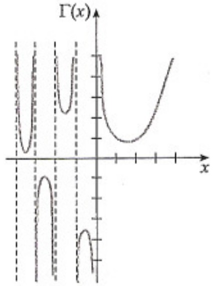

Figura A.1

Quando substituímos $t = u^2$, (3) pode ser escrita como

$$\Gamma(\tfrac{1}{2}) = 2\int_0^\infty e^{-u^2}du.$$

Mas

$$\int_0^\infty e^{-u^2}du = \int_0^\infty e^{v^{-2}}dv,$$

e daí

$$[\Gamma(\tfrac{1}{2})]^2 = \left(2\int_0^\infty e^{-u^2}du\right)\left(2\int_0^\infty e^{-v^2}dv\right)$$

$$= 4\int_0^\infty\int_0^\infty e^{-(u^2+v^2)}du\,dv.$$

Usando coordenadas polares $u = r\cos\theta$, $v = r\,\text{sen}\,\theta$, podemos calcular a integral dupla:

$$4\int_0^\infty\int_0^\infty e^{-(u^2+v^2)}du\,dv = 4\int_0^{\pi/2}\int_0^\infty e^{-r^2}r\,dr\,d\theta = \pi.$$

Logo

$$[\Gamma(\tfrac{1}{2})]^2 = \pi \quad \text{ou} \quad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}.$$

EXEMPLO 1

Calcule $\Gamma(-1/2)$.

Solução Segue-se de (2) que, com $x = -1/2$,

$$\Gamma\left(\frac{1}{2}\right) = -\frac{1}{2}\Gamma\left(-\frac{1}{2}\right).$$

Portanto

$$\Gamma\left(-\frac{1}{2}\right) = -2\Gamma\left(\frac{1}{2}\right) = -2\sqrt{\pi}.$$ ■

Apêndice I EXERCÍCIOS

As respostas dos exercícios selecionados estão na página 466.

1. Calcule **(a)** $\Gamma(5)$ **(b)** $\Gamma(7)$ **(c)** $\Gamma(-3/2)$ **(d)** $\Gamma(-5/2)$
2. Use (1) e o fato de que $\Gamma(6/5) = 0{,}92$ para calcular $\int_0^\infty x^5 e^{-x^5}dx$. [*Sugestão:* Faça $t = x^5$.]
3. Use (1) e o fato de que $\Gamma(5/3) = 0{,}89$ para calcular $\int_0^\infty x^4 e^{-x^3}dx$.
4. Calcule $\int_0^\infty x^3\left(\ln\frac{1}{x}\right)^3 dx$. [*Sugestão:* Faça $t = -\ln x$.]
5. Use o fato de que $\Gamma(x) > \int_0^1 t^{x-1}e^{-t}dt$ para mostrar que $\Gamma(x)$ é ilimitada quando $x \to 0^+$.
6. Use (1) para deduzir (2) quando $x > 0$.

II TRANSFORMADAS DE LAPLACE

	$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
1.	1	$\dfrac{1}{s}$
2.	t	$\dfrac{1}{s^2}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$, n um inteiro positivo
4.	$t^{-1/2}$	$\sqrt{\dfrac{\pi}{s}}$
5.	$t^{1/2}$	$\dfrac{\sqrt{\pi}}{2s^{3/2}}$
6.	t^α	$\dfrac{\Gamma(\alpha+1)}{s^{\alpha+1}}$, $\alpha > -1$
7.	$\text{sen}\, kt$	$\dfrac{k}{s^2+k^2}$
8.	$\cos kt$	$\dfrac{s}{s^2+k^2}$

	$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
9.	$\operatorname{sen}^2 kt$	$\dfrac{2k^2}{s(s^2+4k^2)}$
10.	$\cos^2 kt$	$\dfrac{s^2+2k^2}{s(s^2+4k^2)}$
11.	e^{at}	$\dfrac{1}{s-a}$
12.	$\operatorname{senh} kt$	$\dfrac{k}{s^2-k^2}$
13.	$\cosh kt$	$\dfrac{s}{s^2+k^2}$
14.	$\operatorname{senh}^2 kt$	$\dfrac{2k^2}{s(s^2-4k^2)}$
15.	$\cosh^2 kt$	$\dfrac{s^2-2k^2}{s(s^2-4k^2)}$
16.	te^{at}	$\dfrac{1}{(s-a)^2}$
17.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$, n um inteiro positivo
18.	$e^{at} \operatorname{sen} kt$	$\dfrac{k}{(s-a)^2+k^2}$
19.	$e^{at} \cos kt$	$\dfrac{s-a}{(s-a)^2+k^2}$
20.	$e^{at} \operatorname{senh} kt$	$\dfrac{k}{(s-a)^2-k^2}$
21.	$e^{at} \cosh kt$	$\dfrac{s-a}{(s-a)^2-k^2}$
22.	$t \operatorname{sen} kt$	$\dfrac{2ks}{(s^2+k^2)^2}$
23.	$t \cos kt$	$\dfrac{s^2-k^2}{(s^2+k^2)^2}$
24.	$\operatorname{sen} kt + kt \cos kt$	$\dfrac{2ks^2}{(s^2+k^2)^2}$

