Interior-Point Method for Hydrothermal Dispatch Problem

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***Abstract*— This paper presents a model for the optimization of the hydrothermal dispatch. The model's goal is the minimization of the costs of electricity generation, which can be summarized as the costs of thermal generation and the costs incurred by energy deficits. The methodology proposed to solve this problem is the Interior-Point method which is widely used in several classes of problems, combined with ideas of the well-known Gauss-Newton method in nonlinear programming and Stationary Newton method. The combination of the aforementioned techniques together with the Interior-Point method presents good computational performance and satisfactory results when applied to the hydrothermal dispatch problem for a test system based on the Brazilian Interconnected System.**

***Index Terms*—Hydrothermal dispatch, nonlinear programming, Interior-Point method.**

I. Introduction

IN this paper we present a methodology for the optimization of the hydrothermal dispatch that involves two important and well-known methods: the Gauss-Newton method [1] and the Interior-Point method [2]. The first is generally applied to the minimization of the residue in minimum squares, which is equivalent to a problem of unconstrained minimization (or solving a nonlinear equation system). The second one has been successfully applied in constrained linear and nonlinear problems. Both use Newton's method [3]. The first approximates Newton's direction ignoring the term involving the Hessian matrix of the objective function. In other words, it can approximate Newton's direction in problems where the Hessian matrix has negligible influence. The second uses Newton's method to solve the nonlinear system created in each iteration. The algorithm for this method will be described in the next sections.

The methodology consists in using the Interior- Point method to solve the hydrothermal dispatch problem, which is modeled as a nonlinear, nonconvex problem involving both equality and inequality constraints and bounded variables. Additionally, the problem presents a nonlinear equality constraint whose Hessian matrix is too complex to determine analytically and too computationally expensive to approximate through numerical methods. Due to the model's graph structure, the Hessian matrix is sparse; several tests indicate it has little influence on the solution. Therefore, we discard the term involving the Hessian of nonlinear constraints based on the Gauss-Newton method to solve nonlinear systems. As will be shown, this won't compromise the convergence of the Interior-Point method.

Another feature of the present paper is the use the idea of Stationary Newton's method [4] to solve the nonlinear system created in the iterations of Interior Point method. The Stationary Newton's Method consists in calculating the Jacobian matrix, required by Newton's method in the initial point, and use it in all the iterations instead of calculating it in each new point.

A. Hydrothermal Dispatch

The hydrothermal dispatch of power systems with strong hydro predominance is a particular case of the larger problem of optimal reservoir management and operation. In the extensive state-of-the-art reviews by [5] and more recently by [6], the complexities of this class of problems are detailed. While there is a growing body of works based on nonlinear programming, models based on linear programming, network flow optimization, heuristics and particularly dynamic programming predominate. It is clear that computational feasibility has constrained further applications of nonlinear programming to the problem of hydrothermal dispatch.

This was the reasoning behind [7] and [8]'s decomposition approach. By applying Bender's decomposition to a model based on dynamic programming, one can solve the hydrothermal dispatch problem for large systems with

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stochastic inflows at reasonable computational cost. This feasibility came at the cost of several simplifying hypotheses, such as the need to linearize the hydropower generation function.

As computational power becomes more available, alternative approaches to hydrothermal dispatch optimization which can explicitly consider nonlinearities become increasingly interesting. This was the motivation for the research which resulted in the present paper.

B. The Brazilian Power System

Brazil is a country privileged in terms of water resources. According to official government date [9], in 2010, Brazil produced 509,2 TWh of electricity, of which 403,3 TWh (79,2%) were produced in hydropower plants, 72,2 TWh (14,2%) in fossil-fired or nuclear power plants and the remaining 33,7 TWh (6,6%) came from alternative sources. This strong reliance on hydropower is justified by its low operating cost and by its renewable nature. The downside to this reliance is the uncertainty inherent in energy demand and hydrologic inflows, which subjects a purely hydro system without thermal power backup to unacceptable levels of risk of energy deficits. For this reason, the Brazilian system is complemented by several thermal power plants, which can supply firm energy during drought periods, but do so at higher costs due to fuel consumption. Alternative and intermittent sources such as biomass and wind power are still in an incipient stage.

