Universidade Federal do Paraná

Setor de Ciências Exatas

Departamento de Matematica

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3^{ra} **prova de cálculo II**

Curitiba, 1 de Junho de 2011

1. Determinar o volume do sólido delimitado pelos cilindros $z = x^2$, $y = x^2$ e pelos planos $z = 0$, $y = 4$. Esboce o sólido.

2. Encontre o volume da região no primeiro octante limitada pelos planos coordenados, pelo plano $x + y = 4$ e pelo cilindro $y^2 + 4z^2 = 16$. Esboce o região.

3. Use uma integral tripla para calcular o volume do sólido compreendido entre os parabolóides $z = 5x^2 + 5y^2$ e $z = 6 - x^2 - y^2$. Esboce o sólido.

4. Encontre o volume da região cortada do cilindro elíptico sólido $x^2 + 4y^2 \leq 4$ pelo plano xy e pelo plano $z + x = 2$. Esboce o sólido.

5. Calcule a massa do sólido que tem o formato da região

$$\mathcal{S} = \{(x, y, z) \in \mathbb{R}^3;\ \ \frac{x^2}{4} + \frac{y^2}{9} + z^2 \leq 2z\ \}$$

e cuja densidade é $\rho(x, y, z) = z$. Esboce o sólido $\mathcal{S}$.

BOA SORTE !

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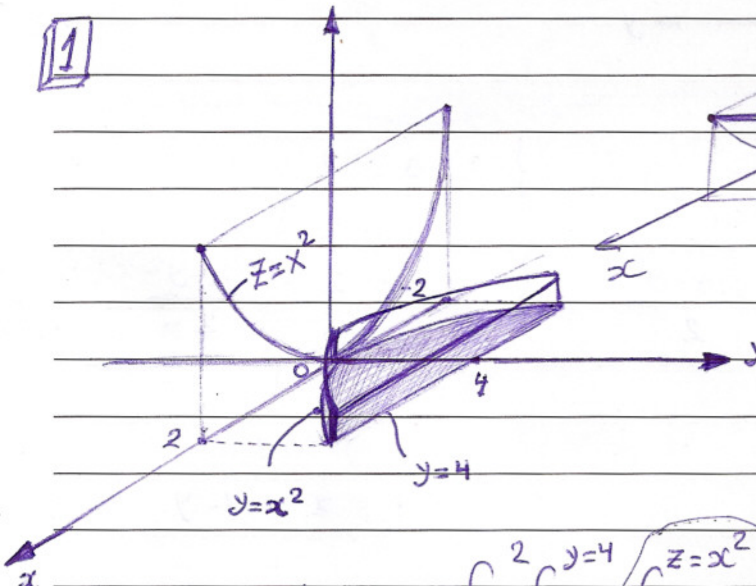
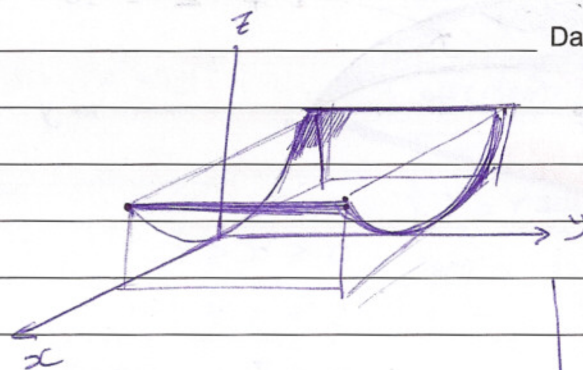
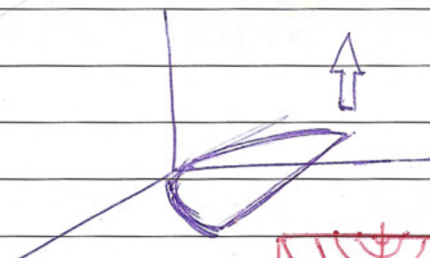
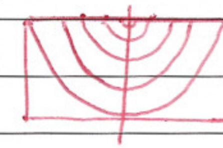
$\boxed{1}$









