

# Ciclos no Simplex

Tecnologia da Decisão I  
TP065

# Exemplo

$$\begin{aligned} \max Z = & \frac{3}{4}x_1 - 20x_2 + \frac{1}{2}x_3 - 6x_4 \\ \text{s. a} & \frac{1}{4}x_1 - 8x_2 - x_3 + 9x_4 \leq 0 \\ & \frac{1}{2}x_1 - 12x_2 - \frac{1}{2}x_3 + 3x_4 \leq 0 \\ & x_3 \leq 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-3/4	20	-1/2	6	0	0	0	0
$F_1$	1/4	-8	-1	9	1	0	0	0
$F_2$	1/2	-12	-1/2	3	0	1	0	0
$F_3$	0	0	1	0	0	0	1	1

a) Escolhendo F2 para sair da base



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-3/4	20	-1/2	6	0	0	0	0
$F_1$	1/4	-8	-1	9	1	0	0	0
$F_2$	1/2	-12	-1/2	3	0	1	0	0
$F_3$	0	0	1	0	0	0	1	1



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	0	2	-5/4	21/2	0	3/2	0	0
$F_1$	0	-2	-3/4	15/2	1	-1/2	0	0
$x_1$	1	-24	-1	6	0	2	0	0
$F_3$	0	0	1	0	0	0	1	1



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	0	2	0	21/2	0	3/2	5/4	5/4
$F_1$	0	-2	0	15/2	1	-1/2	3/4	3/4
$x_1$	1	-24	0	6	0	2	0	1
$x_3$	0	0	1	0	0	0	1	1

Solução ótima encontrada.

b) Escolhendo F1 para sair da base



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-3/4	20	-1/2	6	0	0	0	0
$F_1$	1/4	-8	-1	9	1	0	0	0
$F_2$	1/2	-12	-1/2	3	0	1	0	0
$F_3$	0	0	1	0	0	0	1	1



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	0	-4	-7/2	33	3	0	0	0
$x_1$	1	-32	-4	36	4	0	0	0
$F_2$	0	4	3/2	-15	-2	1	0	0
$F_3$	0	0	1	0	0	0	1	1



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	0	0	-2	18	1	1	0	0
$x_1$	1	0	8	-84	-12	8	0	0
$x_2$	0	1	3/8	-15/4	-1/2	1/4	0	0
$F_3$	0	0	1	0	0	0	1	1



↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	1/4	0	0	-3	-2	3	0	0
$x_3$	1/8	0	1	-21/2	-3/2	1	0	0
$x_2$	-3/64	1	0	3/16	1/16	-1/8	0	0
$F_3$	-1/8	0	0	21/2	3/2	-1	1	1

→

↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-1/2	16	0	0	-1	1	0	0
$x_3$	-5/2	56	1	0	2	-6	0	0
$x_4$	-1/4	16/3	0	1	1/3	-2/3	0	0
$F_3$	5/2	-56	0	0	-2	6	1	1

→

↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-7/4	44	1/2	0	0	-2	0	0
$F_1$	-5/4	28	1/2	0	1	-3	0	0
$x_4$	1/6	-4	-1/6	1	0	1/3	0	0
$F_3$	0	0	1	0	0	0	1	1

→

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-3/4	20	-1/2	6	0	0	0	0
$F_1$	1/4	-8	-1	9	1	0	0	0
$F_2$	1/2	-12	-1/2	3	0	1	0	0
$F_3$	0	0	1	0	0	0	1	1

↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	1/4	0	0	-3	-2	3	0	0
$x_3$	1/8	0	1	-21/2	-3/2	1	0	0
$x_2$	-3/64	1	0	3/16	1/16	-1/8	0	0
$F_3$	-1/8	0	0	21/2	3/2	-1	1	1

→

↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-1/2	16	0	0	-1	1	0	0
$x_3$	-5/2	56	1	0	2	-6	0	0
$x_4$	-1/4	16/3	0	1	1/3	-2/3	0	0
$F_3$	5/2	-56	0	0	-2	6	1	1

→

↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-7/4	44	1/2	0	0	-2	0	0
$F_1$	-5/4	28	1/2	0	1	-3	0	0
$x_4$	1/6	-4	-1/6	1	0	1/3	0	0
$F_3$	0	0	1	0	0	0	1	1

→

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-3/4	20	-1/2	6	0	0	0	0
$F_1$	1/4	-8	-1	9	1	0	0	0
$F_2$	1/2	-12	-1/2	3	0	1	0	0
$F_3$	0	0	1	0	0	0	1	1

Quadro inicial

Ciclos acontecem quando tem-se solução degenerada. Existem regras para se “escapar” de ciclos. Uma destas regras é a **Regra de Bland**:

- Entre todas as variáveis com coeficiente negativo na linha do Z, escolher para entrar na base aquela com menor índice;
- Em caso de empate na saída, escolher a variável com menor índice entre aquelas que empataram.

Aplicando a Regra de Bland:



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-3/4	20	-1/2	6	0	0	0	0
$F_1$	1/4	-8	-1	9	1	0	0	0
$F_2$	1/2	-12	-1/2	3	0	1	0	0
$F_3$	0	0	1	0	0	0	1	1



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	0	-4	-7/2	33	3	0	0	0
$x_1$	1	-32	-4	36	4	0	0	0
$F_2$	0	4	3/2	-15	-2	1	0	0
$F_3$	0	0	1	0	0	0	1	1



Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	0	0	-2	18	1	1	0	0
$x_1$	1	0	8	-84	-12	8	0	0
$x_2$	0	1	3/8	-15/4	-1/2	1/4	0	0
$F_3$	0	0	1	0	0	0	1	1





↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	1/4	0	0	-3	-2	3	0	0
$x_3$	1/8	0	1	-21/2	-3/2	1	0	0
$x_2$	-3/64	1	0	3/16	1/16	-1/8	0	0
$F_3$	-1/8	0	0	21/2	3/2	-1	1	1

→

↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	-1/2	16	0	0	-1	1	0	0
$x_3$	-5/2	56	1	0	2	-6	0	0
$x_4$	-1/4	16/3	0	1	1/3	-2/3	0	0
$F_3$	5/2	-56	0	0	-2	6	1	1

Mudou aqui

→

↓

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	0	24/5	0	0	-7/5	11/5	1/5	1/5
$x_3$	0	0	1	0	0	0	1	1
$x_4$	0	-4/15	0	1	2/15	-1/15	1/10	1/10
$x_1$	1	-112/5	0	0	-4/5	12/5	2/5	2/5

→

Base	$x_1$	$x_2$	$x_3$	$x_4$	$F_1$	$F_2$	$F_3$	$b$
Z	0	2	0	21/2	0	3/2	5/4	5/4
$x_3$	0	0	1	0	0	0	1	1
$F_1$	0	-2	0	15/2	1	-1/2	3/4	3/4
$x_1$	1	-24	0	6	0	2	1	1

# Limitações da Programação Linear:

- Coeficientes constantes;
- Divisibilidade;
- Aditividade;
- Proporcionalidade.