

Universidade Federal do Paraná  
Programa de Pós-Graduação em Geologia

# GEOL7048: Tópicos Especiais em Geologia Exploratória II

## Métodos semiquantitativos

Saulo P. Oliveira

Departamento de Matemática, Universidade Federal do Paraná



# Aula 17

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- Transformada Signum

Anomalia magnética de um dique vertical no polo:

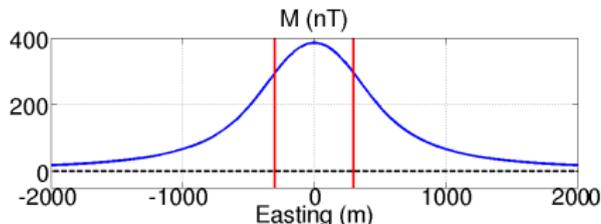
$$M(x) = M(x, z = 0) = A \left( \tan^{-1} \frac{x + a}{h} - \tan^{-1} \frac{x - a}{h} \right)$$

De Souza and Ferreira (2012) On the use of derivatives for interpreting magnetic anomalies due to dyke-like bodies: Qualitative and quantitative analysis. [Intern. Geoph. Conf/ and Oil & Gas Exhib.](#), SEG.

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Para  $A = 300$  nT,  $h = 400$  m e  $a = 300$  m:

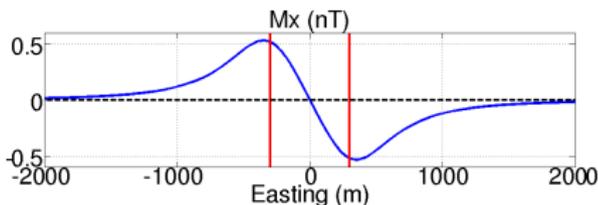
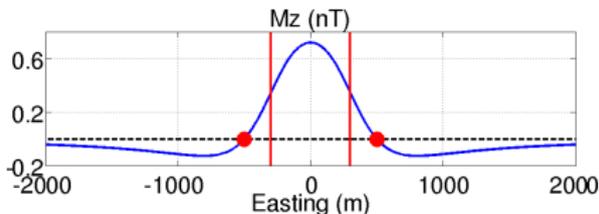


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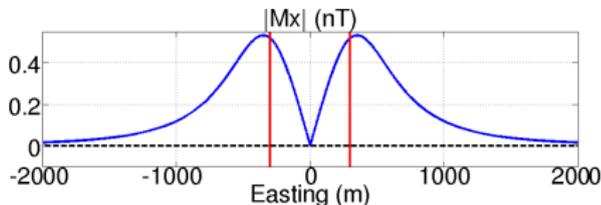
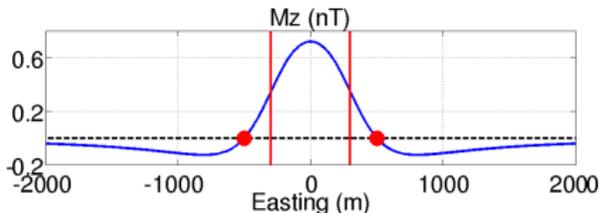


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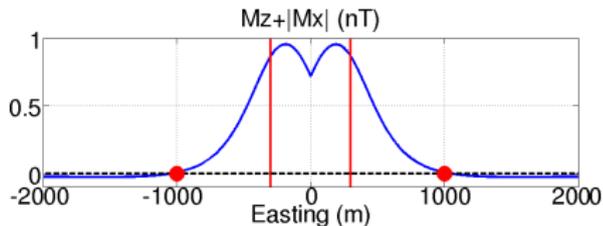
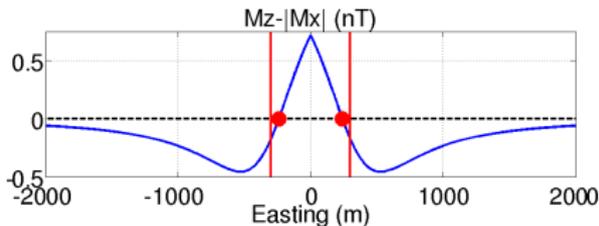
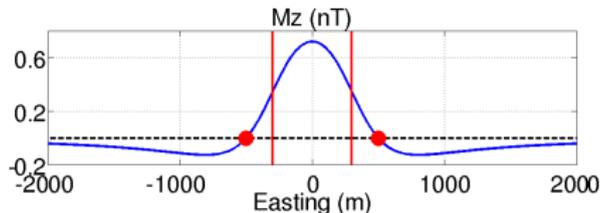


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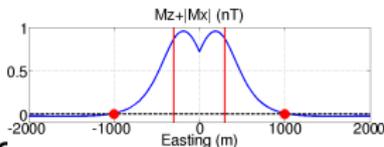
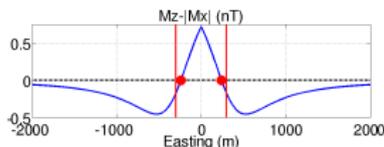
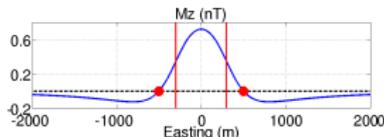
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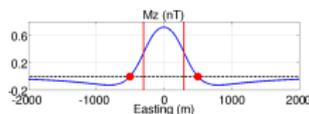
S

$$M_z(x_v) = \frac{\partial M}{\partial z}(x_v) = 0 \implies x_v = \pm \sqrt{a^2 + h^2}$$