Another distinguishing feature of the Brazilian power system is the vast network of transmission lines, which connect nearly all load centers and power plants. In modeling terms, this creates a series of interconnected subsystems, able to transport significant amounts of energy from one area to another. The set of power plants and load centers linked to the network of transmission lines is denominated the Brazilian Interconnected System. Having a large network of transmission lines is advantageous to systems with high reliance on hydropower, as it allows the system operator to take advantage of the various hydrologic regimes. Thus, areas with surplus hydro generation can export energy to areas on drought, reducing spillage and overall thermal power generation.

These features characterize the Brazilian Interconnected System as a hydrothermal system with strong hydro predominance. The operation planning for such systems present a major technical challenge: the need to balance the conflicting goals of minimizing expensive thermal power generation while avoiding excessive water spillage. This defines the problem of hydrothermal dispatch: how much energy should be generated by thermal power plants to avoid energy deficits during droughts while minimizing fuel consumption?

Currently, the hydrothermal dispatch of the Brazilian Interconnected System is optimized through a methodology based on Stochastic Dynamic Dual Programming [7]–[8]. However, this methodology relies on several simplifications to achieve computational feasibility, such as linearization and reservoir aggregation. This paper presents a distinct approach to hydrothermal dispatch optimization based on nonlinear optimization through the Interior-Point method.

C. Paper Organization

The paper is organized as follows. Section II describes the proposed model to optimize the hydrothermal dispatch of predominantly hydro systems. Section III describes the Interior-Point method for the presented model. Section IV describes the application of the methodology to a test system based on a part of the Brazilian Power System and describes the implementation details. Section V presents the conclusions.

II. Mathematical Modeling

The variables involved in the models description are listed below:

$GT_{j,t}$ – generation of the thermal power plant *j* during period *t* [$MWmonth$];
$V_{r,t}$ – volume stored in the reservoir *r* for period *t* [hm^3];
$QVT_{r,t}$ – spilled flow from the reservoir *r* during period *t* [m^3/s];
$QC_{r,t}$ – turbined flow from the reservoir *r* during period *t* [m^3/s];
$INT_{i,t}$ – energy transported at line *i* during period *t* [$MWmonth$];
$DEF_{s,t}$ – energy deficit of the subsystem *s* during period *t* [$MWmonth$].

The nonlinear programming model for the energy optimization problem is:

$$minimize \sum_{t=1}^{T} \lambda_t \left[\sum_{j=1}^{J} CT_j(GT_{j,t}) + \sum_{s=1}^{S} CD_s(DEF_{s,t}) \right] \quad (1)$$

subject to:

$$\begin{aligned} V_{r,t}(10^5/\mathrm{S_m}) = V_{r,t-1}(10^5/\mathrm{S_m}) \\ + \sum_{r \in M_r} \left(QC_{l,t} + QVT_{l,t} \right) - QC_{r,t} \\ - QVT_{r,t} + Y_{r,t} - \sum_{l \in M_r} Y_{l,t} \end{aligned} \quad (2)$$

$$\sum_{j \in J_s} GT_{j,t} + \sum_{r \in R_s} GH_{r,t} + \sum_{i \in I_s} INT_{i,t} = D_{s,t} - DEF_{s,t} \quad (3)$$

$$QVT_{r,t} + QC_{r,t} \geq QMIN_{r,t} \quad (4)$$

$$\begin{gathered} GTMIN_{j,t} \leq GT_{j,t} \leq GTMAX_{j,t} \\ VMIN_{r,t} \leq V_{r,t} \leq VMAX_{r,t} \\ 0 \leq QVT_{r,t} \leq QVTMAX_r \\ QCMIN_r \leq QC_{r,t} \leq QCMAX_r \\ INTMIN_{i,t} \leq INT_{i,t} \leq INTMAX_{i,t} \\ 0 \leq DEF_{s,t} \end{gathered} \quad (5)$$

The objective function (1) minimizes the current value of the sum of costs of thermal power generation (CT_j) and energy deficits (CD_s). CT_j is the cost function of thermal power plant

j, which depends on the fuel used. CD_s is a function that expresses the economic cost of energy deficits in subsystem s. Both CT_j and CD_s are approximated by second degree polynomials; λ_t is the discount factor for period t.

