Supplementary numerical experiments for the article: A trust-region derivative-free algorithm for constrained optimization

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In this material, we present supplementary numerical experiments for [3], where it is considered the problem

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \Omega \end{array}$$

with $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ and $\Omega = \{x \in \mathbb{R}^n \mid h(x) = 0, g(x) \leq 0\}$ where $h : \mathbb{R}^n \to \mathbb{R}^p$ and $g : \mathbb{R}^n \to \mathbb{R}^q$ are differentiable functions. In the original paper, the experiments for the `DFO` algorithm consist in the results extracted from [4] for 28 constrained problems from the Hock-Schittkowski collection [5] . We now present the results for all 219 problems from the Hock-Schittkowski collection [6] that involve at least one constraint beside box constraints. The tests were performed using the `GFortran` or `GCC` version `4.8.2` compiler (32-bits), in Ubuntu operating system with AMD Athlon II X2 270, 2GB of RAM.

In view of this, we are able to present the performance comparison in solving all of these 219 problems by the following algorithms:

- $\mathtt{S}_1$: [3, Alg.1] with $m = 5$ if $n = 2$ and $m = 2n + 3$ otherwise;
- $\mathtt{S}_2$: [3, Alg.1] with $m = \overline{m} = \dfrac{1}{2}(n + 1)(n + 2)$;
- `IR`: Inexact Restoration algorithm, a `C++` code based on [2];
- `DFO`: Derivative Free Optimization Algorithm, a `Fortran` code based on the trust-region approach proposed in [4].

All the algorithms were able to use the derivatives of the constraints for solving subproblems ($\mathtt{S}_1$, $\mathtt{S}_2$ and `DFO`) or for the restoration and construction of the tangent space (`IR`). Among the 219 problems, the authors detected 12 problems for which the derivatives were not available: 332, 348, 356, 362, 363, 364, 365, 366, 369, 390, 392

and 393. In such cases, only $\mathtt{S}_1$ and $\mathtt{S}_2$ were able to easily incorporate the information, which allowed the use of finite differences in the subproblems. All the algorithms used the nonlinear solver ALGENCAN [1] at version 3.0.0 to solve the trust-region or restoration subproblems.

In our analysis, a point $\overline{x} \in \mathbb{R}^n$ is considered feasible if

$$\max\left\{\|h(\overline{x})\|_\infty, \|g^+(\overline{x})\|_\infty\right\} \leq 10^{-8}, \tag{1}$$

where $g_i^+(x) = \max\{0, g_i(x)\}$, for all $i = 1, \cdots, q$. We say that an algorithm solved a problem if it found a feasible point $\overline{x}$ such that

$$\frac{f(\overline{x}) - f_{\min}}{\max\{1, |f(\overline{x})|, |f_{\min}|\}} \leq 10^{-4}, \tag{2}$$

where $f_{\min}$ is the smallest function value found among all the algorithms that are being compared.

Figure 1 shows the performance and data profiles related to the number of objective function evaluations. The $\mathtt{S}_2$ algorithm was the most robust, while DFO was the most efficient solver. Algorithms $\mathtt{S}_1$, $\mathtt{S}_2$, IR and DFO solved 86.8%, 87.2%, 69.4% and 79.9% of the problems. On the other hand, $\mathtt{S}_1, \mathtt{S}_2$, IR and DFO performed fewer function evaluations on 21.5%, 4.6%, 5.5% and 67.6% of the problems, respectively.

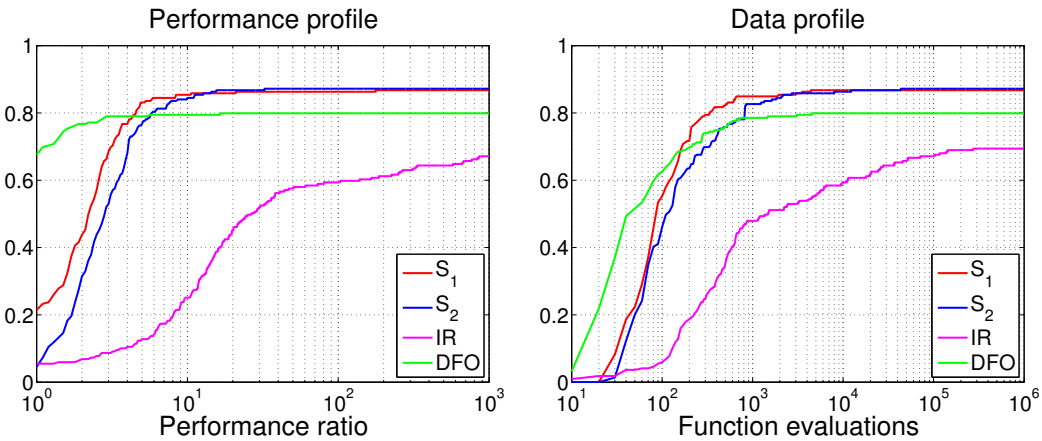

Figure 1: Performance and data profiles.

