Universidade Federal do Paraná

Setor de Ciências Exatas

Departamento de Matematica

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2^{da} prova de cálculo II

Curitiba, 16 de Maio de 2011

1. Classificar os pontos criticos da funcao $f(x,y) = 3xy^2 + x^3 - 3x$.

2. Encontre os extremos absolutos da função $f(x,y) = x^2 - 2xy + 2y^2$ na região

$$\mathcal{R} = \{(x,y) \in \mathbb{R}^2; x^2 + 2y^2 \leq 1\}$$

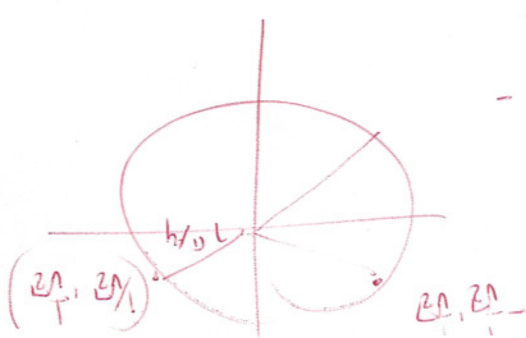
3. As faces de uma caixa rectangular sem tampa têm área total igual a 16 m^2. Determine as dimensões da caixa que maximizam o respectivo volume.

4. Calcule a seguinte integral mudando a ordem de integração:

$$I = \int_0^4 \int_{\sqrt{y}}^2 y \, \cos(x^5) \, dx \, dy$$

5. Determinar a área da região limitada pelas seguintes curvas:

$$y^2 = 4x; \;\; x + y = 3 \;\text{ e }\; y = 0.$$



Ministério da Educação
UNIVERSIDADE FEDERAL DO PARANÁ
Setor de Ciências Exatas

Nota:

Disciplina:

Curso: Professor:

Aluno: 2ª Prova de Cálculo II

Turma: Data: 16 / 05 / 2011.

1 $f(x,y) = 3xy^2 + x^3 - 3x$

Pontos Críticos $\Rightarrow$ $\nabla f = \vec{0}$ ou $\begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$ ou $\begin{cases} 3y^2 + 3x^2 - 3 = 0 & \ldots\ (1) \\ 6xy = 0 & \ldots\ (2) \end{cases}$

De (2) temos:

$x = 0 \Rightarrow$ em (1): $y = \pm 1 \Rightarrow A(0,1),\ B(0,-1)$ «Pontos Críticos»

ou

$y = 0 \Rightarrow$ em (1): $x = \pm 1 \Rightarrow C(1,0),\ D(-1,0)$

$f_x = 3y^2 + 3x^2 - 3$
$f_y = 6xy$
$\Rightarrow$
$f_{xx} = 6x$
$f_{yy} = 6x$
$f_{xy} = 6y$

$$f_{xx} f_{yy} - (f_{xy})^2 = 36x^2 - 36y^2 = H(x,y)$$

Ponto	$H(x,y) = 36x^2 - 36y^2$	f_{xx}	Natureza
$A(0,1)$	$-36 < 0$	0	Sela
$B(0,-1)$	$-36 < 0$	0	Sela
$C(1,0)$	$36 > 0$	6	Mínimo
$D(-1,0)$	$36 > 0$	-6	Máximo

2 $f(x,y) = x^2 - 2xy + 2y^2$

Pontos críticos no interior de R

$\nabla f = \vec{0} \Rightarrow \begin{cases} 2x - 2y = 0 \\ -2x + 4y = 0 \end{cases} \Rightarrow x = 0,\ y = 0 \Rightarrow A(0,0) \in int(R)$

Pontos críticos na fronteira de R (∂R)

$\partial R: \quad x^2 + 2y^2 = 1$

$x = \cos\theta$
$y = \frac{1}{\sqrt{2}} \sin\theta$ com $0 \le \theta \le 2\pi$. $\gamma(\theta) = \left(\cos\theta, \frac{\sin\theta}{\sqrt{2}}\right)$

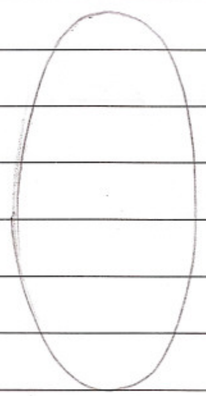
$f\big|_{\partial R} : f(\theta) = 1 - 2\cos\theta\left(\frac{1}{\sqrt{2}}\sin\theta\right)$, com $0 \le \theta \le 2\pi$.