The water balance constraint (2) expresses the relation between stored volume, inflows and outflows in a reservoir, where $Y_{r,t}$ represents the natural inflow of the reservoir r during period t $[m^3/s]$, M_r represents the set of reservoirs immediately upstream of the reservoir r. To be able to perform this operation it requires a change of units, turning the volume from hm^3 to m^3/s. Thus, in the constraint (2), the volume is multiplied by $10^5/S_m$ where S_m is the number of seconds per month.

The energy balance constraint (3) represents the relationship between energy generation, demand and deficit (load shedding) in a subsystem, where $D_{s,t}$ is the energy demand in the subsystem s in the period t [*MWmonth*], I_s represents the set of subsystems connected through transmission lines to subsystem s, J_s is the set of thermal power plants in the subsystem s, R_s is the set of hydropower plants in the subsystem s and $GH_{r,t}$ is the energy produced at the plant r in the period t.

The minimum total outflow constraint (4) guarantees the use of water resources for other activities besides electricity production, such as flood control, river navigability, irrigation, etc., where $QMIN_{r,t}$ represents the minimum total outflow of the reservoir r in the period t $[m^3/s]$.

Variables boundary constraints are represented by (5) and denote the upper and lower bounds for: thermal power generation, reservoirs volume, turbined and spilled volumes, energy transfer through transmission lines between subsystems and energy deficit for each subsystem, respectively.

III. Interior-Point Method

The hydrothermal dispatch presented can be written mathematically as follows:

$$\begin{array}{rl} \textit{minimize} & f(x) \\ \textit{subject to:} & g(x)=0 \\ & Ax=b \\ & Cx\le c \\ & l\le x\le u \end{array} \qquad (6)$$

where $x \in \mathbb{R}^n$ is the decision variables vector that for the model presented in Section II involves: thermal power generation, spilled and turbined flows, reservoir volume, energy transport between subsystems and deficit, $f(x) \in \mathbb{R}$ is the nonlinear objective function, $g(x) \in \mathbb{R}^m$ is the nonlinear constraint that represent load, $Ax = b$ and $Cx \le c$ are linear constraints that represent the water balance and total outflow, respectively, $A, C \in \mathbb{R}^{p\times n}$, $b, c \in \mathbb{R}^p$ and finally $l \le x \le u$ represents the lower and upper bounds of decision variables, $l, u \in \mathbb{R}^n$.

The method used to solve this energetic problem is the Interior-Point method for nonlinear programming. First, positive slack variables r, s, t are used to transform (6)'s inequality constraints into equality constraints, as in [10]–[11].

$$\begin{array}{rl} \textit{minimize} & f(x) \\ \textit{subject to:} & g(x)=0 \\ & Ax=b \\ & Cx-c+r=0 \\ & -x+l+s=0 \\ & x-u+t=0 \\ & r\ge 0, s\ge 0, t\ge 0 \end{array} \qquad (7)$$

The variables that must be non-negatives are penalized by adding the logarithmic barrier function [4] to the objective function:

$$f(x)-\mu\left[\sum_{i=1}^{p}\ln r_i+\sum_{i=1}^{n}(\ln s_i+\ln t_i)\right] \qquad (8)$$

where $\mu > 0$ is the barrier parameter, which has the property of tending to zero when x approaches the optimal solution.

The Lagrangian function associated with the penalized problem is:

$$\begin{aligned} L(x,\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,r,s,t) &= f(x)+\lambda_1^T g(x)+\lambda_2^T(Ax-b) \\ &+\lambda_3^T(Cx-c+r)+\lambda_4^T(-x+l+s) \\ &+\lambda_5^T(x-u+t) \\ &-\mu\left[\sum_{i=1}^{p}\ln r_i+\sum_{i=1}^{n}(\ln s_i+\ln t_i)\right] \end{aligned} \qquad (9)$$

where λ_j are the Lagrange multipliers, $j = 1, \ldots, 5$.

