

Tabelas de Integrais Indefinidas

Observação: Em todas as fórmulas, a constante arbitrária é omitida; a, b, c, α representam números reais e m, n, p, q inteiros positivos. Quando a^2 aparece no integrando, a deve ser tomado como um número positivo, $\ln(\)$ pode sempre ser substituído por $\ln|\ |$.

$$1. \int c dx = cx$$

$$2. \int cf(x) dx = c \int f(x) dx$$

$$3. \int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

$$4. \int f(x)g'(x) dx = f(x)g(x) - \int g(x)f'(x) dx$$

$$5. \int u dv = uv - \int v du$$

$$6. \int x^a dx = \frac{x^{a+1}}{a+1}, \quad a \neq -1$$

$$7. \int \frac{1}{x} dx = \ln|x|$$

$$8. \int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$$

$$9. \int e^{ax} dx = \frac{e^{ax}}{a}$$

$$10. \int a^x dx = \frac{a^x}{\ln(a)}$$

$$11. \int \ln(x) dx = x \ln(x) - x$$

$$12. \int \log_a(x) dx = \frac{1}{\ln(a)} [x \ln(x) - x] = x \log_a(x) - \frac{x}{\ln(a)}; x \neq 1$$

$$13. \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{tg}^{-1}\left(\frac{x}{a}\right) = -\frac{1}{a} \operatorname{Cotg}^{-1}\left(\frac{x}{a}\right)$$

$$14. \int \frac{dx}{x^2 - a^2} = -\frac{1}{a} \operatorname{tgh}^{-1}\left(\frac{x}{a}\right) = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

$$15. \int \frac{dx}{a+bx^2} = \begin{cases} \frac{1}{\sqrt{ab}} \operatorname{tg}^{-1}\left(\frac{x\sqrt{ab}}{a}\right); ab > 0 \\ \frac{1}{2\sqrt{-ab}} \ln \left(\frac{\sqrt{-bx} + \sqrt{a}}{\sqrt{-bx} - \sqrt{a}} \right); a > 0; b < 0 \end{cases}$$

$$16. \int \frac{xdx}{a+bx^2} = \frac{1}{2b} \ln \left(\frac{a+bx^2}{b} \right)$$

$$17. \int \frac{dx}{(a+bx^2)^m} = \frac{1}{2a(m-1)} \times \frac{1}{(a+bx^2)^{m-1}} + \frac{2m-3}{2(m-1)a} \times \int \frac{dx}{(a+bx^2)^{m-1}}; m > 1$$

$$18. \int \frac{xdx}{(a+bx^2)^m} = \frac{-1}{2b(m-1)(a+bx^2)^{m-1}}; m > 1$$

$$19. \int \frac{dx}{\sqrt{a^2 - x^2}} = \text{sen}^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right)$$

$$20. \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2})$$

$$21. \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \cos^{-1}\left(\frac{a}{x}\right)$$

$$22. \int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a} \ln\left(\frac{a + \sqrt{a^2 + x^2}}{x}\right)$$

$$23. \int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left(x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2}) \right)$$

$$24. \int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right)$$

$$25. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \cos^{-1}\left(\frac{a}{x}\right)$$

$$26. \int \frac{xdx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$27. \int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} (x^2 \pm a^2)^{3/2}$$

$$28. \int (x^2 \pm a^2)^{3/2} dx = \frac{1}{8} \left[2x(x^2 \pm a^2)^{3/2} \pm 3a^2 x\sqrt{x^2 \pm a^2} + 3a^4 \ln(x + \sqrt{x^2 \pm a^2}) \right]$$

$$29. \int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}}$$

$$30. \int x^2 \sqrt{(x^2 \pm a^2)} dx = \frac{x}{4} (x^2 \pm a^2)^{3/2} \mp \frac{a^2 x}{8} \sqrt{(x^2 \pm a^2)} - \frac{a^4}{8} \ln(x + \sqrt{(x^2 \pm a^2)})$$

$$31. \int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \operatorname{sen}^{-1} \left(\frac{x}{a} \right) \right]$$

$$32. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right)$$

$$33. \int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \ln \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right)$$

$$34. \int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$$

$$35. \int f(x, \sqrt{x^2 + a^2}) dx = a \int f(a \times \operatorname{tg}(u), a \times \sec(u)) \sec^2(u) du; \quad x = a \times \operatorname{tg}(u)$$

$$36. \int f(x, \sqrt{x^2 - a^2}) dx = a \int f(a \times \sec(u), a \times \operatorname{tg}(u)) \sec(u) \times \operatorname{tg}(u) du; \quad x = a \times \sec(u)$$

$$37. \int f(x, \sqrt{a^2 - x^2}) dx = -a \int f(a \times \cos(u), a \times \operatorname{sen}(u)) \operatorname{sen}(u) du; \quad x = a \times \cos(u)$$