We present in Tables 1–3 some more details on the numerical results. The first columns contain information about the problems. The label prob indicates the number of the problem in the Hock–Schittkowski collection, and n, p, q the number of variables, of equality constraints and of inequality constraints besides bound constraints. The next columns display the minimum function value f^* and the number of function evaluations $\#f$ for each solver.

When an algorithm failed in solving a problem, the following labels were used to indicate the reason:

(1) the problem was not solved because the solution is not feasible according to (1).

(2) the problem was not solved according to (2) considering solvers $\mathtt{S}_1$, $\mathtt{S}_2$, IR and DFO.

		S_1		S_2		IR		DFO	
prob	n, p, q	f^*	$\#f$	f^*	$\#f$	f^*	$\#f$	f^*	$\#f$
6	2, 1, 0	$3.9100E-23$	35	$3.9747E-23$	41	$1.2342E-09$	362	$2.6516E-21$	13
7	2, 1, 0	$-1.7321E+00$	29	$-1.7321E+00$	34	$-1.7321E+00$	150	$-1.7321E+00$	11
8	2, 2, 0	$-1.0000E+00$	25	$-1.0000E+00$	30	$-1.0000E+00$	4	$-1.0000E+00$	3
9	2, 1, 0	$-5.0000E-01$	39	$-5.0000E-01$	50	$-5.0000E-01$	118	$-5.0000E-01$	13
10	2, 0, 1	$-1.0000E+00$	26	$-1.0000E+00$	31	$-1.0000E+00$	260	$-1.0000E+00$	17
11	2, 0, 1	$-8.4985E+00$	26	$-8.4985E+00$	31	$-8.4985E+00$	126	$-8.4985E+00$	21
12	2, 0, 1	$-3.0000E+01$	41	$-3.0000E+01$	41	$-3.0000E+01$	586	$-3.0000E+01$	22
13	2, 0, 1	$9.9575E-01$	34	$9.9575E-01$	40	$1.0347E+00$	73465 (2)	$9.9575E-01$	7
14	2, 1, 1	$1.3935E+00$	25	$1.3935E+00$	30	$1.3935E+00$	20	$1.3935E+00$	9
15	2, 0, 2	$3.0650E+00$	26	$3.0650E+00$	31	$3.0650E+00$	82	$3.6038E+00$	19 (2)
16	2, 0, 2	$3.9821E+00$	33 (2)	$3.9821E+00$	32 (2)	$2.5000E-01$	670	$2.5000E-01$	38
17	2, 0, 2	$1.0000E+00$	46	$1.0000E+00$	49	$1.0000E+00$	118	$1.0000E+00$	16
18	2, 0, 2	$5.0000E+00$	38	$5.0000E+00$	44	$5.0000E+00$	51810	$5.0000E+00$	23
19	2, 0, 2	$-6.9618E+03$	36	$-6.9618E+03$	42	$-6.9618E+03$	135	$-6.9618E+03$	15
20	2, 0, 3	$3.8199E+01$	28	$3.8199E+01$	33	$4.0199E+01$	88 (2)	$4.0199E+01$	13 (2)
21	2, 0, 1	$-9.9960E+01$	34	$-9.9960E+01$	40	$-9.9960E+01$	161	$-9.9960E+01$	26
22	2, 0, 2	$1.0000E+00$	25	$1.0000E+00$	30	$1.0000E+00$	35	$1.0000E+00$	14
23	2, 0, 5	$2.0000E+00$	34	$2.0000E+00$	40	$2.0000E+00$	37	$2.0000E+00$	14
24	2, 0, 3	$-1.0000E+00$	36	$-1.0000E+00$	41	$-1.0000E+00$	131	$-1.0000E+00$	18
26	3, 1, 0	$2.0002E-12$	155	$1.9029E-12$	170	$1.5843E-07$	11250	$2.5370E-08$	50
27	3, 1, 0	$4.0000E+00$	94	$4.0000E+00$	107	$4.0000E+00$	19575	$4.0000E+00$	34
28	3, 1, 0	$2.3120E-16$	60	$2.7008E-16$	66	$1.5873E-24$	530	$1.2473E-18$	27
29	3, 0, 1	$-2.2627E+01$	89	$-2.2627E+01$	69	$-2.2627E+01$	571	$-2.2627E+01$	37
30	3, 0, 1	$1.0000E+00$	58	$1.0000E+00$	64	$1.0000E+00$	788	$1.0000E+00$	36
