EXERCÍCIO 1

Formulation of L.P. Model. Let x_1, x_2, x_3 and x_4 denote the number of units of food of type 1, 2, 3 and 4 respectively.

Objective is to minimize the cost *i.e.*,

$$\text{Minimize } Z = \text{Rs. } (45x_1 + 40x_2 + 85x_3 + 65x_4).$$

Constraints are on the fulfilment of the daily requirements of the various constituents.

i.e., for proteins, $3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800,$

for fats, $2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200,$

and for carbohydrates, $6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700,$

where x_1, x_2, x_3, x_4, each ≥ 0.

EXERCÍCIO 2

Formulation of Linear Programming Model. Let the *percentage contents* of raw materials A, B and C to be used for making the alloy be x_1, x_2 and x_3 respectively.

Objective is to minimize the cost

i.e., $\quad$ minimize $Z = 90x_1 + 280x_2 + 40x_3$.

Constraints are imposed by the specifications required for the alloy. They are

$$0.92x_1 + 0.97x_2 + 1.04x_3 \leq 0.98,$$

$$7x_1 + 13x_2 + 16x_3 \geq 8,$$

$$440x_1 + 490x_2 + 480x_3 \geq 450,$$

and $\quad x_1 + x_2 + x_3 = 100,$

as x_1, x_2 and x_3 are the percentage contents of materials A, B and C in making the alloy.

Also $\quad x_1, x_2, x_3$, each ≥ 0.

EXERCÍCIO 3

Formulation of L.P..Model. Let the number of units of products X, Y and Z produced be x_1, x_2, x_z, where

x_z = number of units of Z produced

= number of units of Z sold + number of units of Z destroyed

= $x_3 + x_4$ (say).

Objective is to maximize the profit. Objective function (profit function) for products X and Y is linear because their profits (Rs. 10/unit and Rs. 20/unit) are constants irrespective of the number of units produced. A graph between the total profit and quantity produced will be a straight line. However, a similar graph for product Z is non-linear since it has slope +6 for first part, while a slope of –4 for the second. However, it is piece-wise linear, since it is linear in the regions (0 to 5) and (5 to 2Y). Thus splitting x_z into two parts, *viz*. the number of units of Z sold (x_3) and number of units of Z destroyed (x_4) makes the objective function for product Z also linear.

Thus the objective function is

$$\text{maximize } Z = 10x_1 + 20x_2 + 6x_3 - 4x_4.$$

Constraints are

on the time available on operation I: $3x_1 + 4x_2 \leq 20$,

on the time available on operation II: $4x_1 + 5x_2 \leq 26$,

on the number of units of product Z sold: $x_3 \leq 5$,

on the number of units of product Z produced: 2Y = Z

or $2x_2 = x_3 + x_4$ or $-2x_2 + x_3 + x_4 = 0$,

where x_1, x_2, x_3, x_4, each ≥ 0.

EXERCÍCIO 4

Formulation of L.P. Model. *Key decision* is to determine the number of units of air coolers to be produced on regular as well as overtime basis together with the number of units of ending inventory in each of the six months.

Let x_{ij} be the number of units produced in month j (j = 1, 2, ..., 6), on a regular or overtime basis (i = 1, 2). Further let y_j represent the number of units of ending inventory in month j (j = 1, 2, ..., 6).

Objective is to minimize the total cost (of production and inventory carrying).

i.e., minimize $Z = (40x_{11} + 42x_{12} + 41x_{13} + 45x_{14} + 39x_{15} + 40x_{16}) + (52x_{21} + 50x_{22} + 53x_{23} + 50x_{24} + 45x_{25} + 43x_{26}) + 12(y_1 + y_2 + y_3 + y_4 + y_5 + y_6)$.

Constraints are

for the first month, $100 + x_{11} + x_{21} - 640 = y_1$,

for the second month, $y_1 + x_{12} + x_{22} - 660 = y_2$,

for the third month, $y_2 + x_{13} + x_{23} - 700 = y_3$,

for the fourth month, $y_3 + x_{14} + x_{24} - 750 = y_4$,

for the fifth month, $y_4 + x_{15} + x_{25} - 550 = y_5$,

and for the sixth month, $y_5 + x_{16} + x_{26} - 650 = y_6$.

Also, the ending inventory constraint is

$$y_6 \geq 150.$$

Further, since regular and overtime production each month is not to exceed 600 and 400 units respectively,

$x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}$, each ≤ 600,

and $x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}$, each ≤ 400.

Also $x_{ij} \geq 0$ (i = 1, 2 ; j = 1, 2,..., 6), $y_j \geq 0$.