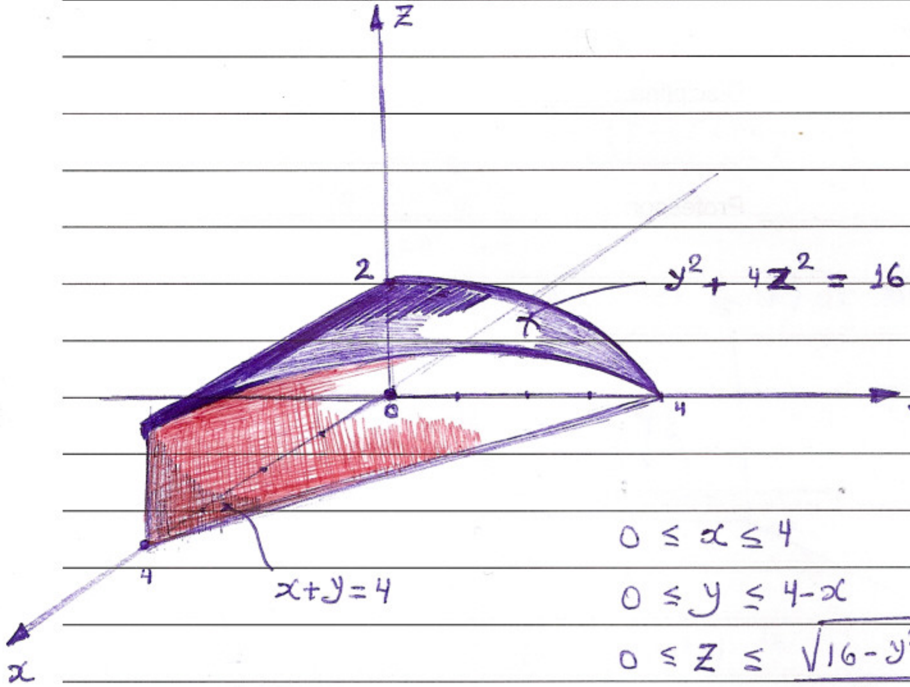
$$Vol(S) = 2\int_0^2 \int_{y=x^2}^{y=4} \int_{z=0}^{z=x^2} dz \, dy \, dx$$

$$= 2\int_0^2 \int_{y=x^2}^{y=4} x^2 \, dy \, dx = 2\int_0^2 x^2(4-x^2)\, dx$$

$$= 2\int_0^2 (4x^2 - x^4)\, dx = 2\left(\frac{4}{3}x^3 - \frac{x^5}{5}\right)\Bigg|_{x=0}^{x=2}$$

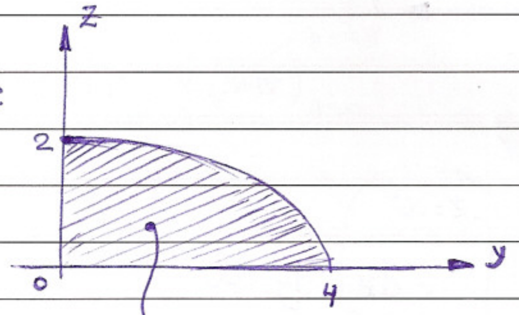
$$= 2\left(\frac{32}{3} - \frac{32}{5}\right) = 2\left(\frac{64}{15}\right) = \frac{128}{15}$$

Sólido no 1º octante.



$$0 \le x \le 4 \qquad\qquad 0 \le y \le 4$$
$$0 \le y \le 4-x \qquad\qquad 0 \le x \le 4-y$$
$$0 \le z \le \frac{\sqrt{16-y^2}}{2} \qquad\qquad 0 \le z \le \frac{\sqrt{16-y^2}}{2}$$