$$38. \int f(x, X) dx = \frac{1}{a} \int f \left(\frac{y-b}{a}, \frac{y^2+d}{a} \right) dy$$

$$x = (y-b)/a; \quad d = ac - b^2; \quad X = ax^2 + 2bx + c$$

$$39. \int x^n (a+bx)^n dx = \frac{x^{n+1} (a+bx)^n}{m+n+1} + \frac{an}{m+n+1} \int x^n (a+bx)^{n-1} dx$$

$$40. \int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx)$$

$$41. \int \frac{xdx}{a+bx} = \frac{1}{b^2} [a+bx - a \ln(a+bx)]$$

$$42. \int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2} \left[\ln(a+bx) + \frac{a}{a+bx} \right]$$

$$43. \int \frac{xdx}{(a+bx)^m} = \frac{1}{b^2} \left[\frac{-1}{(m-2)(a+bx)^{m-2}} + \frac{a}{(m-1)(a+bx)^{m-1}} \right]; \quad m \geq 3$$

$$44. \int \sqrt{a+bx} dx = \frac{2}{3b} \sqrt{(a+bx)^3}$$

$$45. \int x^m \sqrt{a+bx} dx = \frac{2}{(2m+3)b} \left[x^m \sqrt{(a+bx)^3} - ma \int x^{m-1} \sqrt{a+bx} dx \right]$$

$$46. \int \frac{dx}{x^m \sqrt{a+bx}} = \frac{-\sqrt{a+bx}}{a(m-1)x^{m-1}} - \frac{(2m-3)b}{(2m-2)a} \int \frac{dx}{x^{m-1} \sqrt{a+bx}}; \quad m \neq 1$$

$$47. \int f(x, \sqrt{a+bx}) dx = \frac{2}{b} \int f\left(\frac{z^2-a}{b}, z\right) z dz; \quad z^2 = a+bx$$

$$48. \int \frac{dx}{a^2+x^2} = \frac{1}{3a^2} \left[\frac{1}{2} \ln \left[\frac{(a+x)^2}{a^2-ax+x^2} \right] + \sqrt{3} \operatorname{tg}^{-1} \left(\frac{2x-a}{a\sqrt{3}} \right) \right]$$

$$49. \int \operatorname{sen}(x) dx = -\cos(x)$$

$$50. \int \cos(x) dx = \operatorname{sen}(x)$$

$$51. \int \operatorname{tg}(x) dx = -\ln(\cos(x))$$

$$52. \int \cot g(x) dx = \ln(\text{sen}(x))$$

$$53. \int \sec(x) dx = \ln \left[\text{tg} \left(\frac{x}{2} + \frac{\pi}{2} \right) \right]$$

$$54. \int \text{cosec}(x) dx = \ln \left[\text{tg} \left(\frac{x}{2} \right) \right]$$

$$55. \int \text{sen}^2(x) dx = \frac{1}{2} [x - \cos(x)\text{sen}(x)]$$

$$56. \int \text{sen}^m(x) dx = \frac{-\cos(x)\text{sen}(x)^{m-1}}{m} + \frac{m-1}{m} \int \text{sen}(x)^{m-2} dx$$

$$57. \int \cos^2(x) dx = \frac{1}{2} x + \frac{1}{4} \text{sen}(2x)$$

$$58. \int \cos^m(x) dx = \frac{\text{sen}(x)\cos^{m-1}(x)}{m} + \frac{m-1}{m} \int \cos^{m-2}(x) dx$$

$$59. \int \frac{dx}{\cos^2(x)} = \int \sec^2(x) dx = \text{tg}(x)$$

$$60. \int \frac{dx}{\cos^m(x)} = \frac{\text{sen}(x)}{(m-1)\cos^{m-1}(x)} + \frac{m-2}{m-1} \int \frac{dx}{\cos^{m-2}(x)}; \quad m > 1$$

$$61. \int \frac{dx}{\text{sen}^2(x)} = \int \text{cosec}^2(x) dx = -\cot g(x)$$

$$62. \int \frac{dx}{\text{sen}^m(x)} = \frac{-\cos(x)}{(m-1)\text{sen}^{m-1}(x)} + \frac{m-2}{m-1} \int \frac{dx}{\text{sen}^{m-2}(x)}; \quad m > 1$$

$$63. \int \frac{dx}{1 \pm \text{sen}(x)} = \mp \text{tg} \left(\frac{\pi}{4} \mp \frac{x}{2} \right)$$

$$64. \int \frac{dx}{1 + \cos(x)} = \operatorname{tg}\left(\frac{x}{2}\right)$$

$$65. \int \frac{dx}{1 - \cos(x)} = -\operatorname{cot} g\left(\frac{x}{2}\right)$$