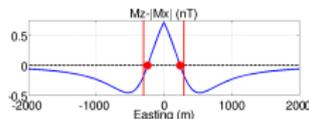
$$M_z(x_{vh-}) - |M_x(x_{vh-})| = 0 \\ \implies x_{vh-} = \pm(h - \sqrt{a^2 + 2h^2})$$

$$M_z(x_{vh+}) + |M_x(x_{vh+})| = 0 \\ \implies x_{vh+} = \pm(h + \sqrt{a^2 + 2h^2})$$

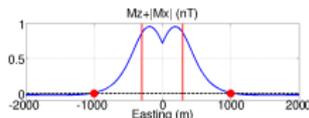
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$$x_v = \pm\sqrt{a^2 + h^2}$$



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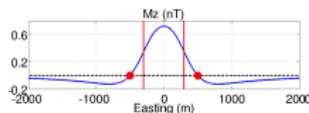
Resolvendo para a profundidade ( $h$ ) e a meia-largura ( $a$ ):

$$h = \frac{x_v^2 - x_{vh-}^2}{2x_{vh-}}$$

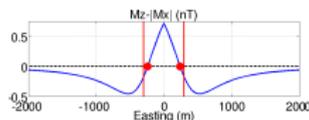
$$a = \sqrt{x_v^2 - h^2}$$

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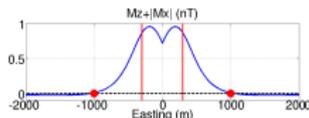
# Motivação



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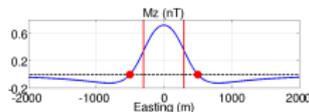
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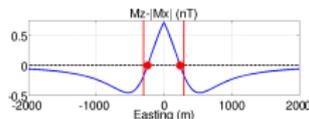
$$h = \frac{x_v^2 - x_{vh-}^2}{2x_{vh-}} \quad \left( \text{ou } h = \frac{x_{vh+} - x_{vh-}}{2} \right) \quad a = \sqrt{x_v^2 - h^2}$$

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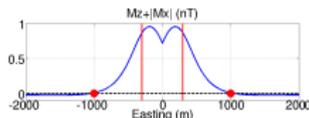
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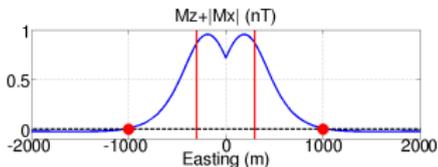
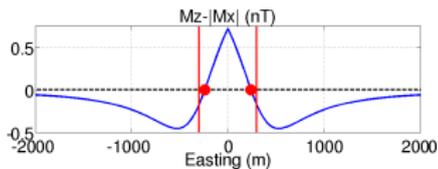
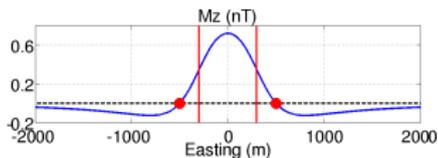
$$h = \frac{x_v^2 - x_{vh-}^2}{2x_{vh-}} \quad \left( \text{ou } h = \frac{x_{vh+} - x_{vh-}}{2} \right) \quad a = \sqrt{x_v^2 - h^2}$$

Como encontrar  $x_v$ ,  $x_{vh+}$  e  $x_{vh-}$  ?

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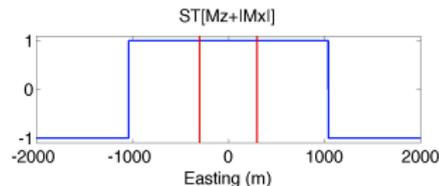
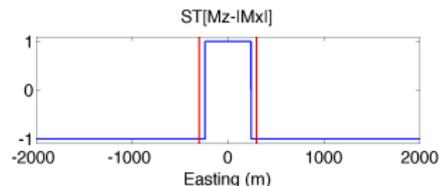
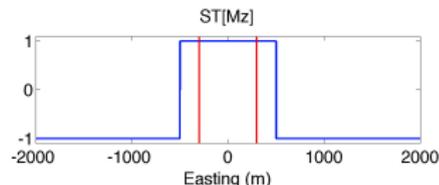
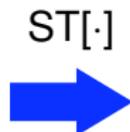
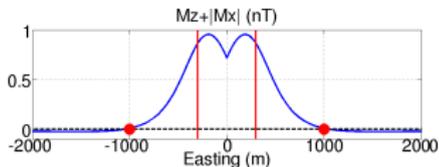
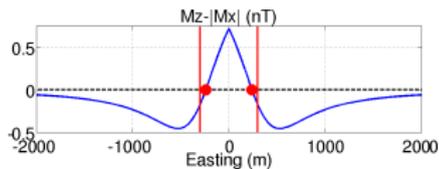
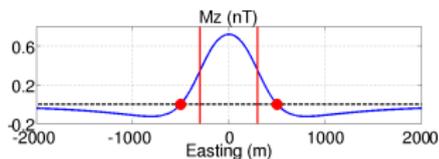
# Signum transform

$$ST[f](x) = \begin{cases} \frac{f(x)}{|f(x)|}, & f(x) \neq 0, \\ 1, & f(x) = 0 \end{cases}$$