(y,z) na elipse



$$\frac{y^2}{16} + \frac{z^2}{4} = 1$$

$$0 \le x \le 4-y$$

ou

$$\boxed{0 \le x \le 4 - 4r\cos\theta}$$

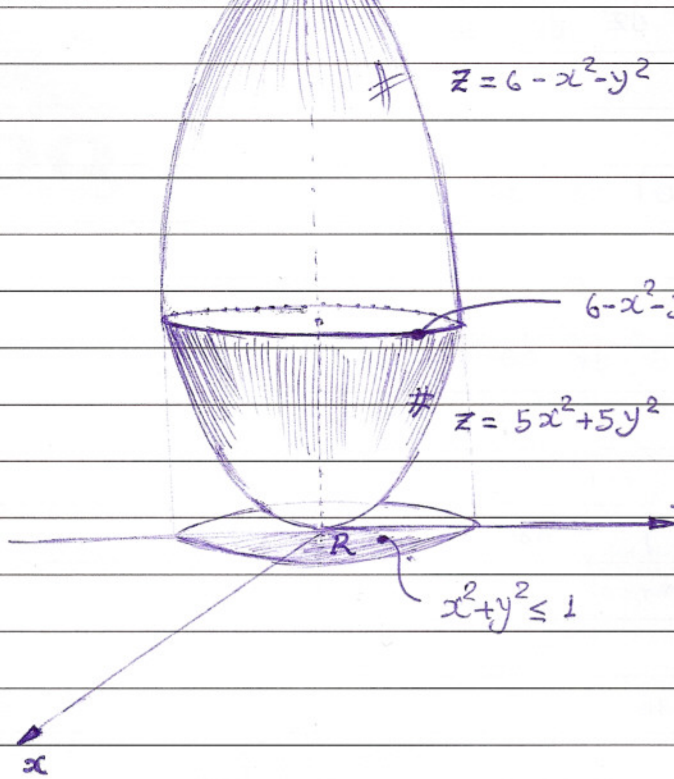
$$y = 4r\cos\theta \qquad 0 \le r \le 1$$
$$z = 2r\sin\theta \qquad 0 \le \theta \le \pi/2$$

$$\mathrm{Vol}(S) = \int_0^{\pi/2}\int_0^1 (4-4r\cos\theta)(4\times 2\times r)\,dr\,d\theta = \int_0^{\pi/2}\int_0^1 4(1-r\cos\theta)\times 8\times r\,dr\,d\theta$$

$$= 32\int_0^{\pi/2}\left(\int_0^1 (r - r^2\cos\theta)\,dr\right)d\theta = 32\int_0^{\pi/2}\left(\frac{r^2}{2} - \frac{r^3}{3}\cos\theta\right)_{r=0}^{r=1} d\theta$$

$$= 32\int_0^{\pi/2}\left(\frac{1}{2} - \frac{1}{3}\cos\theta\right)d\theta = 16\left(\frac{\pi}{2}\right) - \frac{32}{3}\left(\sin\theta\right)_{\theta=0}^{\theta=\pi/2}$$

$$= 8\pi - \frac{32}{3}$$



$6 - x^2 - y^2 = 5x^2 + 5y^2 \Rightarrow x^2 + y^2 = 1$

Coordenadas cilíndricas.

$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$ $\quad 0 \le r \le 1$, $\quad 0 \le \theta \le 2\pi$

com a variável z:

$5x^2 + 5y^2 \le z \le 6 - x^2 - y^2$

substituindo x, y, obtemos:

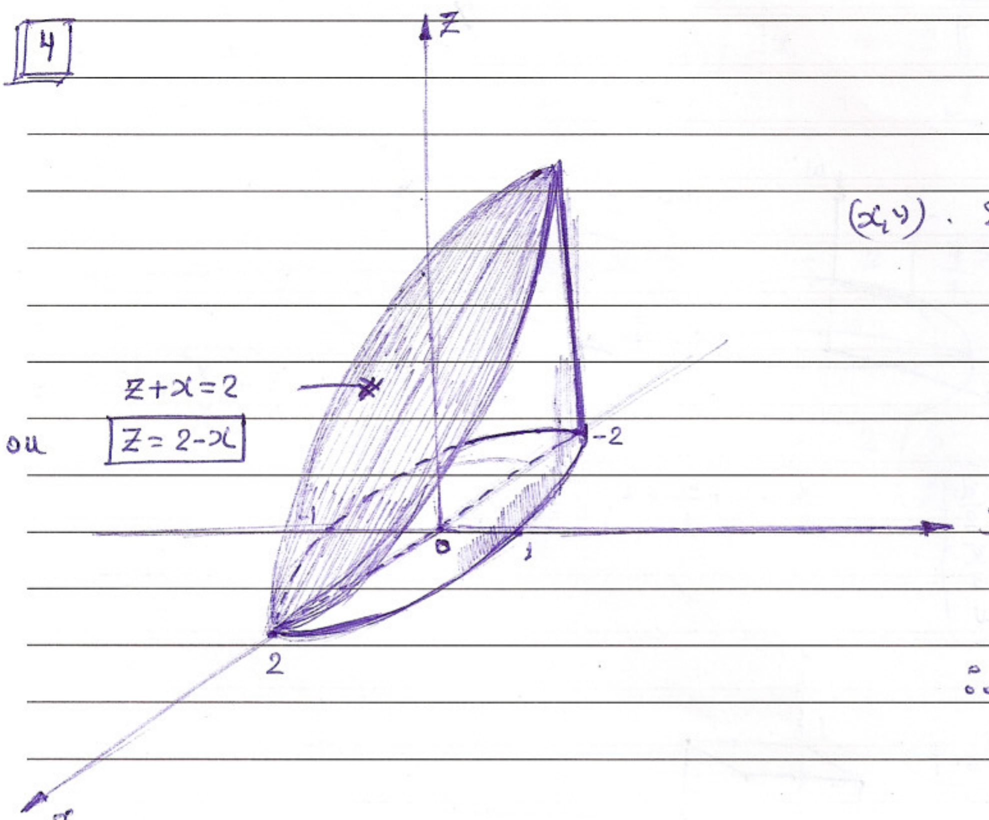
$\boxed{5r^2 \le z \le 6 - r^2}$

$$\text{Vol}(S) = \int_0^{2\pi}\int_0^1 (6 - r^2 - 5r^2)\, r \, dr\, d\theta$$

$$= 2\pi \int_0^1 (6r - 6r^3)\, dr = 2\pi \left(3r^2 - \frac{6r^4}{4}\right)\Bigg|_{r=0}^{r=1}$$

$$= 2\pi\left(3 - \frac{3}{2}\right) = 2\pi\left(\frac{3}{2}\right) = 3\pi$$

$\boxed{4}$



(x, y). satisfaz: $\dfrac{x^2}{4} + y^2 \le 1$

$\therefore$ $\boxed{\begin{array}{l} x = 2r\cos\theta \\ y = r\,\text{sen}\,\theta \end{array}}$

com $0 \le r \le 1$

$0 \le \theta \le 2\pi$

$0 \le z \le 2 - x$

$\therefore$ $\boxed{0 \le z \le 2 - 2r\cos\theta}$

$$Vol(S) = \int_0 \int_0 \int_{z=0} (2\times 1\times r)\, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 2r\,(2-2r\cos\theta)\, dr\, d\theta.$$

$$= \int_0^{2\pi}\int_0^1 \left(4r - 4r^2\cos\theta\right) dr\, d\theta.$$

$$= \int_0^{2\pi} \left(2r^2 - \frac{4}{3} r^3 \cos\theta\right)\Bigg|_{r=0}^{r=1} d\theta$$

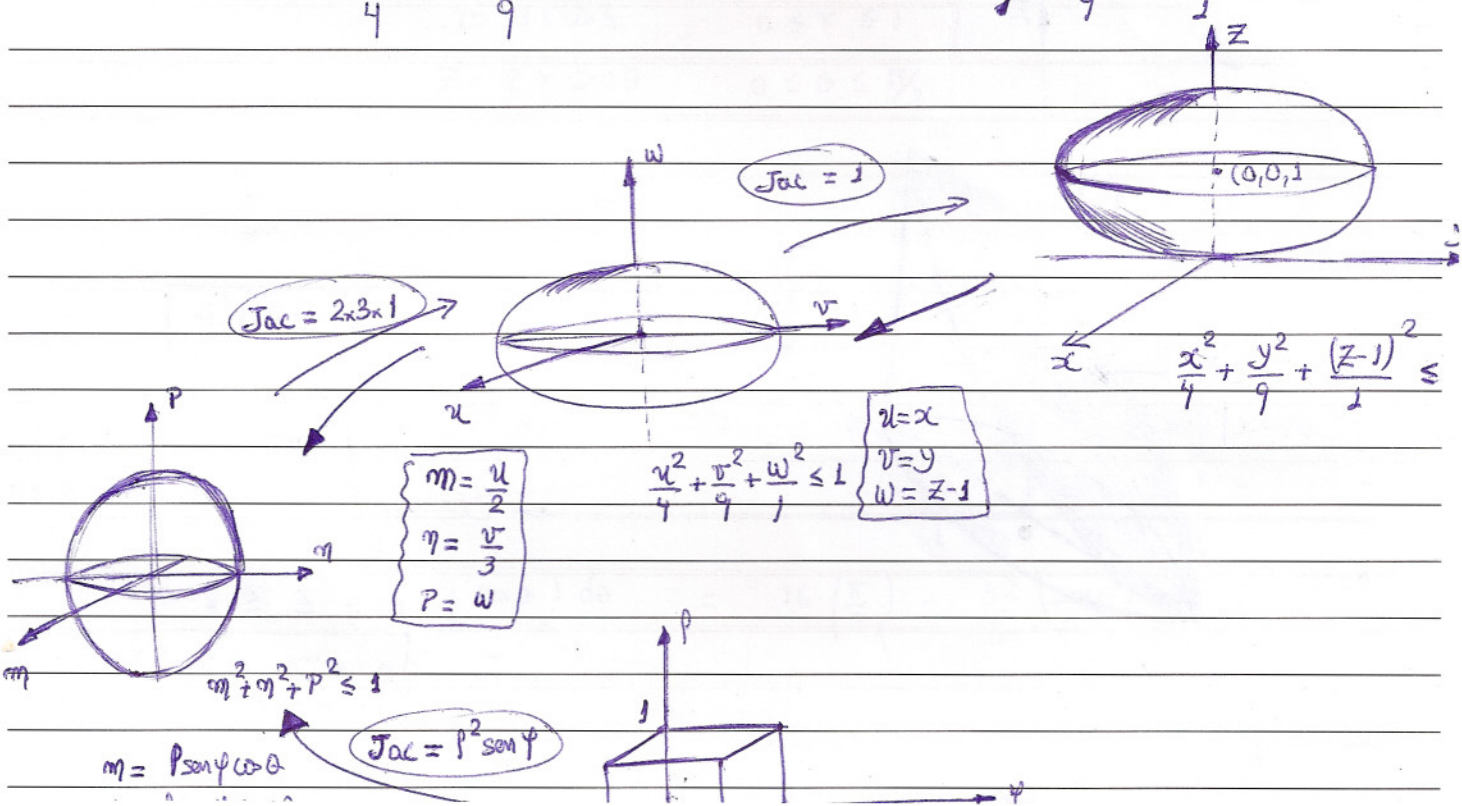
$$= \int_0^{2\pi} \left(2 - \frac{4}{3}\cos\theta\right) d\theta$$

$$= 2(2\pi) - \cancel{\frac{4}{3}\left(\operatorname{sen}\theta\right)\Big|_{\theta=0}^{\theta=2\pi}}^{\,0}$$

$$= 4\pi$$

$\boxed{5}$

Sólido $\quad \dfrac{x^2}{4} + \dfrac{y^2}{9} + z^2 \le 2z \quad$ ou $\quad \dfrac{x^2}{4} + \dfrac{y^2}{9} + \dfrac{(z-1)^2}{1} \le 1$



$x = u \longrightarrow x = 2m \longrightarrow x = 2\rho \operatorname{sen}\varphi \cos\theta$

$y = v \longrightarrow y = 3n \longrightarrow y = 3\rho \operatorname{sen}\varphi \operatorname{sen}\theta$

$z = w+1 \longrightarrow z = p+1 \longrightarrow z = \rho \cos\varphi + 1$

$$\left| \frac{\partial(x,y,z)}{\partial(\theta,\varphi,\rho)} \right| = \left| (1)(2\times 3\times 1)\, \rho^2 \operatorname{sen}\varphi \right| = 6\rho^2 \operatorname{sen}\varphi.$$

$$\text{MASSA} = \int_0^{2\pi}\int_0^{\pi}\int_0^{1} (\rho\cos\varphi + 1)\,(6\rho^2 \operatorname{sen}\varphi)\; d\rho\; d\varphi\; d\theta$$

$$= 2\pi \int_0^{\pi}\int_0^{1} \left(6\rho^3 \operatorname{sen}\varphi\cos\varphi + 6\rho^2 \operatorname{sen}\varphi\right) d\rho \quad d\varphi$$

$$= 12\pi \int_0^{\pi} \left(\frac{\rho^4}{4} \operatorname{sen}\varphi\cos\varphi + \frac{\rho^3}{3}\operatorname{sen}\varphi\right)\Bigg|_{\rho=0}^{\rho=1} d\varphi$$

$$= 12\pi \int_0^{\pi} \left(\frac{\operatorname{sen}\varphi\cos\varphi}{4} + \frac{\operatorname{sen}\varphi}{3}\right) d\varphi.$$

$$= 12\pi \cancel{\left(\frac{\operatorname{sen}^2\varphi}{8}\right)\Bigg|_{\varphi=0}^{\varphi=\pi}}^{\,0} + 12\pi \left(-\frac{\cos\varphi}{3}\right)\Bigg|_{\varphi=0}^{\varphi=\pi}$$

$$= -\frac{12\pi}{3}(-1-1)$$

$$= \frac{-12\pi}{3}(-2)$$

$$= \frac{24\pi}{3}$$

$$= 8\pi$$