$$66. \int \frac{dx}{a + b \operatorname{sen}(x)} = \begin{cases} \frac{1}{\sqrt{b^2 - a^2}} \ln \left(\frac{a \operatorname{tg}\left(\frac{x}{2}\right) + b - \sqrt{b^2 - a^2}}{a \operatorname{tg}\left(\frac{x}{2}\right) + b + \sqrt{b^2 - a^2}} \right); b^2 > a^2 \\ \frac{2}{\sqrt{a^2 - b^2}} \operatorname{tg}^{-1} \left(\frac{a \operatorname{tg}\left(\frac{x}{2}\right) + b}{\sqrt{a^2 - b^2}} \right); a^2 > b^2 \end{cases}$$

$$67. \int \frac{dx}{a + b \cos(x)} = \begin{cases} \frac{1}{\sqrt{b^2 - a^2}} \ln \left(\frac{\sqrt{b^2 - a^2} \operatorname{tg}\left(\frac{x}{2}\right) + a + b}{\sqrt{b^2 - a^2} \operatorname{tg}\left(\frac{x}{2}\right) - a - b} \right); b^2 > a^2 \\ \frac{2}{\sqrt{a^2 - b^2}} \operatorname{tg}^{-1} \left(\frac{\sqrt{a^2 - b^2} \operatorname{tg}\left(\frac{x}{2}\right)}{a + b} \right); a^2 > b^2 \end{cases}$$

$$68. \int \operatorname{sen}(nx) \times \operatorname{sen}(mx) dx = \frac{\operatorname{sen}(m-n)x}{2(m-n)} - \frac{\operatorname{sen}(m+n)x}{2(m+n)}; m^2 \neq n^2$$

$$69. \int \operatorname{sen}(nx) \times \cos(mx) dx = \frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)}; m^2 \neq n^2$$

$$70. \int \cos(nx) \times \cos(mx) dx = \frac{\operatorname{sen}(m-n)x}{2(m-n)} + \frac{\operatorname{sen}(m+n)x}{2(m+n)}; m^2 \neq n^2$$

$$71. \int tg^n(x) dx = \frac{tg^{n-1}(x)}{n-1} - \int tg^{n-2}(x) dx; n \neq 1$$

$$72. \int \frac{dx}{\text{sen}(x) \cos(x)} = \ln(tg(x))$$

$$73. \int \frac{dx}{\text{sen}(x) \cos^m(x)} = \frac{1}{(m-1) \cos^{m-1}(x)} + \int \frac{dx}{\text{sen}(x) \cos^{m-1}(x)}; m > 1$$

$$74. \int x^m \text{sen}(x) dx = -x^m \cos(x) + m \int x^{m-1} \cos(x) dx$$

$$75. \int x^m \cos(x) dx = x^m \text{sen}(x) - m \int x^{m-1} \text{sen}(x) dx$$

$$76. \int \text{sen}^{-1}(x) dx = x \text{sen}^{-1}(x) + \sqrt{1-x^2}$$

$$77. \int \cos^{-1}(x) dx = x \cos^{-1}(x) - \sqrt{1-x^2}$$

$$78. \int tg^{-1}(x) dx = x tg^{-1}(x) - \frac{1}{2} \ln(1+x^2)$$

$$79. \int \cot g^{-1}(x) dx = x \cot g^{-1}(x) + \frac{1}{2} \ln(1+x^2)$$

$$80. \int (\text{sen}^{-1}(x))^2 dx = x (\text{sen}^{-1}(x))^2 - 2x + 2\sqrt{1-x^2} \text{sen}^{-1}(x)$$

$$81. \int (\cos^{-1}(x))^2 dx = x (\cos^{-1}(x))^2 - 2x - 2\sqrt{1-x^2} \cos^{-1}(x)$$

$$82. \int x^n \text{sen}^{-1}(x) dx = \frac{x^{n+1} \text{sen}^{-1}(x)}{n+1} - \frac{1}{n+1} \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx$$

$$83. \int x^n \cos^{-1}(x) dx = \frac{x^{n+1} \cos^{-1}(x)}{n+1} + \frac{1}{n+1} \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx$$

$$84. \int x \ln(x) dx = \frac{x^2}{2} \ln(x) - \frac{x^2}{4}$$

$$85. \int x^m \ln(x) dx = \frac{x^{m+1}}{m+1} - \frac{x^{m+2}}{(m+1)^2}; m \neq -1$$

$$86. \int (\ln(x))^q dx = x(\ln(x))^q - q \int (\ln(x))^{q-1} dx$$

$$87. \int \frac{(\ln(x))^q}{x} dx = \frac{(\ln(x))^{q+1}}{q+1}$$

$$88. \int \frac{dx}{x \ln(x)} = \ln(\ln(x))$$

$$89. \int x^m (\ln(x))^q dx = \frac{x^{m+1} (\ln(x))^q}{m+1} - \frac{q}{m+1} \int x^m (\ln(x))^{q-1} dx; m \neq -1$$

