CM201 - Cálculo Diferencial e Integral I
Lista de Exercícios 8

1. Calcule a derivada $f'(x)$ das funções

   (a) $f(x) = \dfrac{1}{x+2}$ (b) $f(x) = x^2 - 3x + 4$ (c) $f(x) = 1 + \sqrt{x}$

   primeiro pela definição (usando limites) e depois usando as regras de derivação.

2. Usando as regras de derivação, calcule as derivadas das seguintes funções:

   (a) $f(x) = \dfrac{x^3}{3} + \dfrac{x^2}{2} + \dfrac{x}{4}$ (b) $f(x) = \dfrac{1}{3x^2} - \dfrac{5}{2x}$ (c) $f(x) = 2\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$

   (d) $f(x) = \dfrac{2x+5}{3x-2}$ (e) $f(x) = \dfrac{1+3x}{3x}(3-x)$ (f) $f(x) = \sqrt{x} + \dfrac{1}{x^2}$

   (g) $f(x) = \dfrac{1}{x} + \dfrac{2}{x^2} + \dfrac{3}{x^3}$ (h) $f(x) = \sqrt[3]{x} + 3\sqrt[4]{x} - x^{-4}$ (i) $f(x) = \dfrac{3}{x^2} + \dfrac{x^2}{3}$

   (j) $f(x) = 3x^{-4} - 4x^{-3}$ (k) $f(x) = -\sqrt{x} + \sqrt[3]{x^2} + 3x^5$ (l) $f(x) = 8x^{-8} - 7x^{-7} + 6x^{-6}$

3. Calcule a derivada de $y = (2x+3)(5x^2 - 4x)$ das seguintes maneiras:

   (a) pela regra do produto;

   (b) multiplicando os fatores para produzir uma soma de termos mais simples para derivar.

4. Calcule as derivadas das seguintes funções:

   (a) $f(x) = \dfrac{x^2 + 3e^x}{2e^x - x}$ (b) $f(x) = -10x + 3\cos(x)$ (c) $f(x) = x^2\cos(x)$

   (d) $f(x) = x^5 + 5^x$ (e) $f(x) = \ln x + e^x + x + 1$ (f) $f(x) = \operatorname{sen} x - 2\cos x$

5. Calcule $h'(x)$ pela regra da cadeia nos casos abaixo:

   (a) $h(x) = (4 - 3x)^9$ (b) $h(x) = \sqrt{3x^2 - 4x + 6}$ (c) $h(x) = e^{x^2}$

   (d) $h(x) = \operatorname{sen}(x^2)$ (e) $h(x) = 5\cos^{-4}(x)$ (f) $h(x) = 1 + \operatorname{sen}(x)^2 + \operatorname{sen}(x)^4$

   (g) $h(x) = \ln(3x)$ (h) $h(x) = \ln(x^3)$ (i) $h(x) = (\ln(x))^3$ (j) $h(x) = 4\cos\left(\pi x + \dfrac{\pi}{3}\right)$

6. Calcule as derivadas das seguintes funções:

   (a) $f(x) = 3\operatorname{sen} x + \cos\dfrac{x}{2}$ (b) $h(x) = -2\operatorname{sen} 2x - 3\cos 3x + \operatorname{sen} x$

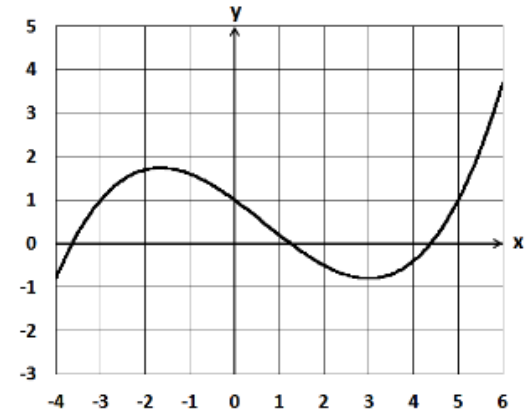
   (c) $f(x) = 3\operatorname{sen}(2x+5) - 4\cos(-x+2) + 2\cos(-3x)$ (d) $f(x) = \cos x + 2\cos 2x + 3\cos 3x$

7. Encontre as retas tangentes à função $f(x)$ no ponto x_0 e represente graficamente no mesmo sistema de coordenadas os gráficos de $f(x)$ e da reta tangente.

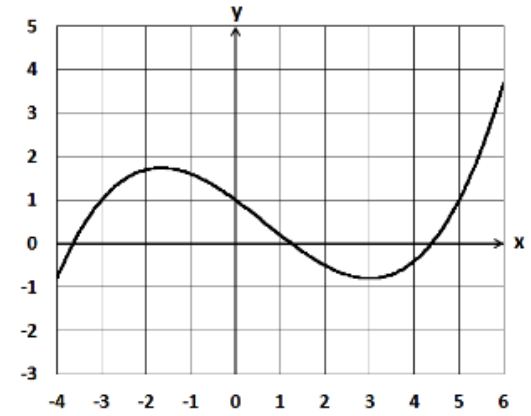
   (a) $f(x) = \sqrt{x}$; $x_0 = 4$ (b) $f(x) = \dfrac{1}{x}$; $x_0 = -1$ (c) $f(x) = \ln(x)$; $x_0 = 1$

8. Abaixo temos esboços do gráfico da função $f(x) = \frac{x^3}{20} - \frac{x^2}{10} - \frac{3x}{4} + 1$. Calcule a reta tangente ao gráfico nos pontos x_0 dados e trace essa reta junto com o gráfico de f.

