EXERCÍCIO 1

$x_i = \begin{cases} 1 & \text{se o jogador i é escolhido} \\ 0 & \text{caso contrário} \end{cases}$

max $z = 3x_1 + 2x_2 + 2x_3 + 1x_4 + 3x_5 + 3x_6 + 1x_7$

s.a. $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 5$

$x_1 + x_3 + x_5 + x_7 >= 3$

$x_3 + x_4 + x_5 + x_6 + x_7 >= 2$

$x_2 + x_4 + x_6 >= 2$

$(3x_1 + 2x_2 + 2x_3 + 1x_4 + 3x_5 + 3x_6 + 3x_7)/5 >= 2$

$(3x_1 + 1x_2 + 3x_3 + 3x_4 + 3x_5 + 1x_6 + 2x_7)/5 >= 2$

$(1x_1 + 3x_2 + 2x_3 + 3x_3 + 3x_4 + 2x_5 + 2x_6)/5 >= 2$

$(3x_1 + 2x_2 + 2x_3 + 1x_4 + 3x_5 + 3x_6 + 1x_7)/5 >= 2$

$x_6 <= M(1 - x_3)$

$x_4 + x_5 – 2x_1 >= 0$

$x_2 + x_3 = 1$

EXERCÍCIO 2

$x_{ij} = \begin{cases} 1 & \text{se o funcionário j é designado ao instrutor i} \\ 0 & \text{caso contrário} \end{cases}$

min $z = (10x_{11} + 4x_{12} + 12x_{13} + 14x_{14} + 18x_{15} + 11x_{16} + 12x_{17}) + (15x_{21} + 13x_{22} + 5x_{23} + 13x_{24} + 20x_{25} + 8x_{26} + 6x_{27}) + (20x_{31} + 30x_{32} + 15x_{33} + 11x_{34} + 13x_{35} + 6x_{36} + 21x_{37})$

s.a. $8x_{11} + 30x_{12} + 7x_{13} + 17x_{14} + 2x_{15} + 8x_{16} + 7x_{17} <= 50$

$10x_{21} + 5x_{22} + 20x_{23} + 4x_{24} + 10x_{25} + 5x_{26} + 3x_{27} <= 70$

$5x_{31} + 1x_{32} + 3x_{33} + 9x_{34} + 18x_{35} + 6x_{36} + 24x_{37} <= 30$

$x_{11} + x_{21} + x_{31} = 1$

$x_{12} + x_{22} + x_{32} = 1$

$x_{13} + x_{23} + x_{33} = 1$

$x_{14} + x_{24} + x_{34} = 1$

$x_{15} + x_{25} + x_{35} = 1$

$x_{16} + x_{26} + x_{36} = 1$

$x_{17} + x_{27} + x_{37} = 1$

EXERCÍCIO 3

xj: quantidade de vezes que o padrão j será utilizado

a)

min z = x1 + x2 + x3 + x4 + x5 + x6 + x7
s.a.

$$\begin{bmatrix}0\\0\\1\end{bmatrix} x1 + \begin{bmatrix}0\\1\\1\end{bmatrix} x2 + \begin{bmatrix}0\\2\\0\end{bmatrix} x3 + \begin{bmatrix}1\\0\\0\end{bmatrix} x4 + \begin{bmatrix}1\\1\\0\end{bmatrix} x5 + \begin{bmatrix}1\\0\\1\end{bmatrix} x6 >= \begin{bmatrix}92\\59\\89\end{bmatrix}$$

x1, x2, x3, x4, x5, x6, x7 pertencem aos inteiros

b)

min z = 3x1 + 4x2 + 0x3 + 1x4 + 5x5 + 2x6 + 1x7
s.a.

$$\begin{bmatrix}0\\0\\1\end{bmatrix} x1 + \begin{bmatrix}0\\1\\1\end{bmatrix} x2 + \begin{bmatrix}0\\2\\0\end{bmatrix} x3 + \begin{bmatrix}1\\0\\0\end{bmatrix} x4 + \begin{bmatrix}1\\1\\0\end{bmatrix} x5 + \begin{bmatrix}1\\0\\1\end{bmatrix} x6 >= \begin{bmatrix}92\\59\\89\end{bmatrix}$$

x1, x2, x3, x4, x5, x6, x7 pertencem aos inteiros

EXERCÍCIO 4

Formulation of L.P. Model. Let us imagine that the person travels from node 1 to 8 along a succession of links. Let x_{ij} represent the link joining node i to j. If the person uses link x_{ij}, then x_{ij} will be given a value 1; if not, a value zero. The person reaches a node, say, j via one of the links, say, x_{ij} and leaves it by another link, say, x_{jk}. So, if a node j has several arrows, such as ij, hj, gj pointing towards it and several arrows, such as jk, jl, jm pointing away from it, then the equation for node j can be written as

$$x_{gj} + x_{hj} + x_{ij} - x_{jk} - x_{jl} - x_{jm} = 0.$$

This is because, for each of the links x_{ij} and x_{jk}, the value is 1 (the person uses this link) and for each of the links x_{gj}, x_{hj}, x_{jl} and x_{jm} the value is zero (the person does not use these links).

