

## EXERCÍCIO 1

**(Capital Budgeting Problem)** A company has six independent projects available. The cash flows are given in the table below, wherein values within parenthesis are cash outflows and without parenthesis are cash inflows.

Project cash flows (Rs. '000)						
Year	A	B	C	D	E	F
0	(100)	-	-	(40)	-	(30)
1	(50)	(60)	-	(60)	(120)	(10)
2	(10)	(70)	(40)	50	100	20
3	70	10	(80)	10	(10)	10
NPV (Rs. '000)	20	15	10	30	10	5

New capital for these projects is limited to	
Year 0	Rs. 120,000
Year 1	Rs. 200,000
Year 2	Nil
Year 3	Nil

Cash generated from these investments can be reinvested in other projects in the same year. Express the problem in a linear programming format if the objective of the company is to maximize NPV and the projects are divisible.

## EXERCÍCIO 2

### Case Study 2

#### Financial programming problem

- Initial amount: € 80000
- Timeframe of investments' decisions: 4 months
- Two-month government bonds: return 3%
- Three-month government bills: return 6.5%
- Bank deposits: interest 1%
- At the beginning of the 5<sup>th</sup> month at least € 40000 are needed
- Maximum amount in two-month or three-month bonds: € 32000

#### Decision variables

$B_j$  = amount to be invested in government bonds at the beginning of the month  $j$

$C_j$  = amount to be invested in government bills at the beginning of the month  $j$

$D_j$  = amount to be invested in bank deposits at the beginning of the month  $j$

(Adaptado do material de Vassilis Kostoglou. Disponível em:  
[http://aetos.it.teithe.gr/~vkostogl/files/Educational%20material/Linear%20Programming\\_case%20studies+solution%20s.pdf](http://aetos.it.teithe.gr/~vkostogl/files/Educational%20material/Linear%20Programming_case%20studies+solution%20s.pdf) )

### EXERCÍCIO 3

**Exemplo 2.9** Uma indústria de papel produz bobinas-jumbo de  $L = 400\text{cm}$  de largura e cada uma pesa  $T = 1$  tonelada. Os jumbos devem ser cortados em bobinas menores nas larguras e quantidades apresentadas na Tabela 2.11, conforme solicitações de diversos clientes.

Na Figura 2.10, listamos 6 padrões de corte que poderiam ser utilizados para cortar os jumbos (outros padrões de corte são deixados a cargo do leitor).

Utilizando os padrões de corte na Figura 2.10 e  $\frac{\ell_1}{L} = 0,1$ ;  $\frac{\ell_2}{L} = 0,1125$ ;  $\frac{\ell_3}{L} = 0,1375$ ;  $\frac{\ell_4}{L} = 0,15$ , o modelo (2.19) é dado por:

**Tabela 2.11**  
**Dados para um problema de corte.**

<i>Dados da demanda</i>	
<i>Larguras (<math>\ell_i</math>)</i>	<i>Quantidades (<math>b_i</math>)</i>
40cm	5 ton
45cm	3,5 ton
55cm	4 ton
60cm	5 ton

*Padrão de corte 1:*  $\mathbf{a}_1 = (10\ 0\ 0\ 0)^T$



*Padrão de corte 2:*  $\mathbf{a}_2 = (1\ 8\ 0\ 0)^T$



*Padrão de corte 3:*  $\mathbf{a}_3 = (0\ 0\ 7\ 0)^T$



*Padrão de corte 4:*  $\mathbf{a}_4 = (1\ 0\ 0\ 6)^T$



*Padrão de corte 5:*  $\mathbf{a}_5 = (0\ 4\ 4\ 0)^T$



*Padrão de corte 6:*  $\mathbf{a}_6 = (0\ 0\ 4\ 3)^T$



#### **EXERCÍCIO 4**

**Exercício 3.2** Uma empresa tem um capital disponível e considera cinco projetos a serem executados nos próximos três anos. Se um projeto é escolhido, ele requer um investimento a cada ano, e resulta em um rendimento anual após o período de três anos. O capital disponível em cada ano é limitado. Os dados são indicados na Tabela 3.3. Formule o problema de selecionar projetos que maximizem o rendimento total.