31	3, 0, 1	$6.0000E+00$	58	$6.0000E+00$	64	$6.0000E+00$	530	$6.0000E+00$	36
32	3, 1, 1	$1.0000E+00$	62	$1.0000E+00$	64	$1.0000E+00$	84	$1.0000E+00$	13
33	3, 0, 2	$-4.0000E+00$	58 (2)	$-4.0000E+00$	64 (2)	$-4.0000E+00$	54 (2)	$-4.5858E+00$	28
34	3, 0, 2	$-8.3403E-01$	61	$-8.3403E-01$	67	$-8.3403E-01$	191	$-8.3403E-01$	20
35	3, 0, 1	$1.1111E-01$	49	$1.1111E-01$	54	$1.1111E-01$	391	$1.1111E-01$	32
36	3, 0, 1	$-3.3000E+03$	61	$-3.3000E+03$	67	$-3.3000E+03$	257	$-3.3000E+03$	29
37	3, 0, 2	$-3.4560E+03$	167	$-3.4560E+03$	92	$-3.4560E+03$	486	$-3.4560E+03$	76
39	4, 2, 0	$-1.0000E+00$	57	$-1.0000E+00$	77	$-1.0000E+00$	504	$-1.0000E+00$	25
40	4, 3, 0	$-2.5000E-01$	58	$-2.5000E-01$	76	$-2.5000E-01$	127	$-2.5000E-01$	16
41	4, 1, 0	$1.9259E+00$	169	$1.9259E+00$	80	$1.9259E+00$	456	$1.9259E+00$	37
42	4, 2, 0	$1.3858E+01$	70	$1.3858E+01$	94	$1.3858E+01$	536	$1.3858E+01$	15
43	4, 0, 3	$-4.4000E+01$	71	$-4.4000E+01$	95	$-4.4000E+01$	1505	$-4.4000E+01$	40
44	4, 0, 6	$-1.5000E+01$	83	$-1.5000E+01$	96	$-1.3000E+01$	264 (2)	$-1.5000E+01$	26
46	5, 2, 0	$2.9277E-12$	231	$3.4118E-12$	354	$5.0366E-07$	5648	$1.7164E-07$	70
47	5, 3, 0	$6.8919E-17$	121	$5.9769E-17$	181	$8.8716E-08$	417	$1.6821E-09$	52
48	5, 2, 0	$6.2924E-09$	111	$1.0928E-17$	132	$7.1504E-25$	853	$4.5258E-18$	34
49	5, 2, 0	$1.0318E-11$	252	$1.4219E-11$	386	$1.2096E-07$	20289	$2.8868E-06$	76
50	5, 3, 0	$1.3418E-19$	153	$8.4683E-18$	234	$4.8357E-28$	611	$1.3791E-07$	51
51	5, 3, 0	$7.6403E-19$	83	$5.3697E-18$	131	$1.8733E-27$	510	$3.1770E-18$	17
52	5, 3, 0	$5.3266E+00$	83	$5.3266E+00$	131	$5.3266E+00$	307	$5.3266E+00$	23
53	5, 3, 0	$4.0930E+00$	82	$4.0930E+00$	130	$4.0930E+00$	306	$4.0930E+00$	18
54	6, 1, 0	$-7.9950E-01$	248	$-7.6033E-01$	524 (2)	$-1.5385E-01$	444 (2)	$-1.5399E-01$	23 (2)
55	6, 6, 0	$6.6667E+00$	75	$6.6667E+00$	140	$6.6667E+00$	18	$6.6667E+00$	9
56	7, 4, 0	$-3.4560E+00$	336	$-3.4560E+00$	223	$-1.8726E-03$	14579 (2)	$-3.4560E+00$	58
57	2, 0, 1	$2.8460E-02$	64	$2.8460E-02$	66	$3.0646E-02$	287 (2)	$2.8460E-02$	27
58	2, 0, 2	$3.1903E+00$	3004	$3.1903E+00$	64	$3.1903E+00$	103	$3.1903E+00$	17
59	2, 0, 3	$-6.7546E+00$	53 (2)	$-6.7546E+00$	57 (2)	$-7.8042E+00$	711	$-6.7546E+00$	28 (2)
60	3, 1, 0	$3.2568E-02$	63	$3.2568E-02$	69	$3.2568E-02$	629	$3.2568E-02$	32
61	3, 2, 0	$-5.8760E+01$	45 (1)	$-5.8760E+01$	50 (1)	$-1.4365E+02$	180	$0.0000E+00$	1 (1)
62	3, 1, 0	$-2.6273E+04$	63	$-2.6273E+04$	70	$-2.6273E+04$	815	$-2.6273E+04$	38
63	3, 2, 0	$9.6172E+02$	59	$9.6172E+02$	53	$9.6172E+02$	171	$9.6172E+02$	10
64	3, 0, 1	$6.2998E+03$	124	$6.2998E+03$	134	$6.2998E+03$	1356	$2.6604E+05$	1 (1)
65	3, 0, 1	$9.5353E-01$	61	$9.5353E-01$	67	$9.5353E-01$	1489	$9.5353E-01$	39