$$90. \int \sin(\ln(x)) dx = \frac{1}{2} x \sin(\ln(x)) - \frac{1}{2} x \cos(\ln(x))$$

$$91. \int \cos(\ln(x)) dx = \frac{1}{2} x \sin(\ln(x)) + \frac{1}{2} x \cos(\ln(x))$$

$$92. \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$93. \int x^m e^{ax} dx = \frac{x^m e^{ax}}{a} - \int x^{m-1} e^{ax} dx; m > 0$$

$$94. \int \frac{e^{ax}}{x^m} dx = -\frac{e^{ax}}{(m-1)x^{m-1}} + \frac{a}{m-1} \int \frac{e^{ax}}{x^{m-1}} dx; m > 1$$

$$95. \int e^{ax} \ln(x) dx = \frac{e^{ax} \ln(x)}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx$$

$$96. \int e^{ax} \operatorname{sen}(nx) dx = \frac{e^{ax} (a \operatorname{sen}(nx) - n \cos(nx))}{a^2 + n^2}$$

$$97. \int e^{ax} \cos(nx) dx = \frac{e^{ax} (a \cos(nx) + n \operatorname{sen}(nx))}{a^2 + n^2}$$

$$98. \int \frac{dx}{a + be^{ax}} = \frac{x}{a} - \frac{1}{aq} \ln(a + be^{ax})$$

$$99. \int \operatorname{senh}(x) dx = \operatorname{cosh}(x)$$

$$100. \int \operatorname{cosh}(x) dx = \operatorname{senh}(x)$$

$$101. \int \operatorname{tgh}(x) dx = \ln \operatorname{cosh}(x)$$

$$102. \int \operatorname{cot gh}(x) dx = \ln \operatorname{senh}(x)$$

$$103. \int \operatorname{sech}(x) dx = 2 \operatorname{tg}^{-1} e^x = \operatorname{tg}^{-1}(\operatorname{senh}(x))$$

$$104. \int \operatorname{cosech}(x) dx = \ln \left(\operatorname{tgh} \left(\frac{x}{2} \right) \right)$$

$$105. \int f(\operatorname{sen}(x))dx = \begin{cases} 2 \int f\left(\frac{2z}{1+z^2}\right) \frac{dz}{1+z^2}; z = \operatorname{tg}\left(\frac{x}{2}\right) \\ \int f(u) \frac{du}{\sqrt{1-u^2}}; u = \operatorname{sen}(x) \end{cases}$$

$$106. \int f(\cos(x))dx = \begin{cases} 2 \int f\left(\frac{1-z^2}{1+z^2}\right) \frac{dz}{1+z^2}; z = \operatorname{tg}\left(\frac{x}{2}\right) \\ - \int f(u) \frac{du}{\sqrt{1-u^2}}; u = \cos(x) \end{cases}$$

$$107. \int f(\operatorname{sen}(x), \cos(x))dx = \begin{cases} 2 \int f\left(\frac{2z}{1+z^2}, \frac{1-z^2}{1+z^2}\right) \frac{dz}{1+z^2}; z = \operatorname{tg}\left(\frac{x}{2}\right) \\ \int f(u, \sqrt{1-u^2}) \frac{du}{\sqrt{1-u^2}}; u = \operatorname{sen}(x) \end{cases}$$

$$108. \int_0^{\pi/2} \operatorname{sen}^n(x)dx = \int_0^{\pi/2} \cos^n(x)dx = \begin{cases} \frac{1}{2} \times \frac{3}{4} \times \dots \times \frac{n-1}{n} \times \frac{\pi}{2}; n = 2; 4; 6; \dots \\ \frac{2}{3} \times \frac{4}{5} \times \dots \times \frac{n-1}{n}; n = 3; 5; 7; \dots \end{cases}$$

$$109. \int_0^{\pi/2} \operatorname{sen}^n(x) \cos^2(x)dx = \begin{cases} \frac{1 \times 3 \times \dots \times (m-1) \times 1 \times 3 \times \dots \times (n-1)}{2 \times 4 \times \dots \times (m+n)} \times \frac{\pi}{2}; m, n = 2; 4; \dots \\ \frac{2 \times 4 \times \dots \times (m-1)}{(n+1) \times (n+3) \times \dots \times (n+m)}; m = 3; 5; 7; \dots, n = 1; 2; 3; \dots \\ \frac{2 \times 4 \times \dots \times (n-1)}{(m+1) \times (m+3) \times \dots \times (m+n)}; m = 1; 2; 3; \dots, n = 3; 5; 7; \dots \end{cases}$$

$$110. \int_{-\infty}^{+\infty} e^{-x^2/2} dx = \sqrt{2\pi}$$