The first-order optimality conditions or Karush-Kuhn-Tucker conditions [12]–[13] are necessary conditions that an optimal solution must satisfy. They are applied to the problem, resulting in a nonlinear system. Thus, Newton's method is applied resulting in the following linear system:

$$J(y)dy=-\nabla L(y) \qquad (10)$$

where: $y = (x, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, r, s, t)^T$,

$$J(y)=\begin{bmatrix} H(x) & J_g(x)^T & A^T & C^T & -I & I & 0 & 0 & 0 \\ J_g(x) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ -I & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & R & 0 & 0 & \Lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & S & 0 & 0 & \Lambda_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & T & 0 & 0 & \Lambda_5 \end{bmatrix},$$

$dy = (dx, d\lambda_1, d\lambda_2, d\lambda_3, d\lambda_4, d\lambda_5, dr, ds, dt)^T$ and

$\nabla L(y) = \left(\nabla_x L, \nabla_{\lambda_1} L, \nabla_{\lambda_2} L, \nabla_{\lambda_3} L, \nabla_{\lambda_4} L, \nabla_{\lambda_5} L, \nabla_r L, \nabla_s L, \nabla_t L\right)^T$ is the Lagrangian function gradient.

$H(x) = \nabla^2 f(x) + \sum_{i=1}^{m} \lambda_{1i} \nabla^2 g_i(x)$, $J_g(x)$ is the constraint $g(x)$Jacobian matrix of the, R, S, T, Λ_3, Λ_4 and Λ_5 are diagonal matrices, with diagonal elements given, respectively, by components of the vectors r, s, t, λ_3, λ_4 and λ_5, and I represents the identity matrix of appropriate size.

First, we isolate *dr*, *ds* and *dt* in system (10):

$$dr = -\left(\nabla_{\lambda_3} L + Cdx\right) \quad (11)$$

$$ds = -\left(\nabla_{\lambda_4} L - dx\right) \quad (12)$$

$$dt = -\left(\nabla_{\lambda_5} L + dx\right). \quad (13)$$

The directions $d\lambda_3$, $d\lambda_4$ and $d\lambda_5$ are also isolated in (10):

$$d\lambda_3 = -R^{-1}(\nabla_r L + \lambda_3 dr) \quad (14)$$

$$d\lambda_4 = -S^{-1}(\nabla_s L + \lambda_4 ds) \quad (15)$$

$$d\lambda_5 = -T^{-1}(\nabla_t L + \lambda_5 dt). \quad (16)$$

Replacing (11), (12) and (13) in (14), (15) and (16) respectively, one obtains $d\lambda_3$, $d\lambda_4$ and $d\lambda_5$ directions, which depend only on dx. Replacing $d\lambda_3$, $d\lambda_4$ and $d\lambda_5$ on the first equation of the system (10) and grouping the terms, we have:

$$\begin{aligned}[H(x) + C^T R^{-1}\lambda_3 C + S^{-1}\lambda_4 + T^{-1}\lambda_5]dx \\ + J_g(x)^T d\lambda_1 + A^T d\lambda_2 = F\end{aligned} \quad (17)$$

where F is a constant vector. The resulting linear system to be solved is:

$$\begin{bmatrix} \bar{H}(x) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} F \\ \nabla_\lambda L \end{bmatrix} \quad (18)$$

where

$$B = \left[J_g(x) \;\; A\right]^T, \;\; d\lambda = [d\lambda_1 \;\; d\lambda_2]^T, \;\; \nabla_\lambda L = -\left[\nabla_{\lambda_1} L \;\; \nabla_{\lambda_2} L\right]^T,$$

$$\begin{aligned}\bar{H}(x) = \nabla^2 f(x) + \sum_{i=1}^{m} \lambda_{1i} \nabla^2 g_i(x) + C^T R^{-1} \Lambda_3 C \\ + S^{-1}\Lambda_4 + T^{-1}\Lambda_5\end{aligned} \quad (19)$$

and R^{-1} is a diagonal matrix composed by the elements of the r^{-1} vector. The matrices S^{-1} and T^{-1} are defined in the same way.

Solving the system (18), one finds *dx*, $d\lambda_1$ and $d\lambda_2$. Thus, the other directions are found through the equations (11), (12), (13), (14), (15) and (16).