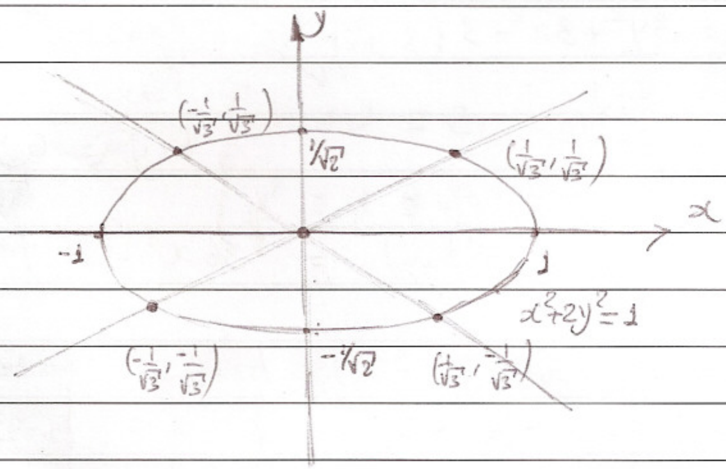
$f(\theta) = 1 - \frac{\sin(2\theta)}{\sqrt{2}}$, com $0 \le \theta \le 2\pi$.

$f'(\theta) = -\frac{2}{\sqrt{2}}\cos(2\theta) = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.





$x = y \Rightarrow 3x^2 = 1$
$x = \pm\frac{1}{\sqrt{3}}$

$\gamma(\pi/4) = \left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$, $\gamma(3\pi/4) = \left(-\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$, $\gamma\left(\frac{5\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, -\frac{1}{2}\right)$

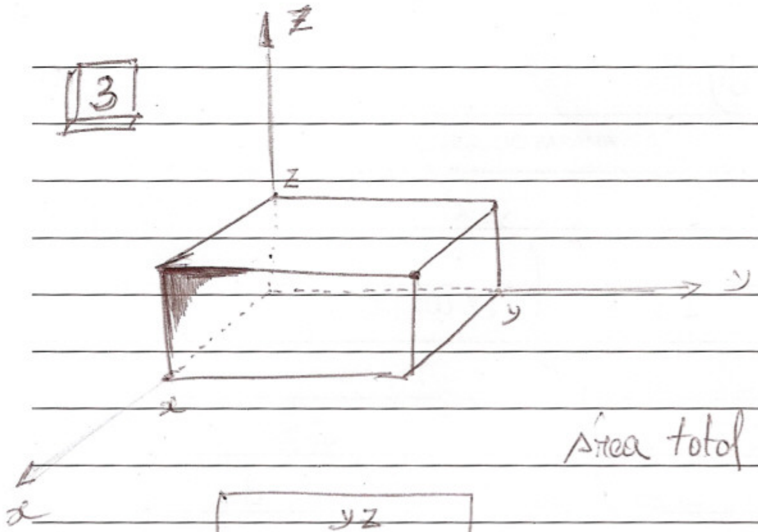
$\gamma\left(\frac{7\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, -\frac{1}{2}\right)$

$f(0,0) = 0$
$f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \frac{1}{3} - \frac{2}{3} + \frac{2}{3} = \frac{1}{3}$
$f\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \frac{1}{3} + \frac{2}{3} + \frac{2}{3} = \frac{5}{3}$
$f\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) = \frac{1}{3} - \frac{2}{3} + \frac{2}{3} = \frac{1}{3}$
$f\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) = \frac{1}{3} + \frac{2}{3} + \frac{2}{3} = \frac{5}{3}$

errado

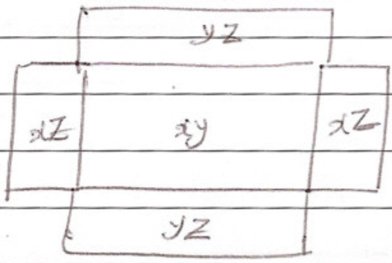
$(0,0)$. Ponto de Mínimo Absoluto
$\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ ponto de Máximo Absoluto
$\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ Ponto de Máximo Absoluto.

$\boxed{3}$



Volume : xyz

Área total : $xy + 2xz + 2yz = 16$



$$\boxed{\begin{cases} f(x,y,z) = xyz & \text{—— Função} \\ g(x,y,z) = xy + 2xz + 2yz - 16 = 0 & \text{—— Restrição.} \end{cases}}$$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases} \Rightarrow \begin{cases} yz = \lambda(y + 2z) \quad \dots\dots (1) \\ xz = \lambda(x + 2z) \quad \dots (2) \\ xy = \lambda(2x + 2y) \quad \dots (3) \\ xy + 2xz + 2yz = 16 \quad \dots (4) \end{cases}$$