66	3, 0, 2	$5.1816E-01$	48	$5.1816E-01$	53	$5.1816E-01$	382	$5.1816E-01$	24
67	3, 0, 14	$-1.1620E+03$	140	$-1.1620E+03$	102	$-1.1620E+03$	26134	$-1.1620E+03$	290
68	4, 2, 0	$-9.2043E-01$	361	$2.4000E-05$	240 (2)	$-9.2042E-01$	5203	$-9.2042E-01$	103
69	4, 2, 0	$-9.5671E+02$	505	$4.0000E-03$	239 (2)	$-9.5671E+02$	4099	$-9.5671E+02$	142
70	4, 0, 1	$7.4994E-03$	4389	$7.4985E-03$	416	$2.6909E-01$	5567 (2)	$7.5007E-03$	448
71	4, 1, 1	$1.7014E+01$	73	$1.7014E+01$	79	$1.7014E+01$	4431	$1.7014E+01$	43
72	4, 0, 2	$7.2768E+02$	80	$7.2768E+02$	103	$7.2768E+02$	2440	$5.0000E+00$	1 (1)
73	4, 1, 2	$2.9894E+01$	58	$2.9894E+01$	78	$2.9894E+01$	211	$2.9894E+01$	18
74	4, 3, 2	$5.1265E+03$	91	$5.1265E+03$	103	$5.1265E+03$	395	$0.0000E+00$	1 (1)
75	4, 3, 2	$5.1744E+03$	77	$5.1744E+03$	101	$5.1744E+03$	139	$0.0000E+00$	1 (1)
76	4, 0, 3	$-4.6818E+00$	72	$-4.6818E+00$	95	$-4.6818E+00$	470	$-4.6818E+00$	31
77	5, 2, 0	$2.4151E-01$	138	$2.4151E-01$	205	$2.4151E-01$	710	$2.4151E-01$	87
78	5, 3, 0	$-2.9197E+00$	69	$-2.9197E+00$	107	$-2.9197E+00$	566	$-2.9197E+00$	28
79	5, 3, 0	$7.8777E-02$	71	$7.8777E-02$	108	$7.8777E-02$	310	$7.8777E-02$	40
80	5, 3, 0	$5.3950E-02$	72	$5.3950E-02$	108	$5.3950E-02$	644	$5.3950E-02$	27
81	5, 3, 0	$5.3950E-02$	86	$5.3950E-02$	177	$5.3950E-02$	780	$5.3950E-02$	24
83	5, 0, 6	$-3.0666E+04$	86	$-3.0666E+04$	134	$-3.0666E+04$	500	$-3.0666E+04$	48
84	5, 0, 6	$-5.2803E+01$	103	$-5.2803E+01$	134	$-5.2803E+01$	548	$-5.2803E+01$	86
85	5, 0, 38	$-1.9052E+00$	105	$-1.9052E+00$	161	$1.0000E+20$	-1 (1)	$-1.9052E+00$	68
86	5, 0, 10	$-3.2349E+01$	93	$-3.2349E+01$	131	$-3.2349E+01$	442	$-3.2349E+01$	37
87	6, 4, 0	$8.9276E+03$	119 (2)	$8.9276E+03$	209 (2)	$8.8535E+03$	861	$4.2090E+04$	1 (1)
88	2, 0, 1	$1.3627E+00$	26	$1.3627E+00$	31	$1.3627E+00$	267	$1.3627E+00$	14
89	3, 0, 1	$1.3627E+00$	46	$1.3627E+00$	51	$1.3627E+00$	650	$1.3627E+00$	18
90	4, 0, 1	$1.3627E+00$	59	$1.3627E+00$	111	$1.3687E+00$	819 (2)	$1.3627E+00$	71
91	5, 0, 1	$1.3627E+00$	68	$1.3627E+00$	129	$1.3702E+00$	1094 (2)	$1.3627E+00$	67
92	6, 0, 1	$1.3627E+00$	95	$1.3627E+00$	143	$1.3659E+00$	1246 (2)	$1.3627E+00$	101
93	6, 0, 2	$1.3508E+02$	215	$1.3508E+02$	176	$1.3508E+02$	138051	$1.3508E+02$	123
95	6, 0, 4	$1.5620E-02$	76	$1.5620E-02$	141	$1.5620E-02$	729	$1.5620E-02$	30
96	6, 0, 4	$1.5620E-02$	76	$1.5620E-02$	141	$1.5620E-02$	641	$1.5620E-02$	30
97	6, 0, 4	$3.1358E+00$	76	$3.1358E+00$	141	$4.0712E+00$	3766 (2)	$3.1358E+00$	26
98	6, 0, 4	$3.1358E+00$	76	$3.1358E+00$	141	$3.1358E+00$	1512	$3.1358E+00$	25
99	7, 2, 0	$-8.3108E+08$	89	$-8.3108E+08$	182	$-8.3087E+08$	4 (2)	$-8.3108E+08$	97
100	7, 0, 4	$6.8063E+02$	113	$6.8063E+02$	225	$8.4900E+02$	2624322 (2)	$6.8063E+02$	240