After finding the Newton's direction, the step length α that will be taken in this direction is computed, in order to not violate the positivity of some variables. All the variables are updated as $y = y + \alpha\, dy$. Finally, the barrier parameter μ is updated. Several heuristics can be found in the literature, in particular, [10]–[11] suggest the following formula:

$$\mu = \frac{r^T\lambda_3 + s^T\lambda_4 + t^T\lambda_5}{2n + p}. \quad (20)$$

The process is repeated until the point found satisfies a stopping criterion that will be described in Section IV.

As the hydrothermal dispatch problem is large, reducing the execution time can be very profitable as the problem dimension increases. Using the idea of Stationary Newton's method rather than the Newton's method in Interior-Point algorithm increases the number of iterations, but decreases the execution time considerably. Computing the matrix of coefficients in the system (18) for the initial point and using it in all iterations has proven itself as a very efficient method.

A great difficulty in applying this method to the hydrothermal dispatch problem as previously described, is the computation of the Hessian matrix for nonlinear constraint (2). This constraint involves at least two nonlinear functions: forebay water level, a nonlinear function of the stored volume; and tailrace water level, a nonlinear function of total outflow (turbined plus spilled). For the Brazilian System, both functions are usually modeled as polynomials up to fourth degree. Those two polynomials are added and the final polynomial is multiplied by the turbined flow variable. Thus, the exact computation of the Hessian matrix is burdensome, demanding the use of numerical approximation. However, an approximation is required for each iteration of the Interior-Point method, which can greatly increase the computational effort for large scale problems.

It is known that if coefficients matrix in (18) is invertible, then the system has solution and it is unique. According to [14], for this to occur, $\bar{H}(x)$ must be positive definite (as will be proved) and B must have full column rank, as it is assumed to be true.

The proposal made in this paper is to use the idea of Gauss-Newton method, that disregard second order information of the problem. In other words, the term involving the Hessian matrix of the nonlinear constraints $[\sum_{i=1}^{m} \lambda_{1_i} \nabla^2 g_i(x)]$ in equation (19) is cut out, i.e.,

$$\bar{H}(x) = \nabla^2 f(x) + C^T R^{-1}\Lambda_3 C + S^{-1}\Lambda_4 + T^{-1}\Lambda_5. \quad (21)$$

Lemma 1: Let $R^{-1} = diag(r^{-1})$, $S^{-1} = diag(s^{-1})$, $T^{-1} = diag(t^{-1})$, $\Lambda_3^{-1} = diag(\lambda_3^{-1})$, $\Lambda_4^{-1} = diag(\lambda_4^{-1})$ and $\Lambda_5^{-1} = diag(\lambda_5^{-1})$ with $(r, s, t, \lambda_3, \lambda_4, \lambda_5) > 0$ given respectively in (8) and (10). Moreover, let $f(x)$ the objective function given in (1) and $C \in \mathbb{R}^{p \times n}$ the coefficients matrix of linear inequalities constraints of the problem (7). Then the following items are true:

i. $S^{-1}\Lambda_4 > 0$ and $T^{-1}\Lambda_5 > 0$, i.e., they are positive definite matrices;
ii. $C^T R^{-1}\Lambda_3 C > 0$, i.e., it is positive definite matrix;
iii. $\nabla^2 f(x) > 0$, i.e., the Hessian matrix of $f(x)$ is positive definite.

Proof:

i. $S^{-1}\Lambda_4$ and $T^{-1}\Lambda_5$ are diagonal matrices with positive elements, then they are positive definite;
ii. Let $D = R^{-1}\Lambda_3$, then:
$x^T C^T D C x = x^T C^T D^{\frac{1}{2}} D^{\frac{1}{2}} C x = x^T C^T \left(D^{\frac{1}{2}}\right)^T D^{\frac{1}{2}} C x = \left(D^{\frac{1}{2}} C x\right)^T \left(D^{\frac{1}{2}} C x\right) = \left\| D^{\frac{1}{2}} C x \right\|_2^2 > 0, \; \forall x \neq 0;$

iii. $\nabla^2 f(x)$ is positive definite matrix because the function $f(x)$ is a sum of two second order polynomials with positive quadratic term coefficients. Then $f(x)$ is convex and its Hessian matrix is positive definite. ■

Theorem 1: The matrix $\overline{H}(x)$ of reduced system (18), given in equation (21), is positive definite.