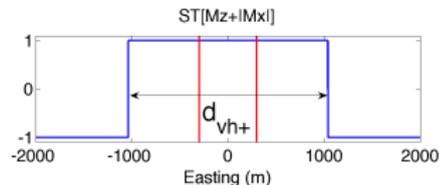
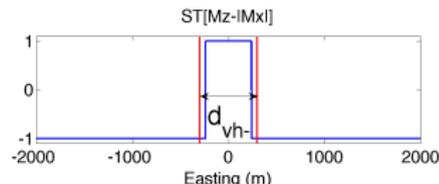
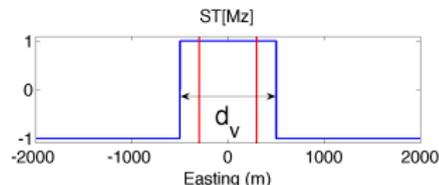
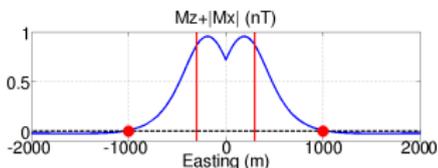
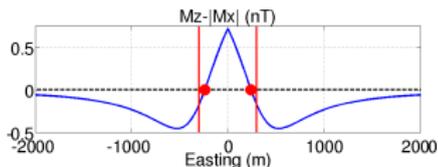
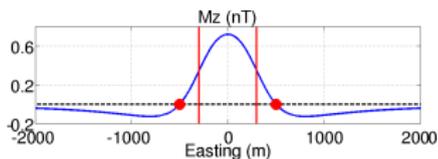
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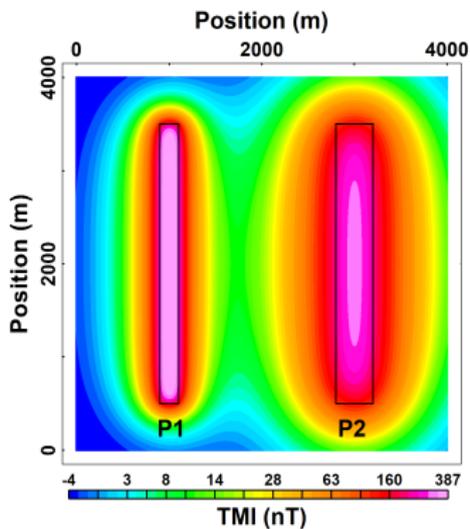


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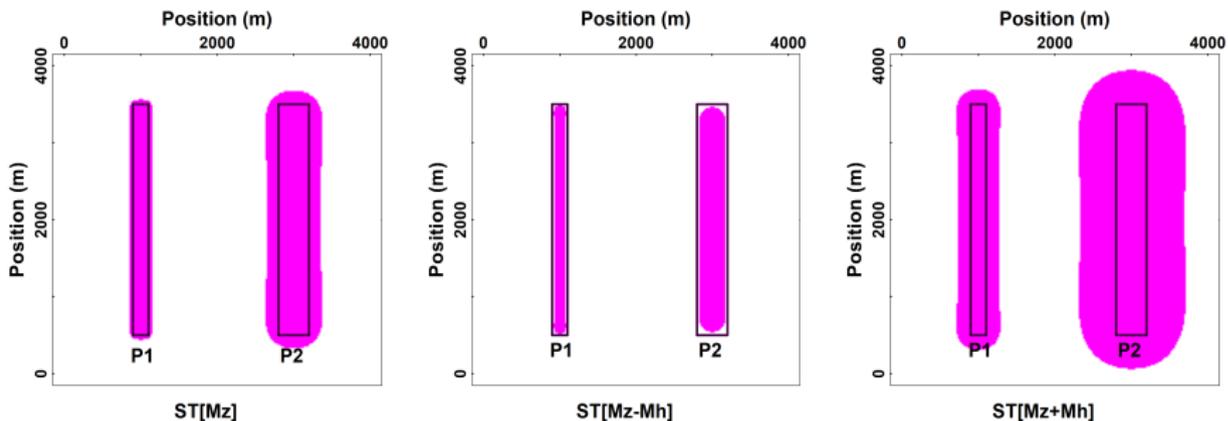
$$d_v = x_v^+ - x_v^-, \quad \begin{cases} d_{vh-} = x_{vh-}^+ - x_{vh-}^- \\ d_{vh+} = x_{vh+}^+ - x_{vh+}^- \end{cases}$$



# Extensão para 2D

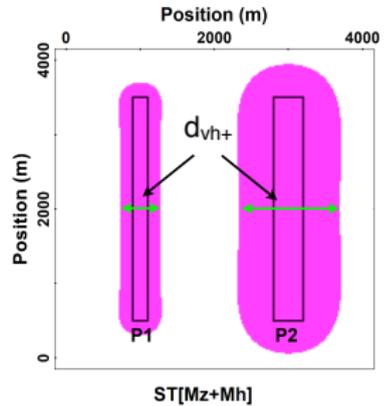
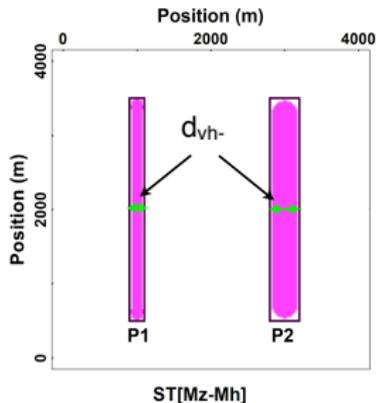
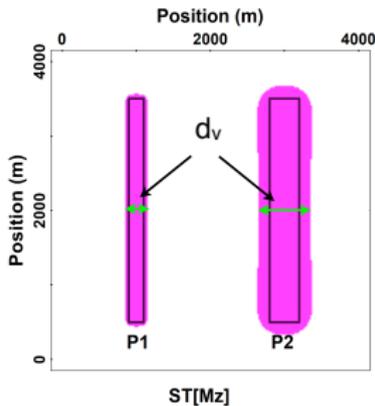