Table 1: Numerical results.

		S_1		S_2		IR		DFO	
prob	$n,\ p,\ q$	f^*	$\#f$	f^*	$\#f$	f^*	$\#f$	f^*	$\#f$
101	7, 0, 6	1.8098E+03	650	1.8098E+03	425	1.8108E+03	53457 (2)	1.8098E+03	735
102	7, 0, 6	9.1188E+02	522	9.1188E+02	426	1.1748E+03	131838 (2)	9.1188E+02	599
103	7, 0, 6	1.0000E+20	-1 (1)	1.0000E+20	-1 (1)	1.0000E+20	-1 (1)	5.4367E+02	489
104	8, 0, 6	3.9512E+00	156	3.9512E+00	276	3.9512E+00	11428	3.9512E+00	186
105	8, 0, 1	1.1384E+03	3518	1.1384E+03	2155	1.1384E+03	25494	1.1387E+03	8000 (2)
106	8, 0, 6	7.0492E+03	131	7.0492E+03	288	7.0492E+03	71464	7.0492E+03	85
107	9, 6, 0	5.0550E+03	107	5.0550E+03	277	5.0550E+03	266	5.0550E+03	25
108	9, 0, 13	−8.6603E−01	156	−8.6603E−01	335	−4.9997E−01	117720 (2)	−8.6603E−01	73
109	9, 6, 4	5.3621E+03	160	5.3621E+03	398	5.5980E+03	461162 (2)	0.0000E+00	1 (1)
111	10, 3, 0	−4.7761E+01	475	−4.7761E+01	1831	−9.1869E+34	1863 (1)	−4.7760E+01	670
112	10, 3, 0	−4.7560E+01	122 (2)	−4.7692E+01	340 (2)	−4.7761E+01	4976	−4.7761E+01	113
113	10, 0, 8	2.4306E+01	145	2.4306E+01	403	2.4306E+01	3270	2.4306E+01	213
114	10, 3, 8	−1.7688E+03	614	−1.7688E+03	413	−1.7594E+03	205808 (2)	−1.7688E+03	167
116	13, 0, 15	9.7591E+01	187	9.7588E+01	645	9.7589E+01	5453	4.5000E+02	1 (1)
117	15, 0, 5	3.2349E+01	1924	3.2349E+01	2074	3.2349E+01	6402	3.2349E+01	3066
118	15, 0, 29	6.6482E+02	206	6.6482E+02	824	6.6482E+02	2220	6.6482E+02	96
119	16, 8, 0	2.4490E+02	299	2.4490E+02	1239	2.4490E+02	1192	2.4490E+02	111
215	2, 0, 1	−5.4291E−11	34	−5.4290E−11	40	−5.3831E−09	99	0.0000E+00	15
216	2 , 1 , 0	9.9938E−01	30	9.9938E−01	35	9.9938E−01	160	9.9938E−01	12
217	2 , 1 , 1	−8.0000E−01	27	−8.0000E−01	32	−8.0000E−01	32	−8.0000E−01	9
218	2 , 0 , 1	0.0000E+00	40	0.0000E+00	46	4.9784E−03	352 (2)	0.0000E+00	28
219	4 , 2 , 0	2.2667E+00	71 (2)	2.2667E+00	95 (2)	−1.0000E+00	409	−1.0000E+00	25
220	2 , 0 , 1	1.1893E+04	47 (2)	1.1893E+04	53 (2)	3.0248E+01	171 (2)	1.0000E+00	200
221	2 , 0 , 1	−1.0000E+00	29	−1.0000E+00	34	−9.5636E−01	46401 (2)	−1.0000E+00	9
222	2 , 0 , 1	−1.5000E+00	27	−1.5000E+00	32	−1.5000E+00	161	−1.5000E+00	12
223	2 , 0 , 2	−8.3403E−01	37	−8.3403E−01	43	−8.3403E−01	144	−8.3403E−01	15
224	2 , 0 , 4	−3.0400E+02	36	−3.0400E+02	42	−3.0400E+02	221	−3.0400E+02	27
225	2 , 0 , 5	2.0000E+00	34	2.0000E+00	40	2.0000E+00	37	2.0000E+00	14
226	2 , 0 , 2	−5.0000E−01	53	−5.0000E−01	33	−5.0000E−01	153	−5.0000E−01	13
227	2 , 0 , 2	1.0000E+00	28	1.0000E+00	33	1.0000E+00	58	1.0000E+00	13
