TP052 – Pesquisa Operacional I
1ª Lista de Exercícios - Formulações
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MÉTODO SIMPLEX

1) **(Simplex Method)** Use simplex method to solve the following problem:

$Maximize\ Z = 2x_1 + 5x_2,$

$Subject\ to$:
$x_1 + 4x_2 \leq 24,$
$3x_1 + x_2 \leq 21,$
$x_1 + x_2 \leq 9,$
$x_1, x_2 \geq 0$

[P.U.B Com. April, 2005]
$(Ans. [basic]\ x_1 = 4, x_2 = 5, s_2 = 4, [nonbasic]\ s_1 = 0, s_3 = 0;\ Z_{max} = 33)$

2) **(Big-M Method)** Use the Big-M method to solve the following problem:

$Maximize\ Z = 3x_1 - x_2,$

$Subject\ to$:
$2x_1 + x_2 \leq 2,$
$x_1 + 3x_2 \geq 3,$
$x_2 \leq 4,$
$x_1, x_2 \geq 0$

[P.U.B.E. (Elect.) Oct., 1993; I.A.S. 1992]
$(Ans. x_1 = {}^{3}/_{5}, x_2 = {}^{4}/_{5};\ Z_{max} = 1)$

3) **(Two-Phase Method)** Use the two-phase method to solve the following problem:

$Maximize\ Z = 5x_1 - 4x_2 + 3x_3,$

$Subject\ to$:
$2x_1 + x_2 - 6x_3 = 20,$
$6x_1 + 5x_2 + 10x_3 \leq 76,$
$8x_1 - 3x_2 + 6x_3 \leq 50,$
$x_1, x_2, x_3 \geq 0$

[H.P.U.B. Tech. (Mech.) Nov., 2007; Kerala M. Sc. (Math) 1984; P.U.B.E. (Elect.) May, 1994]
$(Ans. x_1 = {}^{55}/_{7}, x_2 = {}^{30}/_{7}, x_3 = 0;\ Z_{max} = {}^{155}/_{7})$

4) (Degeneracy)

$Maximize\ Z = 2x_1 + 3x_2 + 10x_3,$

$Subject\ to:$
$x_1 + 2x_3 = 0,$
$x_2 + x_3 = 1,$
$x_1, x_2, x_3 \geq 0$

[Meerut B. Sc. (Math.) 1970]
$(Ans. x_1 = 0\ (nonbasic), x_2 = 1\ (basic), x_3 = 0\ (basic);\ Z_{max} = 3)$

5) (Multiple Optimal Solution)

$Maximize\ Z = 4x_1 + 10x_2,$

$Subject\ to:$
$2x_1 + x_2 \leq 10,$
$2x_1 + 5x_2 \leq 20,$
$2x_1 + 3x_2 \leq 18,$
$x_1, x_2 \geq 0$

(i) Indicate that this problem has an alternate optimal basic feasible solution.
(ii) Find that optimal solution
(iii) Hence show that this problem has multiple optimal solutions

[H.P.U.B. Tech. (Mech.) Nov., 2006, P.U.M.E. (Civil) Nov., 1994; B.E. (Elect.) June, 1993; NIIFT Mohali, 1999; Karn U.B.E. (Mech.) 1995]
$(Ans. x_1 = 0, x_2 = 4, s_1 = 6, s_2 = 0, s_3 = 6;\ Z_{max} = 40)$
$\left(Ans. x_1 = {}^{15}/_4, x_2 = {}^{5}/_2, s_1 = 0, s_2 = 0, s_3 = 3;\ Z_{max} = 40\right)$

6) (Unbounded Solution) Show that there is an unbounded solution to the following L.P. problem:

$Maximize\ Z = 4x_1 + x_2 + 3x_3 + 5x_4,$

$Subject\ to:$
$4x_1 - 6x_2 - 5x_3 - 4x_4 \geq -20,$
$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10,$
$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20,$
$x_1, x_2, x_3, x_4 \geq 0$

[Pbi. U.B. Tech. 1999; Chennai B. Sc. (Math.) 1978]

7) (Infeasible Solution)

$Minimize\ Z = x_1 + 2x_2 + x_3,$

$Subject\ to:$
$x_1 + {}^1\!/_2\, x_2 + {}^1\!/_2\, x_3 \leq 1,$
${}^3\!/_2\, x_1 + 2x_2 + x_3 \geq 8,$
$x_1, x_2, x_3 \geq 0$

8) Now assume some of the numbers in the tableau are parameters. You must decide on the values of these parameters based on the questions asked.

Base	**X1**	**X2**	**X3**	**X4**	**X5**	**b**
Z	0	0	0	**α**	**β**	2
X1	1	0	0	**ε**	2	**γ**
X2	0	1	0	**σ**	-1	**η**
X3	0	0	1	-1	**τ**	**δ**

a. If the above system of equations represent a valid basic feasible solution, what can you say about the sign of the parameters **γ**, **η** and **δ**?
b. For what values of the parameters can you say the tableaux is optimal?
c. For what values of the parameters will X5 be the only choice for entering variable and X3 be the leaving variable?
d. For what values of the parameters can you detect that the LP is unbounded?

$(Ans.[a]\ \gamma, \eta, \delta \geq 0)$
$(Ans.[b]\ \alpha, \beta \geq 0\ )$
$\left(Ans.[c]\ \alpha \geq 0;\ \beta < 0; \tau > 0; \frac{\delta}{\tau} < \frac{\gamma}{2}\right)$
$(Ans.[d]\ \alpha < 0; \varepsilon, \sigma \leq 0)$