De (1), (2) e (3)

$$x(y+2z) = y(x+2z) = z(2x+2y)$$

$\therefore$

$$x(y+2z) = y(x+2z) \qquad\qquad x(y+2z) = z(2x+2y)$$

$$2xz = 2yz \qquad\qquad xy + \cancel{2xz} = \cancel{2xz} + 2zy$$

$$\boxed{x = y} \qquad\qquad \boxed{x = 2z}$$

$\therefore$ Subst. em (4)

$$x^2 + 2x\left(\frac{x}{2}\right) + 2(x)\left(\frac{x}{2}\right) = 16$$

$$3x^2 = 16$$

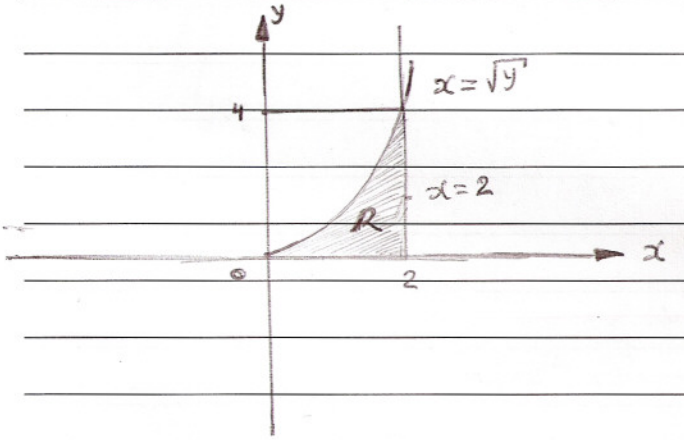
$$x = \frac{16}{3} \Rightarrow \boxed{x = \frac{4}{\sqrt{3}},\ y = \frac{4}{\sqrt{3}},\ z = \frac{2}{\sqrt{3}}}$$

$\boxed{4}$ $$I = \int_0^4 \int_{\sqrt{y}}^2 y \cos(x^5)\, dx\, dy$$

$$R: \begin{cases} 0 \le y \le 4 \\ \sqrt{y} \le x \le 2 \end{cases}$$

Graficar os curvas

$y=0$, $y=4$, $x=\sqrt{y}$, $x=2$



$$R: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{cases}$$

$$I = \int_0^2 \int_{y=0}^{y=x^2} y \cos(x^5)\, dy\, dx$$

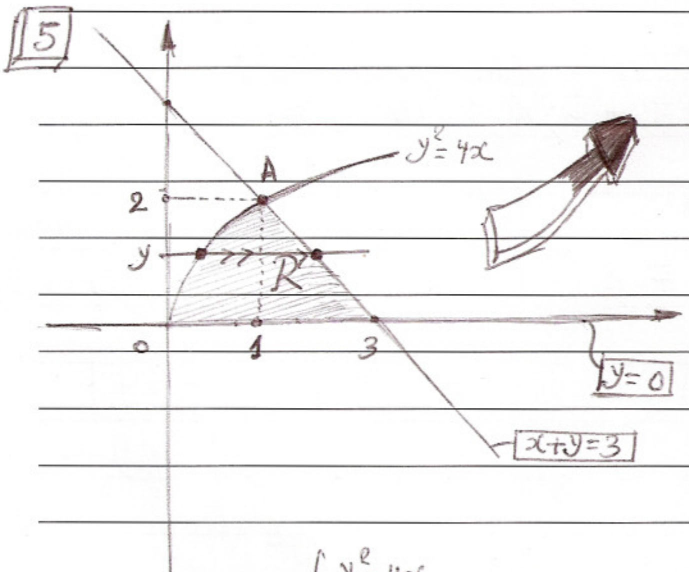
$$= \int_0^2 \cos(x^5) \left(\frac{y^2}{2}\right)\Bigg|_{y=0}^{y=x^2} dx$$

$$= \frac{1}{2} \int_0^2 x^4 \cos(x^5)\, dx$$

$$= \frac{1}{2} \left(\frac{\operatorname{sen}(x^5)}{5}\right)\Bigg|_{x=0}^{x=2}$$

$$= \frac{1}{10} \left[\operatorname{sen}(32) - \operatorname{sen}(0)\right]$$

$\boxed{5}$



Ponto: A $\begin{cases} y^2 = 4x \\ x+y=3 \end{cases}$

$y^2 = 4(3-y)$

$y^2 = 12 - 4y$

$y^2 + 4y - 12 = 0$

$(y+6)(y-2) = 0$

$y = -6$ ∨ $\boxed{y=2}$

$\downarrow$

$\boxed{x=1}$

Região do tipo 2:

$$R: \begin{cases} 0 \le y \le 2 \\ \frac{y^2}{4} \le x \le 3-y \end{cases}$$

$$\text{Área}(R) = \int_0^2 \int_{x=\frac{y^2}{4}}^{x=3-y} dx\, dy$$

$$= \int_0^2 \left(3 - y - \frac{y^2}{4}\right) dy$$

$$= \left(3y - \frac{y^2}{2} - \frac{y^3}{12}\right)\Bigg|_{y=0}^{y=2}$$

$$= 6 - 2 - \frac{2}{3}$$

$$= 4 - \frac{2}{3} = \frac{10}{3} \quad \blacksquare$$