228	2 , 0 , 2	−3.0000E+00	35	−3.0000E+00	41	−3.0000E+00	388	−3.0000E+00	26
230	2 , 0 , 2	3.7500E−01	27	3.7500E−01	32	3.7500E−01	80	3.7500E−01	16
231	2 , 0 , 2	1.3851E−02	702 (2)	8.2649E−17	210	2.7736E−06	24277	1.1487E−07	78
232	2 , 0 , 3	−1.0000E+00	34	−1.0000E+00	41	−1.0000E+00	126	−1.0000E+00	15
233	2 , 0 , 1	1.8725E−05	83	4.8050E−17	78	3.9051E−05	45591	8.5261E−07	67
234	2 , 0 , 1	−8.0000E−01	82	−8.0000E−01	33	−8.0000E−01	3151	−8.0000E−01	19
235	3 , 1 , 0	4.0000E−02	121	4.0000E−02	118	4.0000E−02	9784	4.0000E−02	30
236	2 , 0 , 2	−5.8903E+01	40	−5.8903E+01	53	−8.1975E+00	342 (2)	−8.1975E+00	31 (2)
237	2 , 0 , 3	−5.8903E+01	47	−5.8903E+01	53	−5.8900E+01	17957	−5.8903E+01	27
238	2 , 0 , 3	−5.8903E+01	40	−5.8903E+01	47	−8.1975E+00	700 (2)	−8.1975E+00	31 (2)
239	2 , 0 , 1	−5.8903E+01	40	−5.8903E+01	53	−8.1975E+00	350 (2)	−8.1975E+00	31 (2)
248	3 , 1 , 1	−8.0000E−01	59	−8.0000E−01	65	−8.0000E−01	327	−8.0000E−01	23
249	3 , 0 , 1	1.0000E+00	58	1.0000E+00	64	1.0096E+00	254 (2)	1.0000E+00	36
250	3 , 0 , 2	−3.3000E+03	61	−3.3000E+03	67	−3.3000E+03	335	−3.3000E+03	29
251	3 , 0 , 1	−3.4560E+03	167	−3.4560E+03	92	−3.4560E+03	426	−3.4560E+03	75
252	3 , 1 , 0	4.0000E−02	86	4.0000E−02	94	4.0680E−02	16489 (2)	4.0000E−02	135
253	3 , 0 , 1	6.9282E+01	73	6.9282E+01	79	6.9282E+01	341	6.9282E+01	36
254	3 , 2 , 0	−1.7321E+00	46	−1.7321E+00	51	−1.7282E+00	64 (2)	−1.7321E+00	13
262	4 , 1 , 3	−1.0000E+01	73	−1.0000E+01	97	−1.0000E+01	135	−1.0000E+01	22
263	4 , 2 , 2	−1.0000E+00	74	−1.0000E+00	98	−9.7943E−01	60 (2)	−1.0000E+00	25
264	4 , 0 , 3	−4.4000E+01	71	−4.4000E+01	95	−4.3646E+01	1467 (2)	−4.4000E+01	42
265	4 , 2 , 0	1.9036E+00	55 (2)	1.5915E+00	78 (2)	1.9036E+00	160 (2)	9.7475E−01	13
268	5 , 0 , 5	6.6899E−01	2811 (2)	−2.7348E−13	133	2.0088E−01	332781 (2)	−2.5322E−12	124
269	5 , 3 , 0	4.0930E+00	82	4.0930E+00	130	4.0930E+00	312	4.0930E+00	18
270	5 , 0 , 1	4.9449E−14	101	6.3558E−19	138	2.7526E−11	453	0.0000E+00	81
277	4 , 0 , 4	5.0762E+00	58	5.0762E+00	79	5.0762E+00	503	5.0762E+00	56
278	6 , 0 , 6	7.8385E+00	111	7.8385E+00	201	7.8385E+00	27696	7.8385E+00	121
279	8 , 0 , 8	1.0606E+01	278	1.0606E+01	276	1.0606E+01	273795	1.0606E+01	135
280	10 , 0 , 10	1.3375E+01	166	1.3375E+01	668	1.3375E+01	143010	1.3375E+01	277
284	15 , 0 , 10	−1.8400E+03	202	−1.8400E+03	820	1.0000E+20	-1 (1)	−1.8400E+03	553
285	15 , 0 , 10	−8.2520E+03	202	−8.2520E+03	820	1.0000E+20	-1 (1)	−8.2520E+03	277