EXERCÍCIO 5

Formulation of L.P. Model. *Key decision* is to determine the quantity of milk to be transported from either plant to each distribution centre.

Let x_1, x_2 be the quantity of milk (in million litres) transported from plant 1 to distribution centre no. 1 and 2 respectively. The resulting table representing transportation of milk is shown below.

		Distribution Centres 1	2	3	*Supply*
Plants	1	x_1	x_2	$6 - x_1 - x_2$	6
	2	$7 - x_1$	$5 - x_2$	$9 - (7 - x_1) - (5 - x_2)$	9
Demand		7	5	3	

Objective is to minimize the transportation cost.

i.e., $$\text{minimize } Z = 2x_1 + 3x_2 + 11(6 - x_1 - x_2) + (7 - x_1) + 9(5 - x_2) + 6[9 - (7 - x_1) - (5 - x_2)] = 100 - 4x_1 - 11x_2.$$

Constraints are

$$6 - x_1 - x_2 \geq 0 \quad \text{or} \quad x_1 + x_2 \leq 6,$$
$$7 - x_1 \geq 0 \quad \text{or} \quad x_1 \leq 7,$$
$$5 - x_2 \geq 0 \quad \text{or} \quad x_2 \leq 5,$$

and $9 - (7 - x_1) - (5 - x_2) \geq 0$ or $x_1 + x_2 \geq 3$,

where $x_1, x_2 \geq 0$.

OBS: Esse exercício pode ser resolvido de outra maneira.

Dica:

	Distribution Centre 1	2	3	Supply
Plants	X11	X12	X13	6
	X21	X22	X23	9
Demand	7	5	3	

EXERCÍCIO 6

Formulation of L.P. Model. *Key decision* is to determine how the paper rolls be cut to the required widths so that trim losses (wastage) are minimum.

Let x_j ($j = 1, 2, ..., 6$) represent the number of times each cutting alternative is to be used. These alternatives result/do not result in certain trim loss.

Objective is to minimize the trim losses.

i.e., minimize $Z = x_3 + 2x_4 + 2x_5 + x_6$.

Constraints are on the market demand for each type of roll width:

For roll width of 3cm, $4x_1 + x_3 + 3x_6 \geq 2{,}000$,

for roll width of 4 cm, $3x_1 + 3x_2 + x_3 + 4x_5 + 2x_6 \geq 3{,}600$,

for roll width of 6cm, $2x_2 + x_3 + 2x_4 + x_5 + x_6 \geq 1{,}600$,

and for roll width of 10cm, $x_3 + x_4 \geq 500$.

Since the variables represent the number of times each alternative is to be used, they can not have negative values.

$\therefore x_1, x_2, x_3, x_4, x_5, x_6$, each ≥ 0.

EXERCÍCIO 7

Formulation of L.P. Model. The data given in the problem is shown in the table below. The *key decision* is to determine the number of each type of plane to be dispatched to either of cities *A* and *B*. Let x_{A1}, x_{A2}, x_{A3} and x_{B1}, x_{B2}, x_{B3} denote the number of planes of type I, II and III to be dispatched to cities *A* and *B* respectively.

Aircraft type:	*I*	*II*	*III*
Number:	8	15	12
Tonnage capacity: *(thousands of tons)*	4.5	7	4

City				*Requirement (thousands of tons)*
A	23 x_{A1}	5 x_{A2}	1.4 x_{A3}	20
B	58 x_{B1}	10 x_{B2}	3.8 x_{B3}	30

Cost matrix

Objective is to minimize the total cost of dispatching the planes.

i.e., minimize $Z = 23x_{A1} + 5x_{A2} + 1.4x_{A3} + 58x_{B1} + 10x_{B2} + 3.8x_{B3}$.

Constraints are

on the number of planes of type I to be dispatched to cities *A and B*,

$x_{A1} + x_{B1} \leq 8$.

Similarly, $x_{A2} + x_{B2} \leq 15$,

$x_{A3} + x_{B3} \leq 12$.

Since tonnage requirements (in thousands of tons) are 20 at city *A* and 30 at city *B*, supply cannot be less than these values. As excess tonnage capacity supplied to a city has no value, exactly 20 and 30 (thousands of tons) will be supplied to them. Therefore, the constraints are

$4.5x_{A1} + 7x_{A2} + 4x_{A3} = 20$,

$4.5x_{B1} + 7x_{B2} + 4x_{B3} = 30$,

where $x_{A1}, x_{A2}, x_{A3}, x_{B1}, x_{B2}, x_{B3}$, each ≥ 0.