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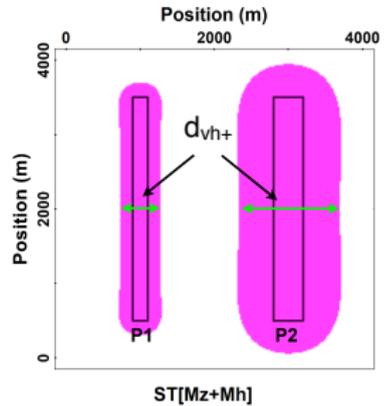
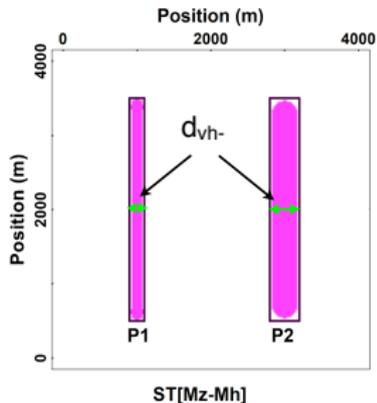
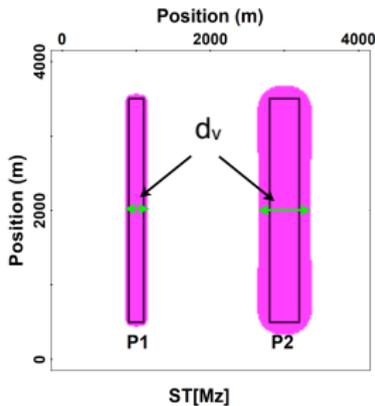
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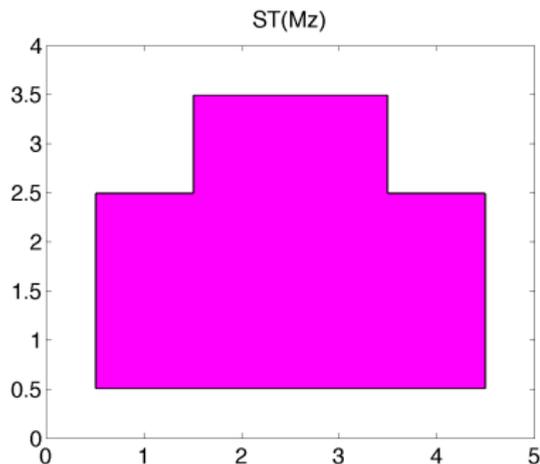
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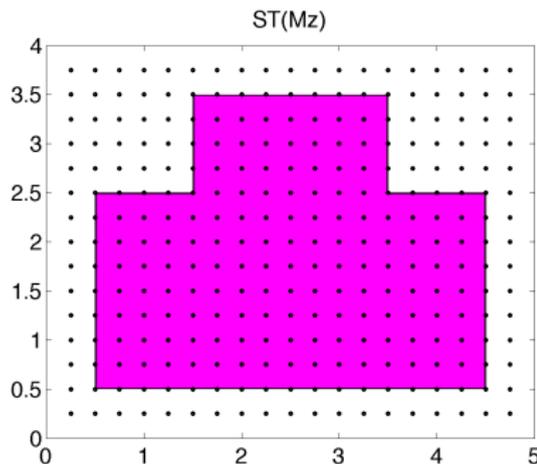
Var. de entrada:  $S=ST[M_z]$



# Algoritmo

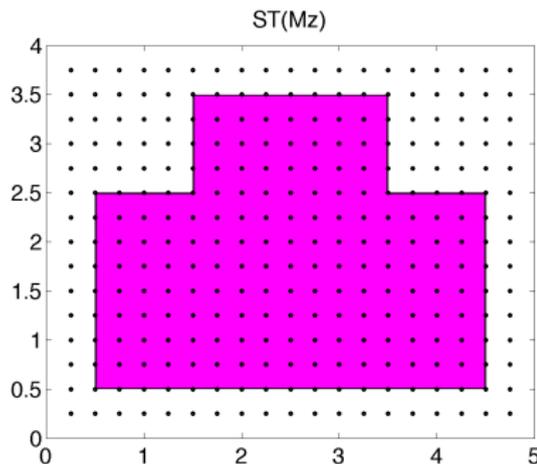


Var. de entrada:  $S=ST[M_z]$  e um grid  $G$



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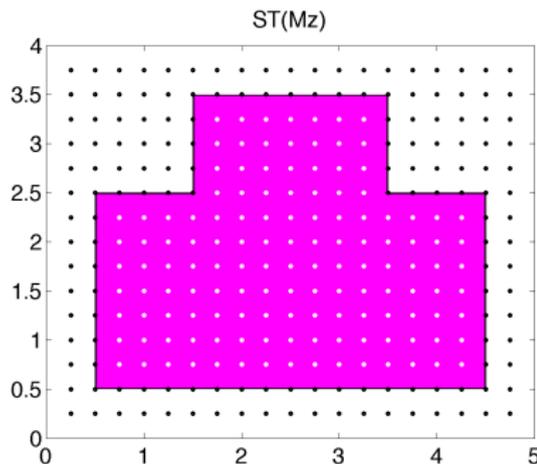


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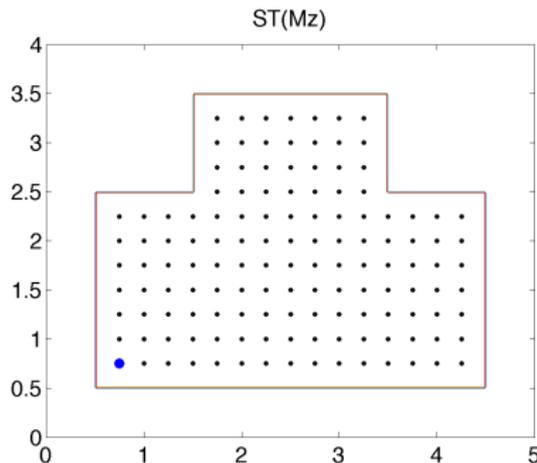