Proof: In fact, let $x \in \mathbb{R}^n$ and $x \neq 0$, then by (21):

$$x^T\overline{H}x = x^T(\nabla^2 f(x) + C^T R^{-1}\Lambda_3 C + S^{-1}\Lambda_4 + T^{-1}\Lambda_5)x$$
$$= x^T\nabla^2 f(x)x + x^T C^T R^{-1}\Lambda_3 Cx + x^T S^{-1}\Lambda_4 x + x^T T^{-1}\Lambda_5 x.$$

By Lemma 1, $x^T\overline{H}x$ is the positive definite matrices sum, then $x^T\overline{H}x > 0, \forall x \in \mathbb{R}^n, x \neq 0$. ■

Note: In systems with the structure as (18), where the positivity of $\overline{H}$ cannot be ensured, $\gamma > 0$ is chosen such that $\overline{\overline{H}} = \overline{H} + \gamma I$ is positive definite, as in [11].

IV. Computational Tests and Results Analysis

In this section, the implementation of the proposed methodology will be shown. Since Theorem 1 guarantees that Newton's direction can be determined, the reduced system (18) and consequently the system (10) has a solution, not considering computational rounding errors.

All the mathematical modeling, as well as the Interior-Point method were implemented in Matlab® 7.10.0 (R2010a).

The methodology was tested in several test systems based on the Brazilian Interconnected System. In this paper, results will be presented for a system composed of 21 hydro plants, 32 thermal power plants and 3 interconnected subsystems. The period considered was January 1952 to January 1957, which was a drought period. Table I shows the composition of the three subsystems considered.

TABLE I
Power plants and demand for the considered subsystems.

Subsystem	# hydro	# thermal	Demand ($MWmonth$)
1	10	18	13740
2	10	14	9918
3	1	0	0

As can be seen in Table I, the subsystem 3 is composed by a single hydro plant, namely, Itaipu. Thus, subsystem 3 is a subsystem whose only purpose is to export energy to subsystems 1 and 2.

The point x^0 for the method follows the premise that all the inflow is turbined. Reservoirs are considered initially full and its final volumes should be between 70% and 100% of the maximum capacity. The parameters used in the algorithm implementation are: $\mu^0 = 100$, Lagrange multipliers and slack variables are initialized as appropriately sized vectors of ones. Adopted stopping criteria are Karush-Kuhn-Tucker conditions or maximum number of iterations.

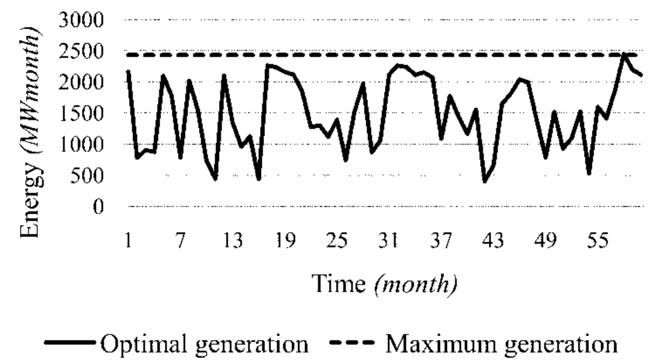

Fig. 1. Hydro generation for subsystem 1.

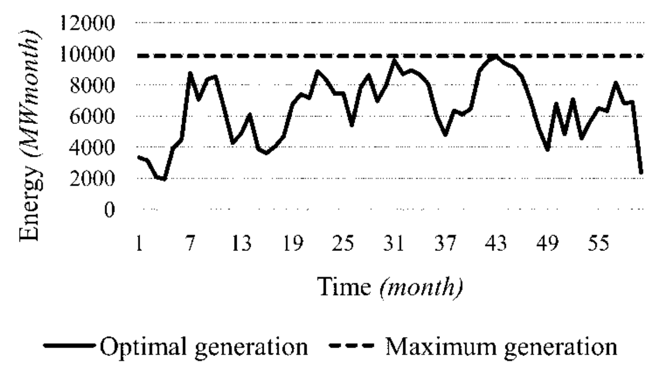

Fig. 2. Hydro generation for subsystem 2.