(a) $x_0 = -3$



(b) $x_0 = 0$



Respostas:

1. (a) $\frac{-1}{(x+2)^2}$ (b) $2x - 3$ (c) $\frac{1}{2\sqrt{x}}$

2. (a) $x^2 + x + \frac{1}{4}$ (b) $\frac{-2}{3x^3} + \frac{5}{2x^2}$ (c) $\frac{1}{\sqrt{x}} - \frac{1}{\sqrt[3]{x}}$ (d) $\frac{-19}{(3x-2)^2}$ (e) $\frac{-1-x^2}{x^2}$

   (f) $\frac{x^{-\frac{1}{2}}}{2} - 2x^{-3}$ (g) $-x^{-2} - 4x^{-3} - 9x^{-4}$ (h) $\frac{x^{-\frac{2}{3}}}{3} + \frac{3x^{-\frac{3}{4}}}{4} + 4x^{-5}$

   (i) $-6x^{-3} + \frac{2x}{3}$ (j) $-12x^{-5} + 12x^{-4}$ (k) $-\frac{x^{-\frac{1}{2}}}{2} + \frac{2x^{-\frac{1}{3}}}{3} + 15x^4$

   (l) $-64x^{-9} + 49x^{-8} - 36x^{-7}$

3. $30x^2 + 14x - 12$

4. (a) $\frac{xe^x - x^2 - 2x^2e^x + 3e^x}{(2e^x - x)^2}$ (b) $-10 - 3\text{sen}\,(x)$ (c) $2x\cos(x) - x^2\text{sen}\,(x)$

   (d) $5x^4 + (\ln 5)5^x$ (e) $\frac{1}{x} + e^x + 1$ (f) $\cos x + 2\text{sen}\,x$

5. (a) $-27(4-3x)^8$ (b) $\dfrac{3x-2}{\sqrt{3x^2-4x+6}}$ (c) $2xe^{x^2}$ (d) $2x\cos(x^2)$

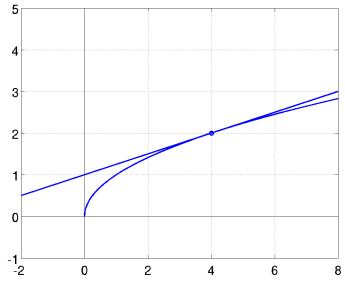
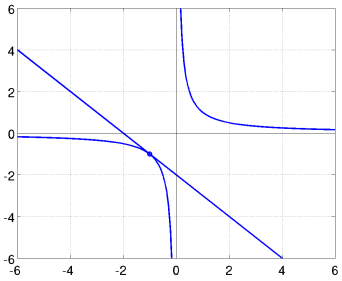
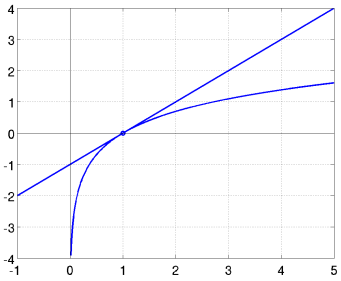
(e) $20\text{sen}\,(x)\cos^{-5}(x)$ (f) $2\text{sen}\,(x)\cos(x)+4\text{sen}^{\,3}(x)\cos(x)$

(g) $\dfrac{1}{x}$ (h) $\dfrac{3}{x}$ (i) $\dfrac{3(\ln(x))^2}{x}$ (j) $-4\pi\text{sen}\left(\pi x+\dfrac{\pi}{3}\right)$

6. (a) $3\cos x-\dfrac{1}{2}\text{sen}\,\dfrac{x}{2}$ (b) $-4\cos 2x+9\text{sen}\,3x+\cos x$

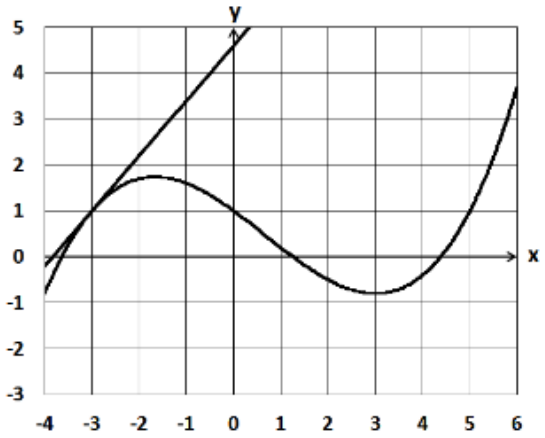
(c) $6\cos(2x+5)-4\text{sen}\,(-x+2)+6\text{sen}\,(-3x)$ (d) $-\text{sen}\,x-4\text{sen}\,2x-9\text{sen}\,3x$

7. (a) $r(x)=\dfrac{x}{4}+1$ (b) $r(x)=-x-2$ (c) $r(x)=x-1$

8.

(a) $y=\dfrac{6x}{5}+\dfrac{23}{5}$



(b) $y=1-\dfrac{3x}{4}$