EXERCÍCIO 8

Formulation as L.P. Problem. *Key decision* is to determine how each of the two standard widths of tin sheets be cut to the required widths so that trim losses are minimum.

From the available widths of 30 cm and 60 cm, several combinations of the three required widths of 15 cm, 21 cm and 27 cm are possible. Let x_{ij} represent these combinations. Each combination results in certain trim loss.

Thus the constraints are

$$2x_{11} + 4x_{21} + 2x_{22} + 2x_{23} + x_{24} \geq 400,$$
$$x_{12} + x_{22} + 2x_{24} + x_{25} \geq 200,$$
and
$$x_{13} + x_{23} + x_{25} + 2x_{26} \geq 300.$$

Objective is to minimize the trim losses.

i.e., minimize $Z = 9x_{12} + 3x_{13} + 9x_{22} + 3x_{23} + 3x_{24} + 12x_{25} + 6x_{26}$,

where $x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26} \geq 0$.

EXERCÍCIO 9

Formulation of L.P. Model. (*a*) *Key decision* to be made is to determine the number of units of products *A*, *B* and *C* to be manufactured through first, second and third routes.

Let the number of units of products *A*, *B* and *C* to be manufactured through first, second and third routes be x_{A1}, x_{A2}, x_{A3}; x_{B1}, x_{B2} and x_{C1}, x_{C2}, x_{C3} respectively, where each is ≥ 0.

Objective is to maximize the sales revenue.

i.e., maximize $Z = 20(x_{A1} + x_{A2} + x_{A3}) + 15(x_{B1} + x_{B2}) + 25(x_{C1} + x_{C2} + x_{C3})$.

Constraints are on the machine hours available for each machine. They are

for lathes: $0.5x_{A1} + 0.7x_{A2} + 0.3x_{A3} + 0.5x_{B2} + 0.6x_{C1} + 0.5x_{C2} + 0.3x_{C3} \leq 200$,

for drills: $0.5x_{A1} + 0.3x_{A2} + 0.2x_{A3} + 0.4x_{B1} + 0.3x_{B2} + 0.7x_{C1} + 0.4x_{C2} + 0.1x_{C3} \leq 250$, and

for grinders: $0.6x_{A1} + 0.4x_{A2} + 0.6x_{A3} + 0.7x_{B1} + 0.5x_{B2} + 0.4x_{C1} + 0.3x_{C2} \leq 300$.

Thus the L.P. model is to maximize Z subject to the constraints and non-negativity restrictions mentioned above.

(*b*) The fixed order is for 250 units of *A*, 200 units of *B* and 150 units of C. The total number of units of product A produced are $x_{A1} + x_{A2} + x_{A3}$ and in order to satisfy the fixed order it must be ≥ 250. *i.e.*,

for lathes: $x_{A1} + x_{A2} + x_{A3} \geq 250$,

for drills: $x_{B1} + x_{B2} \geq 200$,

for grinders: $x_{C1} + x_{C2} + x_{C3} \geq 150$.

These are, then, the additional constaints to be satisfied (along with the three earlier constraints).

The new objective function is slightly more involved and may be written as

$$\text{maximize } Z_1 = 250 \times 20 + 15\,(x_{A1} + x_{A2} + x_{A3} - 250) + 200 \times 15 + 10(x_{B1} + x_{B2} - 200) + 150 \times 25 + 20\,(x_{C1} + x_{C2} + x_{C3} - 150).$$

The problem is, thus, to maximize Z_1 subject to the above six constraints while satisfying the non-negativity conditions.

(*c*) This market limitation results in a new constraint

$$x_{C1} + x_{C2} + x_{C3} \leq 200,$$

and the problem is to maximize Z_1 while satisfying this 7th (additional) constraint also.

EXERCÍCIO 10

(Ans. Maximize $Z = 10x_1 + 8x_2$, subject to $2x_1 + x_2 \leq 40$, $2x_1 + 3x_2 \leq 80$, $x_1, x_2 \geq 0$.)

