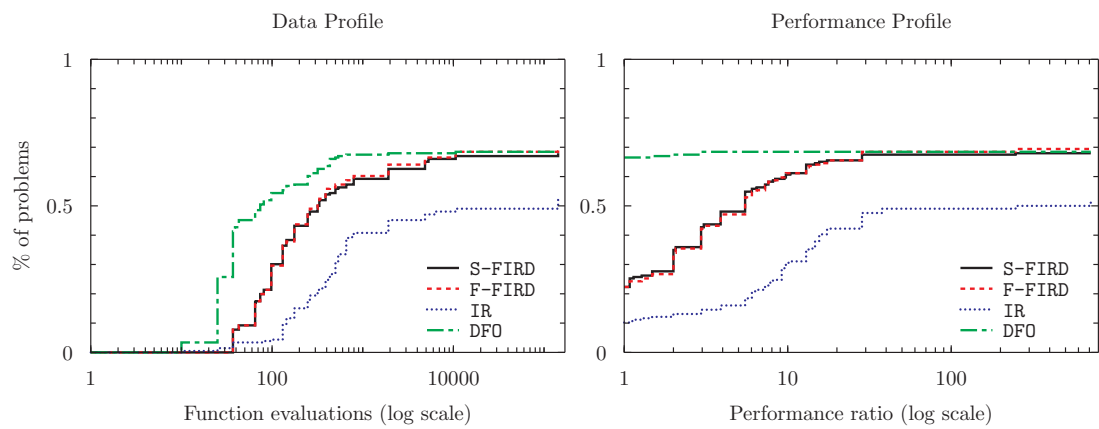


Figure 4: Data and performance profiles obtained when  $\tau = 10^{-8}$  is selected in (1).