EXERCÍCIO 1

Formulation of L.P. Model. Let x_1, x_2, x_3, x_4, x_5 and x_6 denote the *proportion* of projects A, B, C, D, E and F undertaken. The L.P. model can be written as

maximize Z $= 20x_1 + 15x_2 + 10x_3 + 30x_4 + 10x_5 + 5x_6$,

subejct to constraints

$100x_1 + 40x_4 + 30x_6 \leq 120$,

$50x_1 + 60x_2 + 60x_4 + 120x_5 + 10x_6 \leq 200$,

$10x_1 + 70x_2 + 40x_3 \leq 50x_4 + 100x_5 + 20x_6$,

$80x_3 + 10x_5 \leq 70x_1 + 10x_2 + 10x_4 + 10x_6$,

where $x_1, x_2, x_3, x_4, x_5, x_6$, each ≥ 0.

EXERCÍCIO 2

Objective function (total return)

$$\begin{aligned} \text{Max } z = {} & 0.03B_1 + 0.03B_2 + 0.03B_3 + 0.03B_4 \\ & + 0.065C_1 + 0.065C_2 + 0.065C_3 + 0.065C_4 \\ & + 0.01D_1 + 0.01C_2 + 0.01C_3 + 0.01D_4 \end{aligned}$$

Basic rule of capital flow:

Invested amount (start t) + Cash available (start t) =

Available amount (end t-1) + Cash available (end t-1)

Mathematical model

1.		B_1	+	C_1	+	D_1							=	80000
2.		B_2	+	C_2	-	$1.01D_1$	+	D_2					=	0
3.	-	$1.03B_1$	+	B_3	+	C_3	-	$1.01D_2$	+	D_3			=	0
4.	-	$1.03B_2$	+	B_4	-	$1.065C_1$	+	C_4	-	$1.01D_3$	+	D_4	=	0
5.		$1.03B_3$	+	$1.065C_1$	+	$1.01D_4$							>=	40000
6.		B_1											<=	32000
7.		B_1	+	B_2									<=	32000
8.		B_2	+	B_3									<=	32000
9.		B_3	+	B_4									<=	32000
10.		C_1	+										<=	32000
11.		C_1	+	C_2									<=	32000
12.		C_1	+	C_2	+	C_3							<=	32000
13.		C_2	+	C_3	+	C_4							<=	32000

$B_i, C_i, D_i >= 0$, for i = 1, 2, 3, 4

EXERCÍCIO 3

$$\text{Minimizar} \quad g(y_1, y_2, y_3, y_4, y_5, y_6, \ldots) = y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + \ldots$$

$$\begin{bmatrix} 10\times0{,}1 \\ 0 \\ 0 \\ 0 \end{bmatrix} y_1 + \begin{bmatrix} 1\times0{,}1 \\ 8\times0{,}1125 \\ 0 \\ 0 \end{bmatrix} y_2 + \begin{bmatrix} 0 \\ 0 \\ 7\times0{,}1375 \\ 0 \end{bmatrix} y_3 + \begin{bmatrix} 1\times0{,}1 \\ 0 \\ 0 \\ 6\times0{,}15 \end{bmatrix} y_4 + \begin{bmatrix} 0 \\ 4\times0{,}1125 \\ 4\times0{,}1375 \\ 0 \end{bmatrix} y_5 + \begin{bmatrix} 0 \\ 0 \\ 4\times0{,}1375 \\ 3\times0{,}15 \end{bmatrix} y_6 + \ldots = \begin{bmatrix} 5 \\ 3{,}5 \\ 4 \\ 5 \end{bmatrix}$$

$$y_1, y_2, y_3, y_4, y_5, y_6, \ldots \geq 0.$$

EXERCÍCIO 4

$$Xi = \begin{cases} 1 & \text{se é investido em i} \\ 0 & \text{caso contrário} \end{cases}$$

Max z= $30X_1 + 40X_2 + 80X_3 + 110X_4 + 60X_5$

s.a. $7X_1 + 11X_2 + 20X_3 + 19X_4 + 7X_5 <= 55$

$6X_1 + 4X_2 + 10X_3 + 23X_4 + 9X_5 <= 60$

$14X_1 + 18X_2 + 5X_3 + 26X_4 + 4X_5 <= 58$

Xi pertence a B, i=1,...,5

EXERCÍCIO 5

Max z= $10X_1 + 4X_2 + 5X_3 + 9X_4 + 8X_5 + 6X_6$

s.a. $9X_1 + 2X_2 + 6X_3 + 7X_4 + 5X_5 + 7X_6 <= 90$

$8X_1 + 5X_2 + 3X_3 + 6X_4 + 6X_5 + 4X_6 <= 50$

$X_1 <= 5$

$X_2 <= 3$

Xi pertence aos inteiros positivos, i=1,...,6

EXERCÍCIO 6

Formulation of L.P. Model. Let $x_1, x_2, x_3, x_4, x_5, x_6$ be the *percentage* of funds to be invested in alternative 1, 2, 3, 4, 5 and 6 respectively.

Objective is to maximize the return on investment.

i.e., maximize $Z = 0.06x_1 + 0.15x_2 + 0.20x_3 + 0.10x_4 + 0.12x_5 + 0.25x_6$.

Constraints are

on the average risk $x_1 + 3x_2 + 7x_3 + x_4 + x_5 + 2x_6 \leq 4$,

on the lock-up period $15x_1 + 3x_2 + 6x_3 + 3x_4 + 6x_5 + 10x_6 \leq 15$,

on investment in real estate $x_6 \leq 0.3$,

on the total amount invested $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$,

where $x_1, x_2, \ldots x_6$, each ≥ 0.

EXERCÍCIO 7

Thus the linear programming problem is:

Minimize $z = 20x_1 + 10x_2 + x_3 + 19x_4 + 18x_6 + 9x_7 + 18x_9$,

subject to the constraints:

$2x_1 + x_2 + x_3 + x_4 \geq 200$, *(80 cm rolls)*

$2x_2 + x_3 + 4x_5 + 3x_6 + 2x_7 + x_8 \geq 120$, *(45 cm rolls)*

$2x_3 + 3x_4 + x_6 + 3x_7 + 5x_8 + 6x_9 \geq 130$, *(27 cm rolls)*

$x_j \geq 0;\ j = 1, 2, 3, \ldots, 9.]$

EXERCÍCIO 8

(*Ans.* Maximize $Z = \frac{9.5}{100}x_1 + \frac{8.5}{100}x_2 + \frac{12}{100}x_3 + \frac{15}{100}x_4 + \frac{32.5}{100}x_5 + \frac{35}{100}x_6$,

subject to constraints $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 200$,

$0.02x_1 + 0.01x_2 + 0.08x_3 + 0.25x_4 + 0.45x_5 + 0.40x_6 \leq 0.2 \times 200$,

$6x_1 + 4x_2 + 3x_3 + 5x_4 + 3x_5 + 10x_6 \geq 5 \times 200$,

$x_5 + x_6 \leq 0.25 \times 200$,

$x_1, x_2, x_3, x_4, x_5, x_6$, each ≥ 0.)