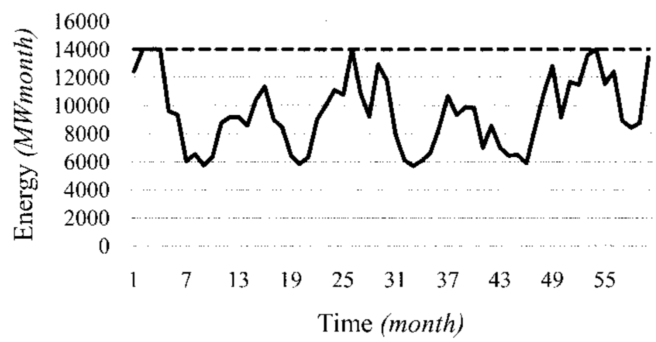

Fig. 3. Hydro generation for subsystem 3.

Figs. 1, 2 and 3 represent, respectively, the sum of energy generated by hydro plants of the subsystems 1, 2 and 3. It is important to remember that plants are being considered individually; their generation was summed for better viewing.

When the plant reservoirs are analyzed individually, it is assumed that spills occur when both reservoirs capacity and turbined flow reach their upper bounds. It is also evident that in periods prior to large inflows, the reservoirs are emptied to accommodate future inflows or, if future inflows are scarce, the reservoirs levels are preserved to ensure a higher head and increase the plant's energy output.

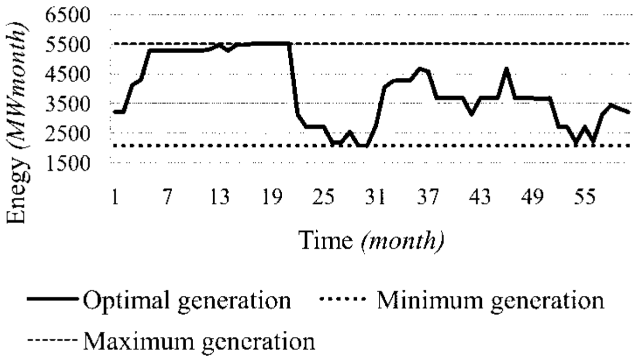

Fig. 4. Thermal generation for subsystem 1.

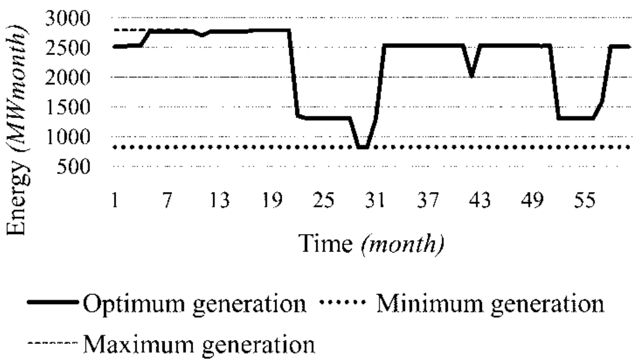

Fig. 5. Thermal generation for subsystem 2.

Figs. 4 and 5 show the thermal power generation of subsystems 1 and 2 respectively. It is noted that maximum possible energy was generated in several periods, in order to satisfy energy demand. This is justified because, as already mentioned, the period considered has low hydrologic inflows.

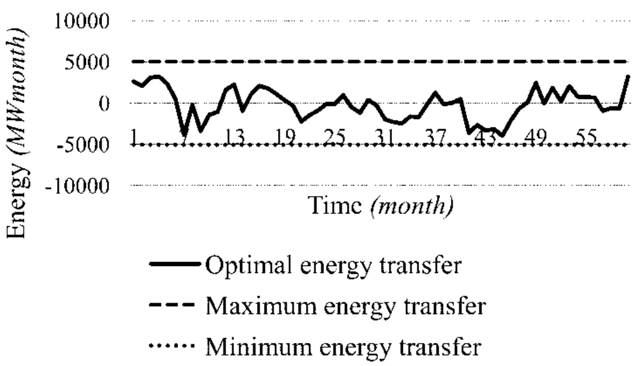

Fig. 6. Energy transfer between subsystems 1 and 2.

Fig. 6 shows the energy transfer between the subsystems 1 and 2. Positive values represent energy exported from subsystem 1 to 2. Negative values represent the opposite, i.e., energy exported from subsystem 2 to 1.