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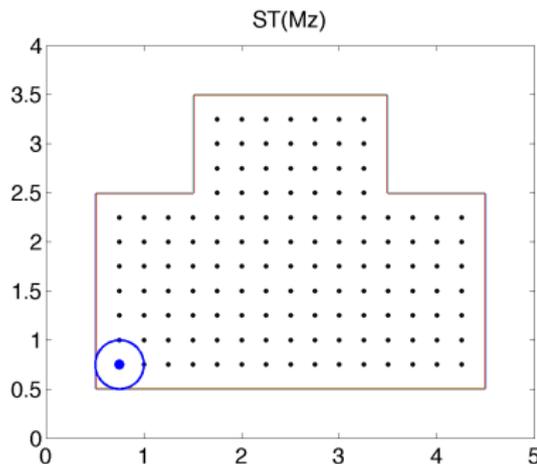


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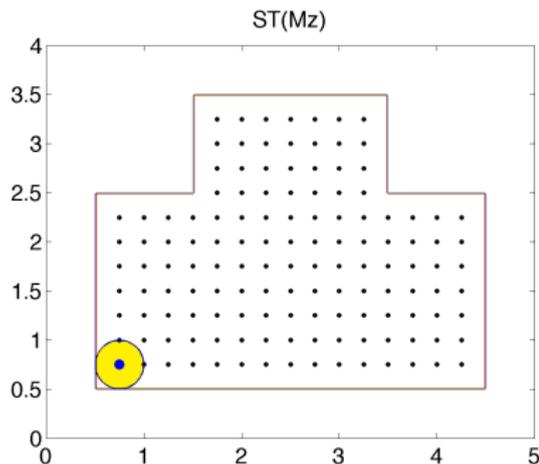


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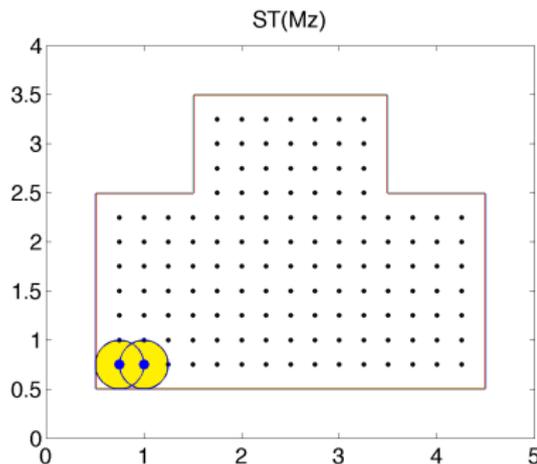


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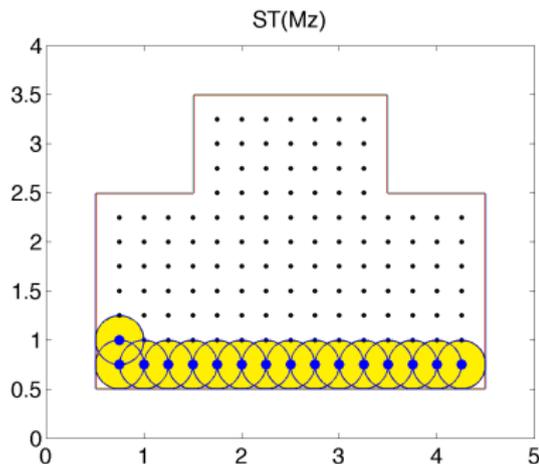


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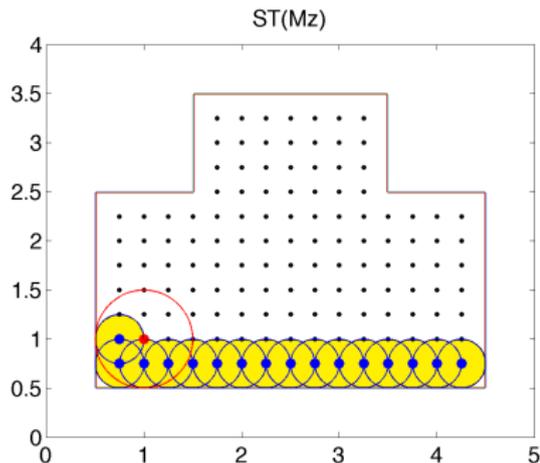


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3. Para cada  $(x_i, y_i)$  no grid  $G^*$ :
  - 3.1 Ache o maior círculo  $C_i$  em  $G^*$  (centro em  $(x_i, y_i)$  e raio  $r_i$ )
  - 3.2  $x_v = \max\{x_v, r_i\}$  em  $C_i$ ;

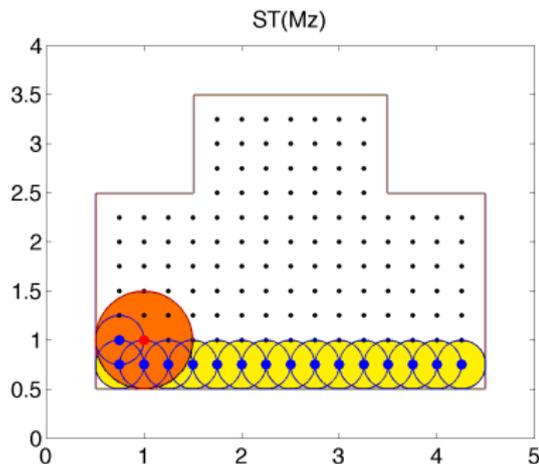


# Algoritmo



Var. de entrada:  $S=ST[M_z]$  e um grid  $G$

1.  $x_v(i) = 0$  se  $(x_i, y_i) \in G$ ;
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  - 3.2  $x_v = \max\{x_v, r_i\}$  em  $C_i$ ;