**Tabela 3.3**

<i>Projeto</i>	<i>Investimento requerido</i>			<i>Retorno anual após três anos</i>
	<i>Ano 1</i>	<i>Ano 2</i>	<i>Ano 3</i>	
1	7	6	14	30
2	11	4	18	40
3	20	10	5	80
4	19	23	26	110
5	7	9	4	60
Capital disponível	55	60	58	

#### **EXERCÍCIO 5**

**Exercício 3.4** Seis itens podem ser colocados em um contêiner. O peso  $w_i$  (em toneladas), o volume  $v_i$  (em  $m^3$ ), e o valor  $r_i$  (em unidades de R\$1.000) de cada unidade do item  $i$  são dados na Tabela 3.5. Existem cinco cópias do item 1, três cópias do item 2, e para os demais itens existe um número ilimitado de cópias. O contêiner suporta uma carga máxima de 90 toneladas, e tem um volume de  $50m^3$ . Formule o problema de maximizar o valor da carga no contêiner.

**Tabela 3.5**

<i>Item <math>i</math></i>	$w_i$	$v_i$	$r_i$
1	9	8	10
2	2	5	4
3	6	3	5
4	7	6	9
5	5	6	8
6	7	4	6

## EXERCÍCIO 6

**Example 2.24 (Investment Problem).** Mr. Krishnamurty, a retired Govt. officer, has recently received his retirement benefits, viz., provident fund, gratuity, etc. He is contemplating as to how much funds he should invest in various alternatives open to him so as to maximize return on investment. The investment alternatives are: government securities, fixed deposits of a public limited company, equity shares, time deposits in banks, national saving certificates and real estate. He has made a subjective estimate of the risk involved. The data on the return on investment, the number of years for which the funds will be blocked to earn this return on investment and the subjective risk involved are as follows:

Investment alternatives	Return	No. of years	Risk
Govt. securities	6%	15	1
Company deposits	15%	3	3
Equity shares	20%	6	7
Time deposits	10%	3	1
N.S.C.	12%	6	1
Real estate	25%	10	2

He was wondering what percentage of funds he should invest in each alternative so as to maximize the return on investment. He decided that average risk should not be more than 4, and funds should not be locked up for more than 15 years. Formulate an L.P. model for the problem if he does not want more than 30% of the investment to be put in the real estate.

[Bombay MMS, 1980]

## EXERCÍCIO 7

47. Rolls of paper having a fixed length and width of 180 cm are being manufactured by a paper mill. These rolls have to be cut to satisfy the following demand:

Width:	80 cm	45 cm	27 cm
No. of rolls:	200	120	130

Obtain the linear programming formulation of the problem to determine the cutting pattern, so that the demand is satisfied and wastage of paper is a minimum.

[Hint: Various alternatives for the number of rolls are given below:

Feasible patterns of cutting	No. of rolls cut	Wastage per roll (cm)	Rolls obtained from each mother roll of width 180 cm		
			80 cm.	45 cm.	27 cm.
80 + 80	$x_1$	20	2	—	—
80 + 45 + 45	$x_2$	10	1	2	—
80 + 45 + 27 + 27	$x_3$	1	1	1	2
80 + 27 + 27 + 27	$x_4$	19	1	—	3
45 + 45 + 45 + 45	$x_5$	0	—	4	—
45 + 45 + 45 + 27	$x_6$	18	—	3	1
45 + 45 + 27 + 27 + 27	$x_7$	9	—	2	3
45 + 27 + 27 + 27 + 27 + 27	$x_8$	0	—	1	5
27 + 27 + 27 + 27 + 27 + 27	$x_9$	18	—	—	6

## EXERCÍCIO 8

49. A mutual fund company has cash resources of Rs. 200 million for investment in a diversified portfolio. Table below shows the opportunities available, their estimated annual yields, risk factors and term period details.

Formulate an L.P. model to find the optimal portfolio that will maximize return, considering the following policy guidelines:

<i>Investment type</i>	<i>Annual yield (%)</i>	<i>Risk factor</i>	<i>Time period (years)</i>
Bank deposit	9.5	0.02	6
Treasury notes	8.5	0.01	4
Corporate deposit	12.0	0.08	3
Blue-chip stock	15.0	0.25	5
Speculative stocks	32.5	0.45	3
Real estate	35.0	0.40	10

- (i) All the funds available may be invested.
- (ii) Weighted average period of at least 5 years should be the planning horizon.
- (iii) Weighted average risk factor must not exceed 0.20.
- (iv) Investment in real estate and speculative stocks together must not be more than 25% of the money invested in total.