315	2 , 0 , 3	−8.0000E−01	35	−8.0000E−01	41	−7.9853E−01	159 (2)	−8.0000E−01	13
316	2 , 1 , 0	8.0000E+02	25 (1)	8.0000E+02	30 (1)	3.3431E+02	157	8.0000E+02	1 (1)
317	2 , 1 , 0	8.0000E+02	25 (1)	8.0000E+02	30 (1)	3.7247E+02	151	8.0000E+02	1 (1)
318	2 , 1 , 0	8.0000E+02	25 (1)	8.0000E+02	30 (1)	4.1275E+02	151	8.0000E+02	1 (1)
319	2 , 1 , 0	8.0000E+02	25 (1)	8.0000E+02	30 (1)	4.5240E+02	202	8.0000E+02	1 (1)
320	2 , 1 , 0	8.0000E+02	25 (1)	8.0000E+02	30 (1)	4.8553E+02	128	8.0000E+02	1 (1)
321	2 , 1 , 0	8.0000E+02	25 (1)	8.0000E+02	30 (1)	4.9611E+02	240	8.0000E+02	1 (1)
322	2 , 1 , 0	4.9996E+02	28	4.9996E+02	33	4.9996E+02	229	4.9996E+02	18
323	2 , 0 , 2	3.7989E+00	28	3.7989E+00	33	3.7989E+00	184	3.7989E+00	14
324	2 , 0 , 2	5.0000E+00	38	5.0000E+00	44	5.0000E+00	51920	5.0000E+00	23
325	2 , 1 , 2	3.7913E+00	35	3.7913E+00	41	7.4641E+00	42 (2)	9.0000E+00	2 (2)
326	2 , 0 , 2	−7.9808E+01	34	−7.9808E+01	40	−7.9808E+01	101	−7.9808E+01	13
327	2 , 0 , 1	2.8460E−02	71	2.8460E−02	72	3.0646E−02	287 (2)	2.8460E−02	26
329	2 , 0 , 3	−6.9618E+03	37	−6.9618E+03	43	−6.9618E+03	127	−6.9618E+03	14
330	2 , 0 , 1	1.6206E+00	61	1.6206E+00	65	1.6206E+00	244	1.0000E+20	-1 (1)
331	2 , 0 , 1	4.2584E+00	62	4.2584E+00	37	4.2584E+00	281	4.2584E+00	24
332	2 , 0 , 2	1.1541E+02	28	1.1541E+02	33	3.9268E+01	1012048 (1)	1.2198E+02	22 (2)
335	3 , 2 , 0	−3.7719E−03	45 (2)	−3.7719E−03	50 (2)	6.7614E−01	6312141 (1)	−4.4721E−03	19
336	3 , 2 , 0	−3.3790E−01	46	−3.3790E−01	51	−3.3790E−01	249	−3.3790E−01	17
337	3 , 0 , 1	6.0000E+00	58	6.0000E+00	64	6.0000E+00	516	6.0000E+00	36
338	3 , 2 , 0	−7.2057E+00	59 (2)	−7.2057E+00	65 (2)	−1.0993E+01	6140	−7.2057E+00	15 (2)
339	3 , 0 , 1	3.3617E+00	86	3.3617E+00	83	3.3617E+00	1150	1.0000E+20	-1 (1)
340	3 , 0 , 1	−5.4000E−02	151	−5.4000E−02	64	−5.4000E−02	390	−5.4000E−02	37
341	3 , 0 , 1	−2.2627E+01	89	−2.2627E+01	69	−2.2627E+01	658	−2.2627E+01	35
342	3 , 0 , 1	−2.2627E+01	90	−2.2627E+01	69	−2.2627E+01	658	−2.2627E+01	35
343	3 , 0 , 2	−5.6848E+00	74	−5.6848E+00	79	−5.6848E+00	157	−5.6848E+00	34
344	3 , 1 , 0	3.2568E−02	61	3.2568E−02	68	3.2568E−02	636	3.2568E−02	32
345	3 , 1 , 0	3.2568E−02	67	3.2568E−02	83	3.2595E−02	102623	3.2568E−02	32
346	3 , 0 , 2	−5.6848E+00	74	−5.6848E+00	79	−5.6848E+00	157	−5.6848E+00	34
347	3 , 1 , 0	1.7375E+04	62	1.7375E+04	68	1.7375E+04	287	1.7375E+04	18
348	3 , 1 , 0	3.6971E+01	63	3.6971E+01	69	5.0689E+01	2527562 (1)	5.0689E+01	1 (1)

