QUESTÃO 1)

$$\begin{aligned} \min \quad & D = 6y_1 + 3y_2 \\ sa \quad & 3y_1 + 6y_2 \geq 6 \\ & 4y_1 + 1y_2 \geq 3 \\ & y_1, y_2 \geq 0 \end{aligned}$$

Resolvendo o ppl primal, obtém-se o seguinte quadro ótimo.

Variáveis do Dual	E_1^*	E_2^*	y_1^*	y_2^*	
Variáveis do Primal	x_1	x_2	x_3	x_4	b
Z	0	0	4/21	5/21	13/7
x_2^*	0	1	2/7	-1/7	9/7
x_1^*	1	0	-1/21	4/21	2/7

Solução ótima do primal

$x_1^* = 2/7$

$x_2^* = 9/7$

$F_1^* = 0$

$F_2^* = 0$

$Z^* = 13/7$

Solução ótima do dual

$y_1^* = 4/21$

$y_2^* = 5/21$

$E_1^* = 0$

$E_2^* = 0$

$D^* = 13/7$

QUESTÃO 2)

$$\begin{aligned} \max \ Z = \ & 5x_1 + 2x_2 \\ s.a \quad & x_1 \leq 3 \\ & x_2 \leq 4 \\ & x_1 + 2x_2 \geq 9 \\ & x_1, \ x_2 \geq 0 \end{aligned}$$

Resolvendo o ppl primal, obtém-se o seguinte quadro ótimo.

Variáveis do Dual	E_1^*	E_2^*	y_1^*	y_2^*	y_3^*	
Variáveis do Primal	x_1	x_2	F_1	F_2	E	B
Z	0	0	5	2	0	23
E	0	0	1	2	1	2
x_2	0	1	0	1	0	4
x_1	1	0	1	0	0	3

Solução ótima do primal

$x_1^* = 3$

$x_2^* = 4$

$F_1^* = 0$

$F_2^* = 0$

$E^* = 2$

$Z^* = 23$

Solução ótima do dual

$y_1^* = 5$

$y_2^* = 2$

$y_3^* = 0$

$E_1^* = 0$

$E_2^* = 0$

$D^* = 23$

QUESTÃO 3)

- Acha a solução do dual através do teorema da folga complementar
- Como Z(x) = D(y), para os valores dados de x e para os valores encontrados de y, pode-se afirmar que x=(4,1) é solução ótima do problema.

QUESTÃO 4)

Answer: The dual of this LPP is:

$$\begin{array}{l} \text{Minimize } z' = 50w_1 + 60w_2 + 5w_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} w_1 & + & 4w_2 & + & w_3 & \geq & 5 \\ 3w_1 & + & 2w_2 & & & \geq & 10 \\ w_1, & & w_2, & & w_3 & \geq & 0 \end{array} \end{array}$$

Therefore $w_1 = 10/3$, $w_2 = 0$, and $w_3 = 5/3$ gives an optimal solution to the dual problem.

QUESTÃO 5)

Solution . (*i*) Let x_1, x_2, and x_3 denote the number of units of tables, chairs and racks to be produced by the company. Then the L.P. problem is

$$\begin{array}{ll} \text{maximize} & Z = 12x_1 + 3x_2 + x_3, \\ \text{subject to} & 10x_1 + 2x_2 + x_3 \leq 100, \\ & 7x_1 + 3x_2 + 2x_3 \leq 77, \\ & 2x_1 + 4x_2 + x_3 \leq 80, \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

(*ii*) Dual problem can be expressed as

$$\begin{array}{ll} \text{minimize} & Z' = 100y_1 + 77y_2 + 80y_3, \\ \text{subject to} & 10y_1 + 7y_2 + 2y_3 \geq 12, \\ & 2y_1 + 3y_2 + 4y_3 \geq 3, \\ & y_1 + 2y_2 + y_3 \geq 1, \\ & y_1, y_2, y_3 \geq 0. \end{array}$$

(*iii*) The primal problem can be solved by using the simplex method.

Optimal solution to the given problem is

$$x_1 = 73/8,\ x_2 = 35/8,\ x_3 = 0\ ;\ Z_{max} = \text{Rs. } (876/8 + 105/8) = \text{Rs. } 122.63.$$

(*iv*) Optimal solution to the dual problem is

$$y_1 = 15/16,\ y_2 = 3/8,\ y_3 = 0\ ;\ Z'_{min} = \text{Rs. } [100 \times 15/16 + 77 \times 3/8 + 80 \times 0] = \text{Rs. } 122.63.$$

(*v*) Shadow prices of the resources are

timber = Rs.15/16 per cubit foot,

time in manufacturing deptt. = Rs. 3/8 per hour,

time in finishing deptt. = Nil (because there is already unused capacity of 177/4 hours in this department).

Note that racks are not to be produced as $c_j - Z_j$ value in the optimal table (–11/16) indicates that every rack produced would cause a loss of Rs. 11/16 in the profit.

(*vi*) Economic interpretation of the dual problem can be explained as follows :

Suppose this manufacturing company is thinking of renting its production facilities and selling out timber to some other firm, say *ABC* company, instead of using them by itself and then selling the products — tables, chairs and racks to get a profit of Rs. 122.63. Then *ABC* company is interested in minimizing the sum to be paid (cost to it), while the parent manufacturing company will be interested in knowing the rates it should charge for timber/cubic feet and production time/hour in manufacturing as well as finishing departments.

Let y_1 be the price of timber/cubic feet, y_2 be the charges/hour of manufacturing department and y_3 be the charges/hour of finishing department. Then the total amount *ABC* company would have to pay is $100\,y_1 + 77y_2 + 80\,y_3$ and its objective is to

minimize $Z' = 100y_1 + 77y_2 + 80\,y_3$.

Having known this total cost, *ABC* company would be interested in knowing the values of y_1, y_2 and y_3 respectively. The total this company has to pay to make a table is Rs. $(10y_1 + 7y_2 + 2y_3)$. As the parent company can earn a profit of Rs. 12 / table if it produces by itself, *ABC* company has no option but to settle for paying a minimum of Rs. 12 / table.

QUESTÃO 6)

$x_1 = 4$, $x_2 = 3$ and $Z_{max} = -3 \times 4 - 2 \times 3 = -18$.

QUESTÃO 7)

- Escreve o dual do problema e resolve por dual simplex
- Soluções encontradas: $x_1 = x_2 = x_3 = x_4 = 0$, $F_1 = -10$, $F_2 = -2$, $F_3 = -15$ e $Z = 0$