EXERCÍCIO 11

(Ans. Maximize $Z = 3x_1 + 4x_2$,

$$\begin{aligned} \text{subject to } 5x_1 + 4x_2 &\leq 200, \\ 3x_1 + 5x_2 &\leq 150, \\ 5x_1 + 4x_2 &\geq 100, \\ 8x_1 + 4x_2 &\geq 80, \\ x_1, x_2 &\geq 0.) \end{aligned}$$

EXERCÍCIO 12

(Ans. Maximize $Z = (2{,}000 - 50 - 100)x_1 + (2{,}250 - 50 - 120)x_2 + (2{,}000 - 25 - 100)x_3$

$$= 1{,}850x_1 + 2{,}080x_2 + 1{,}875x_3,$$

$$\begin{aligned} \text{subject to } x_1 + x_2 + x_3 &\leq 100, \\ 5x_1 + 6x_2 + 5x_3 &\leq 400, \\ x_1, x_2, x_3 &\geq 0.) \end{aligned}$$

EXERCÍCIO 13

(Ans. Maximize $Z = 0.03x_{A_1} + 0.025x_{A_2} + 0.035x_{B_1} + 0.04x_{B_2} + 0.05x_{C_1} + 0.045x_{C_2}$,

$$\begin{aligned} \text{subject to } x_{A_1} + x_{A_2} + x_{B_1} + x_{B_2} + x_{C_1} + x_{C_2} &= 10{,}000, \\ x_{A_1} + x_{A_2} &\geq 4{,}000, \\ x_{B_1} + x_{B_2} &\leq 3{,}500, \\ x_{C_1} + x_{C_2} &\leq 3{,}500, \\ x_{A_1}, x_{A_2}, x_{B_1}, x_{B_2}, x_{C_1}, x_{C_2} &\geq 0.) \end{aligned}$$

EXERCÍCIO 14

[**Hint:** Let x_1, x_2, x_3, x_4, x_5 and x_6 denote the number of drivers joining duty at 00, 04, 08, 12, 16 and 20 hours respectively. The objective is to minimize the number of drivers required *i.e.*, minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$.

Drivers who join duty at 00 hours and 04 hours shall be available between 04 and 08 hours. As the number of drivers required during this interval is 10, we have the constraint

$$x_1 + x_2 \geq 10.$$

Likewise,

$$\begin{aligned} x_2 + x_3 &\geq 20, \\ x_3 + x_4 &\geq 12, \\ x_4 + x_5 &\geq 22, \\ x_5 + x_6 &\geq 8, \end{aligned}$$

and

$$x_6 + x_1 \geq 5.]$$

EXERCÍCIO 15

Minimizar $f(t_A, t_B, \ldots, t_H) = t_H + 3$

$$t_A \geq 0$$
$$t_B \geq 0$$
$$t_C \geq t_A + 6$$
$$t_D \geq t_B + 5$$
$$t_D \geq t_C + 4$$
$$t_E \geq 0$$
$$t_F \geq t_E + 2$$
$$t_F \geq t_D + 2$$
$$t_G \geq t_F + 3$$
$$t_H \geq t_G + 72.$$

EXERCÍCIO 16

Example 2.1 (Production Allocation Problem) *A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.*

TABLE 2.1

Machine	*Time per unit (minutes)*			*Machine capacity (minutes/day)*
	Product 1	*Product 2*	*Product 3*	
M_1	2	3	2	440
M_2	4	—	3	470
M_3	2	5	—	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is Rs. 4, Rs. 3 and Rs. 6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (L.P.) model that will maximize the daily profit. [*H.P.U. MCA 1999*]

Formulation of Linear Programming Model

Step 1: From the study of the situation find the *key-decision* to be made. It this connection, looking for variables helps considerably. In the given situation key decision is to decide the extent of products 1, 2 and 3, as the extents are permitted to vary.

Step 2: Assume symbols for variable quantities noticed in step 1. Let the extents (amounts) of product 1, 2 and 3 manufactured daily be x_1, x_2 and x_3 units respectively.

Step 3: Express the *feasible alternatives* mathematically in terms of variables. Feasible alternatives are those which are physically, economically and financially possible. In the given situation feasible alternatives are sets of values of x_1, x_2 and x_3,

where $\quad x_1, x_2, x_3 \geq 0,$

since negative production has no meaning and is not feasible.

Step 4: Mention the *objective* quantitatively and express it as a linear function of variables. In the present situation, objective is to maximize the profit.

i.e., maximize $\quad Z = 4x_1 + 3x_2 + 6x_3.$

Step 5: Put into words the *influencing factors* or *constraints*. These occur generally because of constraints on availability (resources) or requirements (demands). Express these constraints also as linear equations/inequalities in terms of variables.

Here, constraints are on the machine capacities and can be mathematically expressed as

$$2x_1 + 3x_2 + 2x_3 \leq 440,$$
$$4x_1 + 0x_2 + 3x_3 \leq 470,$$
$$2x_1 + 5x_2 + 0x_3 \leq 430.$$