Table 2: Numerical results.

		S_1		S_2		IR		DFO	
prob	n, p, q	f^*	$\#f$	f^*	$\#f$	f^*	$\#f$	f^*	$\#f$
349	3 , 9 , 0	$-9.4471E-01$	59	$-9.4399E-01$	64 (2)	$-8.7610E-01$	681 (2)	$-9.4106E-01$	28 (2)
353	4 , 1 , 2	$-3.9934E+01$	56	$-3.9934E+01$	76	$-3.9934E+01$	495	$-3.9934E+01$	13
354	4 , 0 , 1	$1.1378E-01$	144	$1.1378E-01$	157	$1.2756E-01$	415 (2)	$1.1378E-01$	101
355	4 , 1 , 0	$6.9675E+01$	102	$6.9675E+01$	99	$6.9675E+01$	36927	$6.9675E+01$	48
356	4 , 0 , 5	$8.9945E+00$	57 (2)	$8.9945E+00$	77 (2)	$1.5815E+01$	9481 (2)	$2.4736E+00$	37
359	5 , 0 ,14	$-5.2803E+06$	90	$-5.2803E+06$	137	$-3.2334E+06$	7 (2)	$-5.2803E+06$	135
360	5 , 0 , 2	$-5.2803E-01$	103	$-5.2803E-01$	134	$-2.5413E-01$	161 (2)	$-5.2803E-01$	80
361	5 , 0 , 6	$-7.7641E+05$	84	$-7.7641E+05$	130	$-7.7641E+05$	4	$-7.7641E+05$	66
362	5 , 0 , 4	$2.6120E-01$	130 (2)	$4.1700E-02$	452 (2)	$2.0080E-01$	4521374 (1)	$4.1400E-02$	215
363	5 , 3 , 0	$6.8869E-02$	65 (1)	$6.8869E-02$	105 (1)	$-4.4760E+00$	3016225 (1)	$6.8869E-02$	1 (1)
364	6 , 0 , 4	$6.2488E-02$	10500 (2)	$6.0600E-02$	607	$8.8959E-02$	3803 (2)	$6.0970E-02$	577 (2)
365	7 , 0 , 5	$2.3314E+01$	379	$2.3314E+01$	221	$0.0000E+00$	6012247 (1)	$6.0000E+00$	1 (1)
366	7 , 0 ,14	$7.0431E+02$	117	$7.0431E+02$	230	$-6.6307E+03$	4031233 (1)	$2.0019E+03$	45 (2)
367	7 , 2 , 3	$-3.7413E+01$	336	$-3.7413E+01$	224	$-3.7413E+01$	1161	$-3.7413E+01$	69
369	8 , 0 , 6	$7.0492E+03$	132	$7.0492E+03$	287	$1.0000E+20$	-1 (1)	$1.5000E+04$	1 (1)
372	9 , 0 ,12	$1.1001E+05$	159	$1.1001E+05$	342	$1.1001E+05$	2240	$1.5334E+05$	135 (2)
373	9 , 6 , 0	$1.3390E+04$	138	$1.3390E+04$	342	$1.3390E+04$	4592	$7.5289E+05$	1 (1)
374	10 , 0 ,35	$2.3326E-01$	142	$2.3326E-01$	400	$3.0438E-01$	296 (2)	$2.3326E-01$	144
375	10 , 9 , 0	$-1.5161E+01$	143 (2)	$-1.5161E+01$	401 (2)	$-1.5624E+01$	20160	$-1.5161E+01$	17 (2)
376	10 , 1 ,14	$-4.4301E+03$	354	$-4.4301E+03$	1350	$-4.4234E+03$	104340 (2)	$-4.4301E+03$	322
377	10 , 3 , 0	$-7.9499E+02$	172	$-7.9499E+02$	473	$-7.9499E+02$	11017	$-7.9499E+02$	33
378	10 , 3 , 0	$-4.7761E+01$	654	$-4.7761E+01$	2708	$-4.7744E+01$	962352 (2)	$-4.7730E+01$	221 (2)
380	12 , 0 , 3	$3.1687E+05$	259 (2)	$3.1682E+05$	43522	$3.1687E+05$	370066 (2)	$3.1688E+05$	7177 (2)
381	13 , 1 , 3	$1.0149E+00$	149	$1.0149E+00$	529	$1.0149E+00$	2395	$1.0149E+00$	70
382	13 , 1 , 3	$1.0383E+00$	148	$1.0383E+00$	528	$1.0383E+00$	6062	$1.0000E+20$	-1 (1)
383	14 , 1 , 0	$7.2859E+00$	229	$7.2859E+00$	849	$1.0000E+20$	-1 (1)	$7.2867E+00$	237 (2)
384	15 , 0 ,10	$-8.3103E+03$	202	$-8.3103E+03$	820	$1.0000E+20$	-1 (1)	$-8.3103E+03$	280
385	15 , 0 ,10	$-8.3153E+03$	202	$-8.3153E+03$	820	$1.0000E+20$	-1 (1)	$-8.3153E+03$	277
386	15 , 0 ,11	$-8.1644E+03$	202	$-8.1644E+03$	820	$-8.1227E+03$	1128 (2)	$-8.1644E+03$	518
387	15 , 0 ,11	$-8.2501E+03$	202	$-8.2501E+03$	820	$-8.2500E+03$	9487	$-8.2501E+03$	277
388	15 , 0 ,15	$-5.8211E+03$	203	$-5.8211E+03$	821	$1.0000E+20$	-1 (1)	$-5.8211E+03$	533
389	15 , 0 ,15	$-5.8097E+03$	203	$-5.8097E+03$	821	$-5.1780E+03$	28036 (2)	$-5.8097E+03$	377
390	19 , 1 ,11	$1.0000E+20$	-1 (1)	$1.0000E+20$	-1 (1)	$1.0000E+20$	-1 (1)	$2.2191E+05$	1 (1)
392	30 , 0 ,45	$-1.6961E+06$	872 (2)	$-1.6961E+06$	2990 (2)	$1.0000E+20$	-1 (1)	$-1.0758E+07$	4485
393	48 , 2 , 1	$8.7393E-01$	1112 (2)	$8.6338E-01$	12271	$1.0000E+20$	-1 (1)	$1.8623E+00$	1 (1)
394	20 , 1 , 0	$1.9167E+00$	265	$1.9167E+00$	1630	$1.9167E+00$	36692	$1.9167E+00$	1464
395	50 , 1 , 0	$1.9167E+00$	625	$1.9167E+00$	7963	$1.9167E+00$	116147	$1.0000E+20$	-1 (1)

Table 3: Numerical results.

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