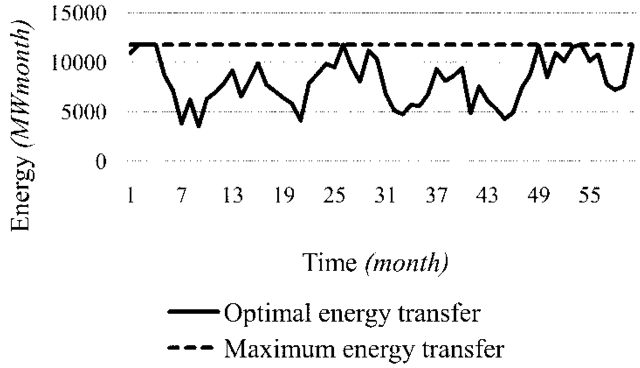

Fig. 7. Energy transfer between subsystems 3 and 1.

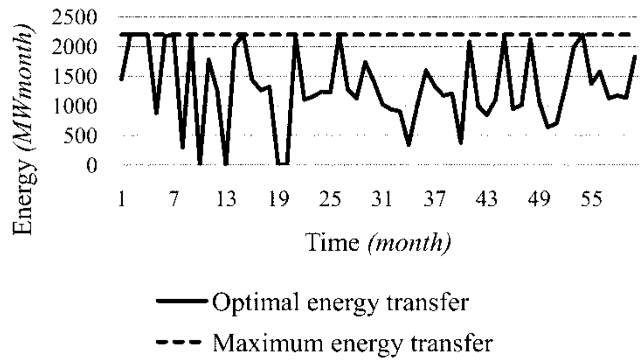

Fig. 8. Energy transfer between subsystems 3 and 2.

Figs. 7 and 8 show energy exported from subsystem 3 to 1 and 2. As subsystem 3 is composed solely of Itaipu, it is a purely generating subsystem.

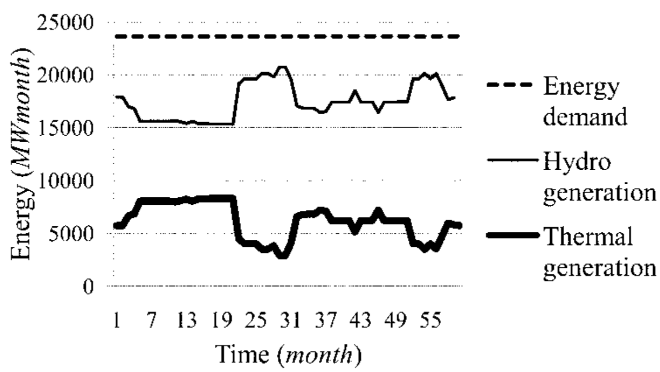

Fig. 9. Energy balance between hydro and thermal generation.

Fig. 9 shows the energy generated distribution between the hydro and thermal power plants for a test system. The sum of the two kinds of energy gives exactly the system demand, because in this case there was no energy deficit. It is observed that in the periods where hydro plants operate at reduced capacity (low inflows), the thermal power plants are more required and vice versa.

Several tests were carried out, changing the period considered and varying the demand too. The processing time on a conventional computer to solve the presented problem was approximately 20 minutes.

V. Conclusion

Many works on hydrothermal dispatch tend to simplify the problem with the aim of achieving a simpler, possibly linear model. To tackle the hydrothermal dispatch problem through nonlinear optimization, an efficient search algorithm with fast and accurate answers is needed. The Interior-Point method combined with ideas of Gauss-Newton and Stationary Newton method were shown to be very effective when applied to this problem. As shown, disregarding the second order information of the nonlinear constraint did not affect the method's convergence, and greatly improved the computational feasibility of the proposed method.

From the energy point of view, it can be said that the proposed methodology meets all requirements of the problem. All the generation bounds, reservoirs capacity and power transmission lines were respected, as well as meeting the total demand. It was observed that the energy from thermal power plants was only necessary in periods where the inflows were low. The Interior-Point method held high levels of reservoirs, allowing its use in periods of low inflows.

The methodology proposed in this research was tested for several periods and different demands of energy. The result was very good and, for further research, the main goal would be apply it to the entire Brazilian Interconnected System.

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