Universidade Federal do Paraná
Programa de Pós-Graduação em Geologia

GEOL7048: Tópicos Especiais em Geologia Exploratória II

Métodos semiquantitativos

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Aula 14

- Derivada via FFT (continuação)

- Derivada vertical

Cálculo da derivada via FFT

$$\frac{df}{dx} = \mathcal{T}^{-1}(i2\pi\kappa\hat{f}) = \mathcal{T}^{-1}\left(i2\pi\kappa\mathcal{T}(f)\right)$$

Passos do cálculo:

```
>> k = (-N/2+1):(N/2);
>> kappa = (k - 1)/(b-a);
>> F = dx*fft(f);
>> F = fftshift(F);
>> F = exp(-I*2*pi*a*kappa).*F;

>> dF = I*(2*pi)*kappa.*F;
>> dF = exp(I*2*pi*a*kappa).*dF;
>> dF = ifftshift(dF);
>> df = (1/dx)*ifft(dF);
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OBS: compare `fftshift(fftshift(1:11))` com `ifftshift(fftshift(1:11))`

Cálculo da derivada via FFT

Correção nas bordas: buscamos uma extensão de $f(x)$ da forma

$$F(x) = \left(A + Bx + Cx^2\right) e^{Dx}$$

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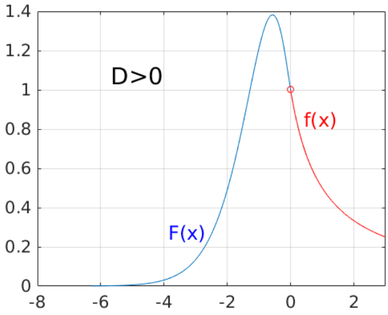
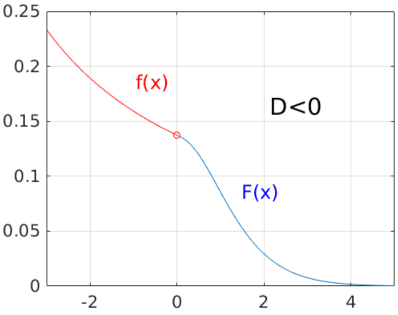
Escolhemos o parâmetro D para decaimento/crescimento em $x = 0$





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Escolhemos A, B, C tais que

$$F(0) = f(0), \qquad \frac{dF}{dx}(0) = \frac{df}{dx}(0), \qquad \frac{d^2F}{dx^2}(0) = \frac{d^2f}{dx^2}(0)$$

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Resultado: $A = f(0)$, $B = \frac{df}{dx}(0) - AD$, $C = \frac{d^2f}{dx^2}(0) - 2BD - AD^2$

Cálculo da derivada via FFT

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Resultado: $A = f(0)$, $B = \frac{df}{dx}(0) - AD$, $C = \frac{d^2f}{dx^2}(0) - 2BD - AD^2$

Em geral: $F(x) = \left(A + B(x - \bar{x}) + C(x - \bar{x})^2\right) e^{D(x-\bar{x})}$,

$$A = f(\bar{x}), \qquad B = \frac{df}{dx}(\bar{x}) - AD, \qquad C = \frac{d^2f}{dx^2}(\bar{x}) - 2BD - AD^2$$

Derivada vertical

Vimos que

$$\frac{\partial^2 f}{\partial z^2}(x, y, 0) = -\frac{\partial^2 f}{\partial x^2}(x, y, 0) - \frac{\partial^2 f}{\partial y^2}(x, y, 0)$$

Derivada vertical

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Transformada de Fourier 2D:

$$\mathcal{T}(f) = \hat{f}(\kappa_x, \kappa_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x) e^{-i2\pi\kappa_x x} e^{-i2\pi\kappa_y y}\, dxdy$$

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Transformadas das derivadas parciais:

$$\mathcal{T}\left(\frac{\partial f}{\partial x}\right) = (i2\pi\kappa_x)\mathcal{T}(f), \quad \mathcal{T}\left(\frac{\partial f}{\partial y}\right) = (i2\pi\kappa_y)\mathcal{T}(f)$$

Derivada vertical

Vimos que