	$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
25.	$\operatorname{sen} kt - kt \cos kt$	$\dfrac{2k^3}{(s^2+k^2)^2}$
26.	$t \operatorname{senh} kt$	$\dfrac{2ks}{(s^2-k^2)^2}$
27.	$t \cosh kt$	$\dfrac{s^2+k^2}{(s^2-k^2)^2}$
28.	$\dfrac{e^{at}-e^{bt}}{a-b}$	$\dfrac{1}{(s-a)(s-b)}$
29.	$\dfrac{ae^{at}-be^{bt}}{a-b}$	$\dfrac{s}{(s-a)(s-b)}$
30.	$1 - \cos kt$	$\dfrac{k^2}{s(s^2+k^2)}$
31.	$kt - \operatorname{sen} kt$	$\dfrac{k^3}{s^2(s^2+k^2)}$
32.	$\dfrac{a \operatorname{sen} bt - b \operatorname{sen} at}{ab(a^2-b^2)}$	$\dfrac{1}{(s^2+a^2)(s^2+b^2)}$
33.	$\dfrac{\cos bt - \cos at}{a^2-b^2}$	$\dfrac{s}{(s^2+a^2)(s^2+b^2)}$
34.	$\operatorname{sen} kt \operatorname{senh} kt$	$\dfrac{2k^2 s}{s^4+4k^4}$
35.	$\operatorname{sen} kt \cosh kt$	$\dfrac{k(s^2+2k^2)}{s^4+4k^4}$
36.	$\cos kt \operatorname{senh} kt$	$\dfrac{k(s^2-2k^2)}{s^4+4k^4}$
37.	$\cos kt \cosh kt$	$\dfrac{s^3}{s^4+4k^4}$
38.	$J_0(kt)$	$\dfrac{1}{\sqrt{s^2+k^2}}$
39.	$\dfrac{e^{bt}-e^{at}}{t}$	$\ln \dfrac{s-a}{s-b}$

	$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
40.	$\dfrac{2(1 - \cos kt)}{t}$	$\ln \dfrac{s^2 + k^2}{s^2}$
41.	$\dfrac{2(1 - \cosh kt)}{t}$	$\ln \dfrac{s^2 - k^2}{s^2}$
42.	$\dfrac{\operatorname{sen} at}{t}$	$\operatorname{arctg}\left(\dfrac{a}{s}\right)$
43.	$\dfrac{\operatorname{sen} at \cos bt}{t}$	$\dfrac{1}{2}\operatorname{arctg}\dfrac{a+b}{s} + \dfrac{1}{2}\operatorname{arctg}\dfrac{a-b}{s}$
44.	$\dfrac{1}{\sqrt{\pi t}}\, e^{-a^2/4t}$	$\dfrac{e^{-a\sqrt{s}}}{\sqrt{s}}$
45.	$\dfrac{a}{2\sqrt{\pi t^3}}\, e^{-a^2/4t}$	$e^{-a\sqrt{s}}$
46.	$\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{s}$
47.	$2\sqrt{\dfrac{t}{\pi}}\, e^{-a^2/4t} - a\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{s\sqrt{s}}$
48.	$e^{ab}e^{b^2t}\operatorname{erfc}\left(b\sqrt{t} + \dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{\sqrt{s}(\sqrt{s} + b)}$
49.	$-e^{ab}e^{b^2t}\operatorname{erfc}\left(b\sqrt{t} + \dfrac{a}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{be^{-a\sqrt{s}}}{s(\sqrt{s} + b)}$
50.	$\delta(t)$	1
51.	$\delta(t - t_0)$	e^{-st_0}
52.	$e^{at}f(t)$	$F(s - a)$
53.	$f(t - a)\,\mathscr{U}(t - a)$	$e^{-as}F(s)$
54.	$\mathscr{U}(t - a)$	$\dfrac{e^{-as}}{s}$
55.	$f^{(n)}(t)$	$s^nF(s) - s^{(n-1)}f(0) - \ldots - f^{(n-1)}(0)$
56.	$t^nf(t)$	$(-1)^n \dfrac{d^n}{ds^n}F(s)$
57.	$\displaystyle\int_0^t f(\tau)g(t - \tau)\, d\tau$	$F(s)G(s)$

III REVISÃO DE DETERMINANTES

O determinante de uma matriz **A** 2×2 é definido por

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

Menores e Cofatores

Para uma matriz **A** $n \times n$, seja a_{ij} a entrada na i-ésima linha e j-ésima coluna. O **menor** M_{ij} associado a a_{ij} é o determinante da matriz $(n - 1) \times (n - 1)$ obtida eliminando-se a i-ésima linha e a j-ésima coluna da matriz. O **cofator** C_{ij} associado a a_{ij} é um *assinalado*, especificamente

$$\mathbf{C_{ij} = (-1)^{i+j}M_{ij}}.$$

EXEMPLO 1

Os cofatores da matriz 3×3

$$\begin{pmatrix} 2 & 4 & 7 \\ 1 & 2 & 3 \\ 1 & 5 & 3 \end{pmatrix} \qquad \textbf{(1)}$$

são

$$C_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 2 & 3 \\ 5 & 3 \end{vmatrix} = -9 \qquad C_{12} = (-1)^{1+2}M_{12} = (-1)\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 0 \qquad C_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} = 3$$

$$C_{21} = (-1)^{2+1}M_{21} = (-1)\begin{vmatrix} 4 & 7 \\ 5 & 3 \end{vmatrix} = 23 \qquad C_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 2 & 7 \\ 1 & 3 \end{vmatrix} = -1 \qquad C_{23} = (-1)^{2+3}M_{23} = (-1)\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} = -6$$

$$C_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} 4 & 7 \\ 2 & 3 \end{vmatrix} = -2 \qquad C_{32} = (-1)^{3+2}M_{32} = (-1)\begin{vmatrix} 2 & 7 \\ 1 & 3 \end{vmatrix} = 1 \qquad C_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 0.$$

■

Cálculo do Determinante por Cofatores

Pode ser provado que um *determinante* pode ser calculado em termos de cofatores: