

### EXERCICIO 1

**Example 2.2 (Diet Problem)** . A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in table 2.2.

TABLE 2.2

Food type	Yield per unit			Cost per unit (Rs.)
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Formulate linear programming model for the problem.

### EXERCICIO 2

**Example 2.3 (Blending Problem)**. A firm produces an alloy having the following specifications:

- (i) specific gravity  $\leq 0.98$ ,
- (ii) chromium  $\geq 8\%$ ,
- (iii) melting point  $\geq 450^\circ\text{C}$ .

Raw materials A, B and C having the properties shown in the table can be used to make the alloy.

TABLE 2.3

Property	Properties of raw material		
	A	B	C
Specific gravity	0.92	0.97	1.04
Chromium	7%	13%	16%
Melting point	440°C	490°C	480°C

Costs of the various raw materials per ton are: Rs. 90 for A, Rs. 280 for B and Rs. 40 for C. Formulate the L.P. model to find the proportions in which A, B and C be used to obtain an alloy of desired properties while the cost of raw materials is minimum.

[P.U.B.E. (E. and Ec.) 1998]

### EXERCÍCIO 3

**Example 2.6 (Product Mix Problem).** A chemical company produces two products, X and Y. Each unit of product X requires 3 hours on operation I and 4 hours on operation II, while each unit of product Y requires 4 hours on operation I and 5 hours on operation II. Total available time for operations I and II is 20 hours and 26 hours respectively. The production of each unit of product Y also results in two units of a by-product Z at no extra cost.

Product X sells at profit of Rs. 10/unit, while Y sells at profit of Rs. 20/unit. By-product Z brings a unit profit of Rs. 6 if sold; in case it cannot be sold, the destruction cost is Rs. 4/unit. Forecasts indicate that not more than 5 units of Z can be sold. Formulate L.P. model to determine the quantities of X and Y to be produced, keeping Z in mind, so that the profit earned is maximum.

[P.U.B. Com. April, 2006; Jammu U.B.E. (Mech.) 2004; P.T.U.B.Tech. 2000; R.E.C. Hamirpur, 1998]

### EXERCÍCIO 4

**Example 2.11 (Production Planning Problem).** A company manufacturing air coolers has, at present, firm orders for the next 6 months. The company can schedule its production over the next 6 months to meet orders on either regular or overtime basis. The order size and production costs over the next six months are as follows:

Month	1	2	3	4	5	6
Orders	640	660	700	750	550	650
Cost/unit (Rs.) for regular production	40	42	41	45	39	40
Cost/unit (Rs.) for overtime production	52	50	53	50	45	43

With 100 air coolers in stock at present, the company wishes to have at least 150 air coolers in stock at the end of 6 months. The regular and overtime production in each month is not to exceed 600 and 400 units respectively. The inventory carrying cost for air coolers is Rs. 12 per unit per month. Formulate the L.P. model to minimize the total cost.

### EXERCÍCIO 5

**Example 2.12 (Transportation Problem).** A dairy firm has two milk plants with daily milk production of 6 million litres and 9 million litres respectively. Each day the firm must fulfil the needs of its three distribution centres which have milk requirement of 7, 5 and 3 million litres respectively. Cost of shipping one million litres of milk from each plant to each distribution centre is given, in hundreds of rupees below. Formulate the L.P. model to minimize the transportation cost.

		Distribution Centres			Supply
		1	2	3	
Plants	1	2	3	11	6
	2	1	9	6	9
Demand		7	5	3	

**Formulation of L.P. Model.** Key decision is to determine the quantity of milk to be transported from either plant to each distribution centre.

## EXERCÍCIO 6

**Example 2.8 (Trim Loss Problem).** A paper mill produces rolls of paper used in making cash registers. Each roll of paper is 100m in length and can be used in widths of 3, 4, 6 and 10cm

The company's production process results in rolls that are 24 cm in width. Thus the company must cut its 24cm roll to the desired widths. It has six basic cutting alternatives as follows:

Cutting alternatives	Width of rolls (cm)				Waste (cm)
	3	4	6	10	
1	4	3	—	—	—
2	—	3	2	—	—
3	1	1	1	1	1
4	—	—	2	1	2
5	—	4	1	—	2
6	3	2	1	—	1

The minimum demand for the four rolls is as follows:

Roll width (cm)	Demand
2	2,000
4	3,600
6	1,600
10	500

The paper mill wishes to minimize the waste resulting from trimming to size. Formulate the L.P. model. [P.U.B. Com. April, 2006]

## EXERCÍCIO 7

**Example 2.22 (Flight Scheduling Problem).** An aircraft company, which operates out of a central terminal has 8 aircrafts of Type I, 15 aircrafts of Type II and 12 aircrafts of Type III available for today's flights. The tonnage capacities (in thousands of tons) are 4.5 for Type I, 1.7 for Type II and 4 for Type III.

The company dispatches its planes to cities A and B. Tonnage requirements (in thousands of tons) are 20 at city A and 30 at city B; excess tonnage capacity supplied to a city has no value. A plane can fly once only during the day.

The cost of sending a plane from the terminal to each city is given by the following table:

	Type I	Type II	Type III
City A	23	5	1.4
City B	58	10	3.8

Formulate the LP model to minimize the air transportation cost.

## EXERCÍCIO 8

**Example 2.33 (Trim Loss Problem).** A manufacturer of cylindrical containers receives tin sheets in widths of 30 cm and 60 cm respectively. For these containers the sheets are to be cut to three different widths of 15 cm, 21 cm and 27 cm respectively. The number of containers to be manufactured from these three widths are 400, 200 and 300 respectively. The bottom plates and top covers of the containers are purchased directly from the market. There is no limit on the lengths of standard tin sheets. Formulate the L.P. model for the production schedule that minimizes the trim losses.

The possible cutting combinations (plans) for both types of sheets are shown in the table below.

Width (cm)	$i = 1$ (30cm)			$i = 2$ (60 cm)					
	$x_{11}$	$x_{12}$	$x_{13}$	$x_{21}$	$x_{22}$	$x_{23}$	$x_{24}$	$x_{25}$	$x_{26}$
15	2	0	0	4	2	2	1	0	0
21	0	1	0	0	1	0	2	1	0
27	0	0	1	0	0	1	0	1	2
Trim loss (cm)	0	9	3	0	9	3	3	12	6

### EXERCÍCIO 9

**Example 2.35 (Production Scheduling Problem).** A company wants to plan the next week's production of its three products A, B and C. These products are made on three machines—lathes, drills and grinders. Time available on lathes, drills and grinders for the next week is 200 hrs., 250 hrs. and 300 hrs. respectively. The products can be made through different alternative routes shown in the table below. The products sell in the market at Rs. 20, Rs. 15 and Rs. 25 per unit respectively.

- Formulate the L.P. model assuming unlimited market demand for the products.
- There is a fixed order (that has to be satisfied) of 250 units of A, 200 units of B and 150 units of C.

The customer pays Rs. 20, Rs. 15 and Rs. 25 per unit of products A, B and C in the fixed order and is willing to pay Rs. 15, Rs. 10 and Rs. 20 per unit for the extra units of A, B and C respectively. Construct the model that maximizes the sales revenue.

- If not more than 200 units of C can be sold in the market, what modifications would be required in the model?

TABLE 2.15

Machine	Product A			Product B		Product C			Machine hours available
	Route			Route		Route			
	1	2	3	1	2	1	2	3	
Lathes	0.5	0.7	0.3	—	0.5	0.6	0.5	0.3	200
Drills	0.5	0.3	0.2	0.4	0.3	0.7	0.4	0.1	250
Grinders	0.6	0.4	0.6	0.7	0.5	0.4	0.3	—	300

### EXERCÍCIO 10

4. A small manufacturer employs 5 skilled men and 10 semi-skilled men and makes an article in two qualities, a deluxe model and an ordinary model. The making of a deluxe model requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. The ordinary model requires 1 hour work by a skilled man and 3 hours work by a semi-skilled man. By union rules no man can work more than 8 hours per day. The manufacturer's clear profit of the deluxe model is Rs. 10 and of the ordinary model Rs. 8. Formulate the model of the problem. [NIIFT Mohali, 2000, 01]

### EXERCÍCIO 11

7. The manager of an oil refinery has to decide upon the optimal mix of two possible blending processes, of which the inputs and outputs per production run are as follows:

Process	Input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amounts available of crude A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 3 and Rs. 4 respectively. Formulate the problem as a linear programming problem.

[P.U. MBA, 2000; Karn. U.B. Tech. (Mech.) May, 1989; P.U.B.E. (Mech.) 1977; August, 1978; Nov., 1980, May, 1982, 1995; B.Com. Sept., 2005]

### EXERCÍCIO 12

10. A farmer has a 100-acre farm. He can sell all the tomatoes, lettuce or radishes he can raise. The price he can obtain is Re. 1 per kg for tomatoes, Re. 0.75 a head for lettuce and Rs. 2 per kg for radishes. The average yield per acre is 2,000 kg of tomatoes, 3,000 heads of lettuce and 1,000 kg of radishes. Fertilizer is available at Re. 0.50 per kg and the amount required per acre is 100 kg each for tomatoes and lettuce and 50kg for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour are available at Rs. 20 per man-day.

Formulate an L.P. model for this problem in order to maximize the farmer's total profit.

[Nagpur U.B.E. 2003; Jammu U.B.E. (Mech.) 2004; Delhi M.B.A. 1976; P.U. B.E. (Mech.) May, 1984; Nov., 2002]

### EXERCÍCIO 13

12. The financial secretary of a firm wants to invest a sum of Rs. 10,000 so as to maximize its yield. He has the following alternatives:

Investment type	A <sub>1</sub>	A <sub>2</sub>	B <sub>1</sub>	B <sub>2</sub>	C <sub>1</sub>	C <sub>2</sub>
Yield	3%	2.5%	3.5%	4%	5%	4.5%

It is the firm's policy that at least 40% of the whole amount be invested in units of type A and not more than 35% in any of the other two types. Make a model for the investment plan if the whole of Rs. 10,000 is to be invested.

[P.T.U. M.B.A. May 2002; P.U.B.E. (Mech.) 1983, 2001]

### **EXERCÍCIO 14**

13. A truck company requires the following number of drivers for its trucks during 24 hours:

<i>Time</i>	<i>No. required</i>
00 - 04 hr.	5
04 - 08 hr.	10
08 - 12 hr.	20
12 - 16 hr.	12
16 - 20 hr.	22
20 - 24 hr.	8

According to the shift schedule a driver may join for duty at midnight, 04, 08, 12, 16, 20 hours and work continuously for 8 hours. Formulate the problem as L.P. problem for optimal shift plan.

[P.U.B.E. (Mech.) 2002, 1983; B.E.(Elect.) Oct., 1993]

### **EXERCÍCIO 15**

Considere o problema de construir pilares de edificação, a qual é constituída basicamente por oito atividades que estão relacionadas na tabela abaixo, juntamente com a descrição dessas atividades, a duração e os predecessores imediatos de cada uma. Formule o modelo de programação linear que determine o menor tempo necessário para que todas as oito atividades sejam concluídas.

**Tabela 2.9**  
**Dados das atividades do projeto.**

<i>Atividades</i>	<i>Descrição</i>	<i>Predecessor imediato</i>	<i>Duração (horas)</i>
<b>A</b>	<i>Preparo da armadura</i>	—	6
<b>B</b>	<i>Preparo da forma</i>	—	5
<b>C</b>	<i>Lançamento da armadura</i>	<b>A</b>	4
<b>D</b>	<i>Lançamento da forma</i>	<b>B, C</b>	2
<b>E</b>	<i>Providências para concretagem</i>	—	2
<b>F</b>	<i>Aplicação do concreto</i>	<b>E, D</b>	3
<b>G</b>	<i>Cura do concreto</i>	<b>F</b>	72
<b>H</b>	<i>Desforma do pilar</i>	<b>G</b>	3

[Adaptado de: Arenales, Marcos, Vinicius Armentano, and Reinaldo Morabito. *Pesquisa operacional: para cursos de engenharia*. 2007]

### **EXERCÍCIO 16**

- 1) **(Production Allocation Problem)** A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.

Machine	Time per unit (minutes)			Machine Capacity (minutes/day)
	Product 1	Product 2	Product 3	
M <sub>1</sub>	2	3	2	440
M <sub>2</sub>	4	-	3	470
M <sub>3</sub>	2	5	-	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is Rs. 4, Rs. 3 and Rs. 6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (L.P.) model that will maximize the daily profit.

[H.P.U. MCA 1999]