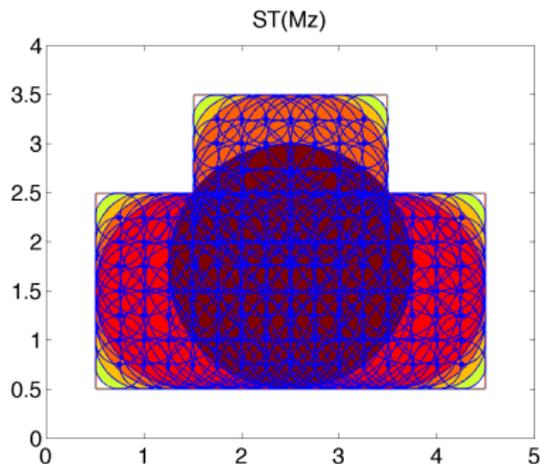


# Algoritmo



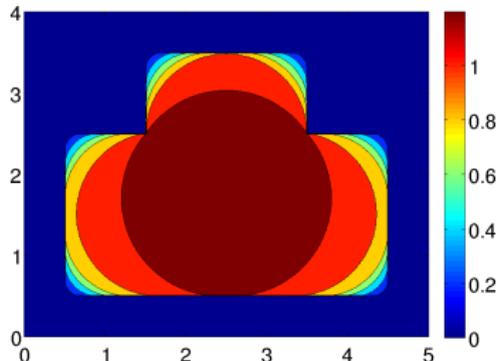
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1.  $x_v(i) = 0$  se  $(x_i, y_i) \in G$ ;
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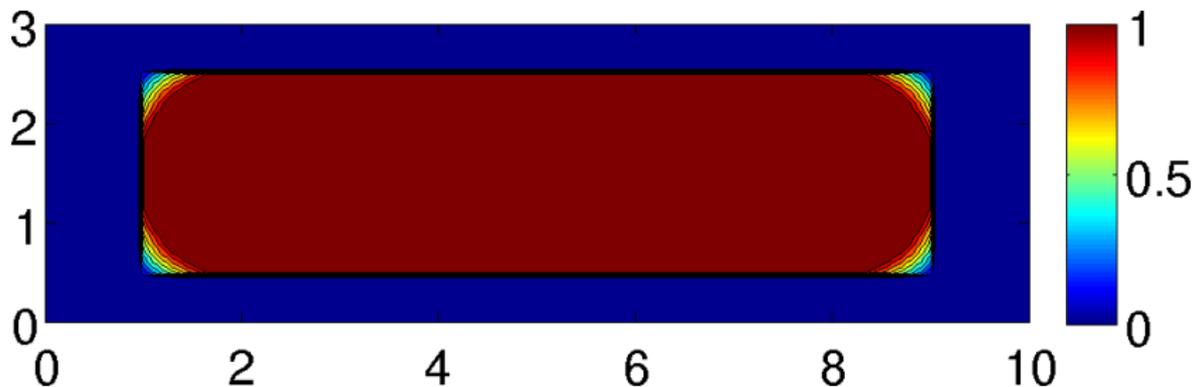


Var. de entrada:  $S=ST[M_z]$  e um grid  $G$

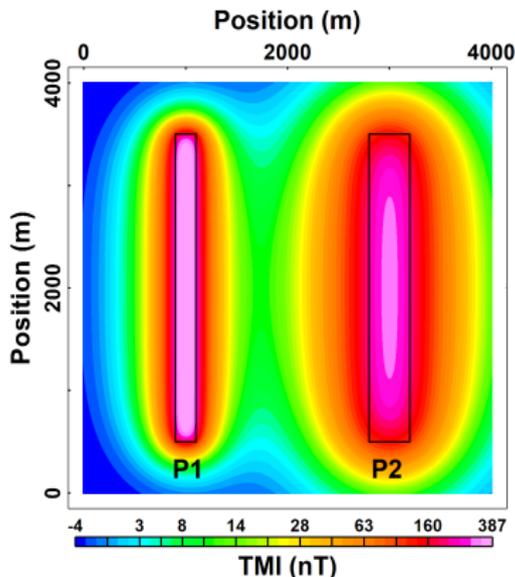
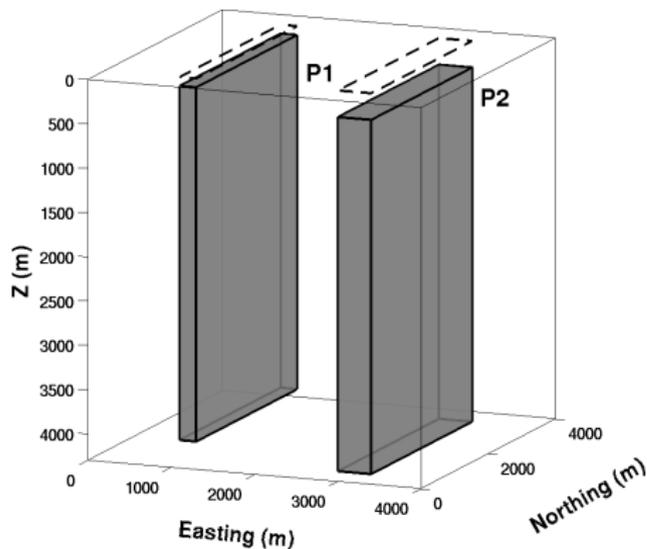
1.  $x_v(i) = 0$  se  $(x_i, y_i) \in G$ ;
2.  $G^* = \{(x_i, y_i) \in G ; S(x_i, y_i) = 1\}$
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  - 3.2  $x_v = \max\{x_v, r_i\}$  em  $C_i$ ;