$$\frac{\partial^2 f}{\partial z^2}(x, y, 0) = -\frac{\partial^2 f}{\partial x^2}(x, y, 0) - \frac{\partial^2 f}{\partial y^2}(x, y, 0)$$

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$$\mathcal{T}(f) = \hat{f}(\kappa_x, \kappa_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x) e^{-i2\pi(\kappa_x x + \kappa_y y)}\, dxdy$$

Transformadas das derivadas parciais:

$$\mathcal{T}\left(\frac{\partial^2 f}{\partial x^2}\right) = (i2\pi\kappa_x)^2 \mathcal{T}(f), \quad \mathcal{T}\left(\frac{\partial^2 f}{\partial y^2}\right) = (i2\pi\kappa_y)^2 \mathcal{T}(f)$$

Derivada vertical

Vimos que

$$\frac{\partial^2 f}{\partial z^2}(x, y, 0) = -\frac{\partial^2 f}{\partial x^2}(x, y, 0) - \frac{\partial^2 f}{\partial y^2}(x, y, 0)$$

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OBS: $\boldsymbol{\kappa} = (\kappa_x, \kappa_y)$ é o vetor de onda $\left(|\boldsymbol{\kappa}| = \sqrt{\kappa_x^2 + \kappa_y^2}\right)$

Derivada vertical

$$\frac{\partial^2 f}{\partial z^2}(x,y,0) = -\frac{\partial^2 f}{\partial x^2}(x,y,0) - \frac{\partial^2 f}{\partial y^2}(x,y,0)$$

No domínio de Fourier,

$$\mathcal{T}\left(\frac{\partial^2 f}{\partial z^2}\right) = \mathcal{T}\left(-\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2}\right)$$

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$$\mathcal{T}\left(\frac{\partial^2 f}{\partial z^2}\right) = -(i)^2(2\pi\kappa_x)^2\mathcal{T}(f) - (i)^2(2\pi\kappa_y)^2\mathcal{T}(f)$$

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No domínio de Fourier,

$$\mathcal{T}\left(\frac{\partial^2 f}{\partial z^2}\right) = (2\pi\kappa_x)^2\mathcal{T}(f) + (2\pi\kappa_y)^2\mathcal{T}(f) = (2\pi|\boldsymbol{\kappa}|)^2\mathcal{T}(f)$$

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Segundo Blakely (p. 326),

$$\mathcal{T}\left(\frac{\partial f}{\partial z}\right) = 2\pi|\boldsymbol{\kappa}|\mathcal{T}(f), \quad |\boldsymbol{\kappa}| = \sqrt{\kappa_x^2 + \kappa_y^2}$$

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$$\mathcal{T}\left(\frac{\partial f}{\partial z}\right) = 2\pi|\boldsymbol{\kappa}|\mathcal{T}(f), \quad |\boldsymbol{\kappa}| = \sqrt{\kappa_x^2 + \kappa_y^2}$$

Usando a transformada inversa,

$$\frac{\partial f}{\partial z} = \mathcal{T}^{-1}\Big(2\pi|\boldsymbol{\kappa}|\mathcal{T}(f)\Big).$$

Derivada vertical

Em 1D, $\dfrac{\partial f}{\partial z} = \mathcal{T}^{-1}\Big(2\pi|\kappa|\mathcal{T}(f)\Big)$.

Exemplo: anomalia gerada por um dique infinito

$$f(x,z) = A\left[\cos Q\left(\tan^{-1}\frac{x+a}{h} - \tan^{-1}\frac{x-a}{h}\right) + \sin Q \frac{1}{2}\ln\frac{(x-a)^2+h^2}{(x+a)^2+h^2}\right]$$

- A: amplitude
- Q: ângulo efetivo
- a: meia-largura
- h: profundidade