For the network shown in Fig. 2.1, we get for

node 2,	$x_{12} - x_{24} - x_{25}$	$= 0,$
node 3,	$x_{13} - x_{35}$	$= 0,$
node 4,	$x_{24} - x_{45} - x_{46}$	$= 0,$
node 5,	$x_{25} + x_{35} + x_{45} - x_{56} - x_{57}$	$= 0,$
node 6,	$x_{46} + x_{56} - x_{68}$	$= 0,$
and node 7,	$x_{57} - x_{78}$	$= 0.$

Further, since the person has to start from node 1, along one of the paths (links) starting from it, we have

$$- x_{12} - x_{13} = - 1.$$

Similarly, as the person has to reach node 8 along one of the paths, we get

$$x_{68} + x_{78} = 1.$$

The above eight are, then, the constraints (equality type) that must be satisfied. The objective is to minimize the total cost of travelling from node 1 to 8, given by

$$\text{Minimize } Z = 5x_{12} + 7x_{13} + 10x_{24} + 3x_{25} + 8x_{35} + x_{45} + 6x_{46} + 7x_{56} + 4x_{57} + 5x_{68} + 3x_{78}.$$

This is, then, the linear programming model for the network problem wherein every variable has a value 1 or 0.

EXERCÍCIO 5

x_i: quantidade de funcionários que entram no dia i, i = 1(segunda), 2(terça), 3(quarta), 4(quinta), 5(sexta), 6(sábado), 7(domingo).

Min $z = 30 (x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7)$

s.a. $x_1 + x_7 + x_6 + x_5 + x_4 >= 10$

$x_2 + x_1 + x_7 + x_6 + x_5 >= 6$

$x_3 + x_2 + x_1 + x_7 + x_6 >= 8$

$x_4 + x_3 + x_2 + x_1 + x_7 >= 5$

$x_5 + x_4 + x_3 + x_2 + x_1 >= 9$

$x_6 + x_5 + x_4 + x_3 + x_2 >= 4$

$x_7 + x_6 + x_5 + x_4 + x_3 >= 6$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7$ pertencem aos inteiros

EXERCÍCIO 6

Variables

Let

x_i be the number of patterns of type i (i=1,2,3,4) stamped per week

y be the number of cans produced per week

Note $x_i >= 0$ i=1,2,3,4 and $y >= 0$ and again we assume that the x_i and y are large enough for fractional values not to be significant.

Constraints

- time available

$t_1x_1 + t_2x_2 + t_3x_3 + t_4x_4 <= T$

- sheet availability

$x_1 + x_3 + x_4 <= L_1$ (sheet 1)

$x_2 <= L_2$ (sheet 2)

- number of cans produced

$y = \min[\ (7x_1+4x_2+3x_3+9x_4)/2,\ (x_1+4x_2+2x_3)\ ]$

where the first term in this expression is the limit imposed upon y by the number of can ends produced and the second term in this expression is the limit imposed upon y by the number of can bodies produced. This constraint (because of the min[,] part) is not a linear constraint.

Objective

Presumably to maximise profit - hence

maximise

revenue - cost of scrap - unused main bodies stock - holding cost - unused ends stock - holding cost

i.e. maximise

$Py - C(s_1x_1 + s_2x_2 + s_3x_3 + s_4x_4) - c_1(x_1 + 4x_2 + 2x_3 - y) - c_2((7x_1 + 4x_2 + 3x_3 + 9x_4) - 2y)$

As noted above this formulation of the problem is not an LP - however it is relatively easy (for this *particular* problem) to turn it into an LP by replacing the y = min[,] non-linear equation by *two* linear equations.

Suppose we replace the constraint

$y = \min[\ (7x_1+4x_2+3x_3+9x_4)/2,$
$(x_1+4x_2+2x_3)\ ]$ (A)

by the two constraints

$y <= (7x_1+4x_2+3x_3+9x_4)/2$ (B)

$y <= (x_1+4x_2+2x_3)$ (C)