Em corpos quase 1D, o algoritmo retorna  $x_v \approx \text{const}$



# Exemplos (dados sintéticos)



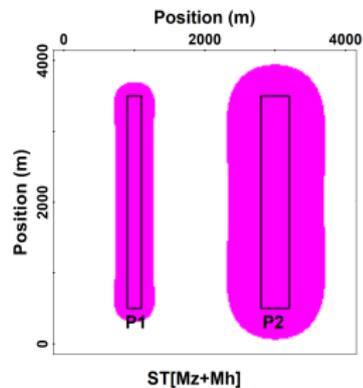
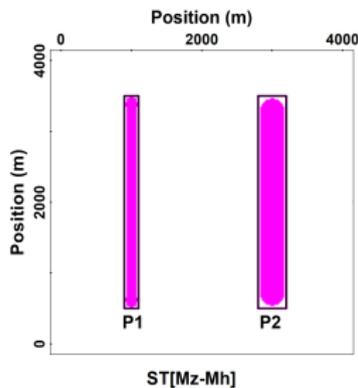
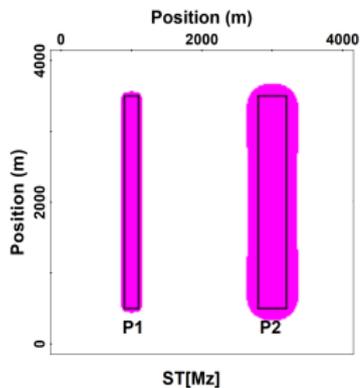
**P1:**  $h = 100m$ ,  $a = 200m$ ,  $L_y = 3000m$ ,  $L_z = 4000m$

**P2:**  $h = 300m$ ,  $a = 400m$ ,  $L_y = 3000m$ ,  $L_z = 4000m$

# Exemplos (dados sintéticos)



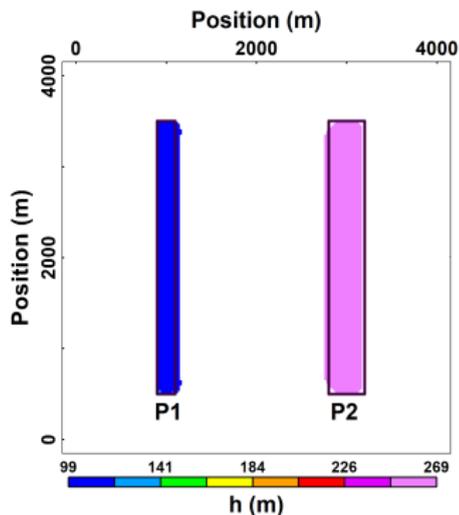
Transformadas Signum:



# Exemplos (dados sintéticos)

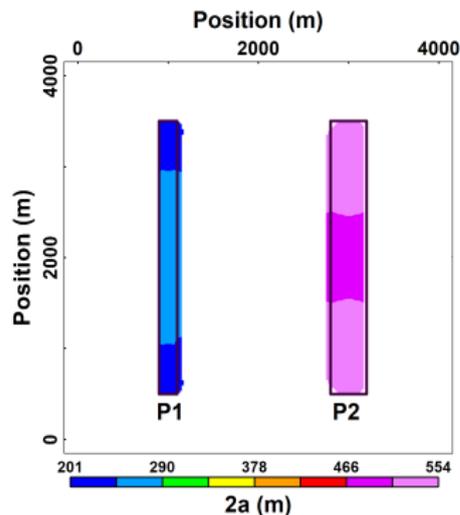


Prof.



$$h = \frac{x_{vh+} - x_{vh-}}{2}$$

1/2 larg:

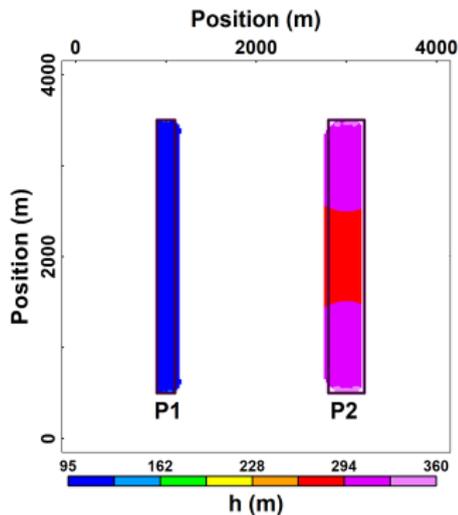


$$a = \sqrt{x_v^2 - h^2}$$

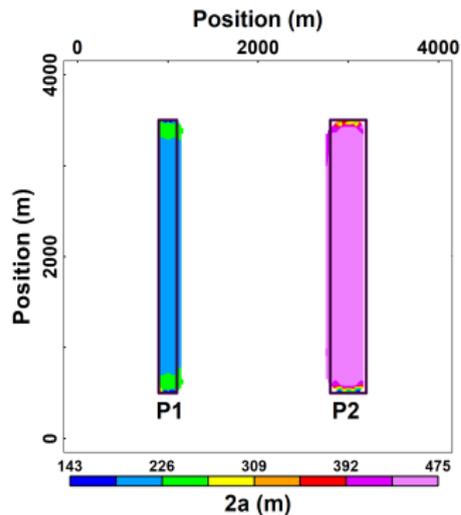
# Exemplos (dados sintéticos)



Prof.



1/2 larg:



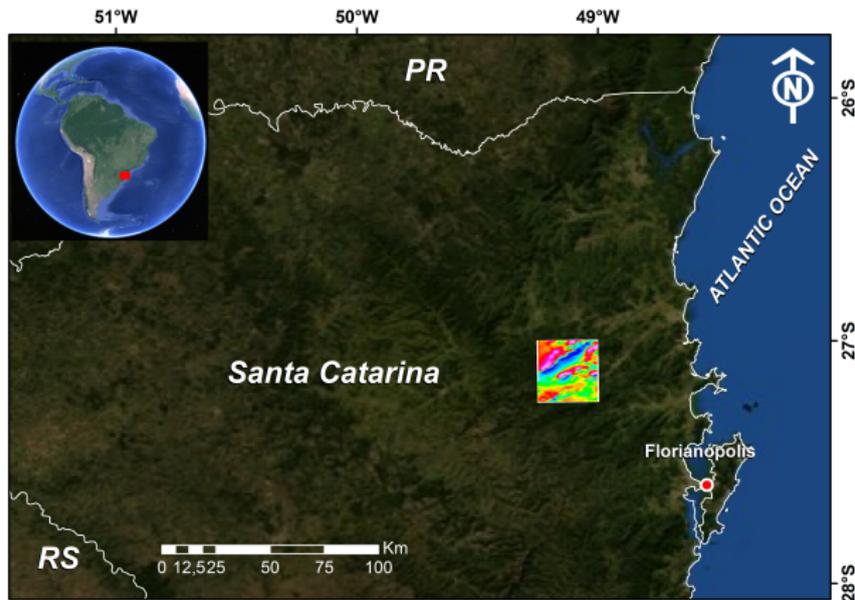
$$h = \frac{x_v^2 - x_{vh-}^2}{2x_{vh-}}$$

$$a = \sqrt{x_v^2 - h^2}$$

# Exemplos (dados reais)



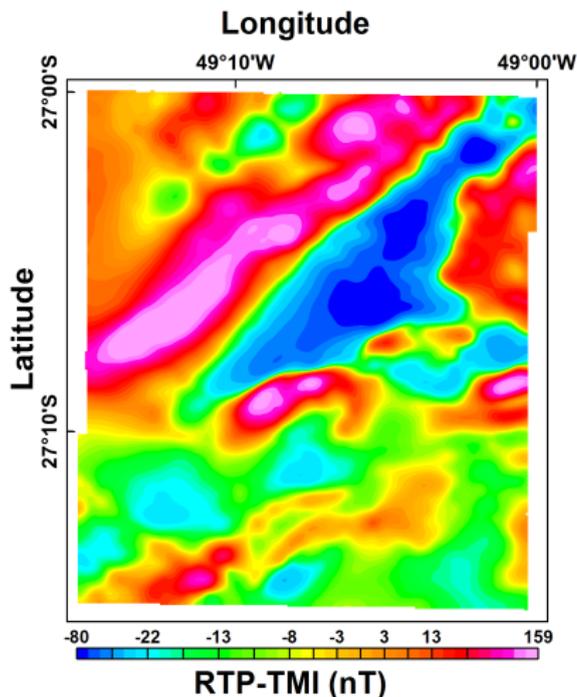
Localização:



# Exemplos (dados reais)



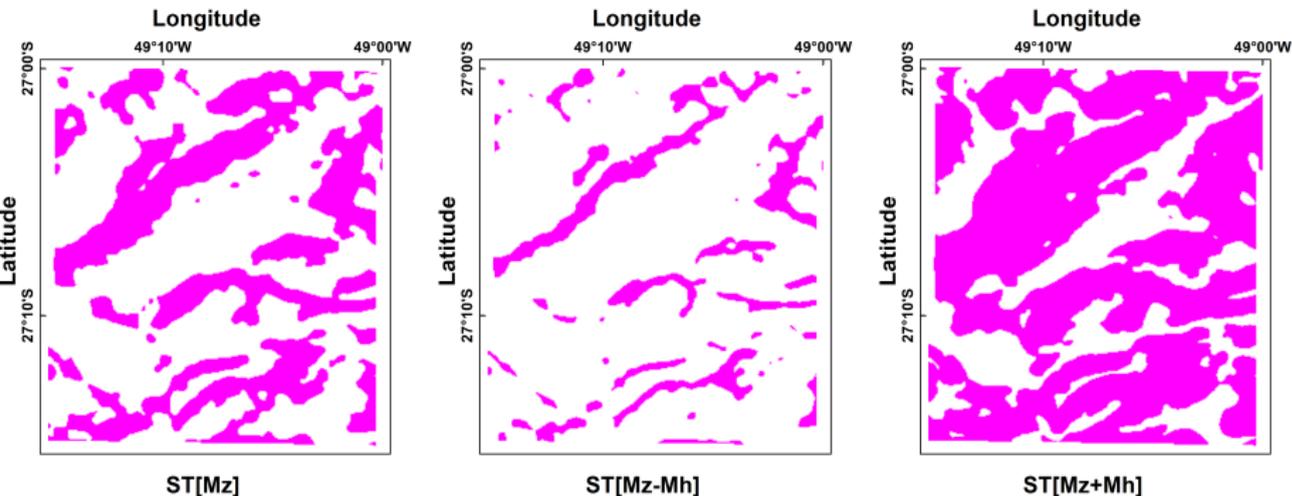
Anomalia magnética (reduzida ao polo, upward continued / 500m):



# Exemplos (dados reais)



Transformadas Signum:

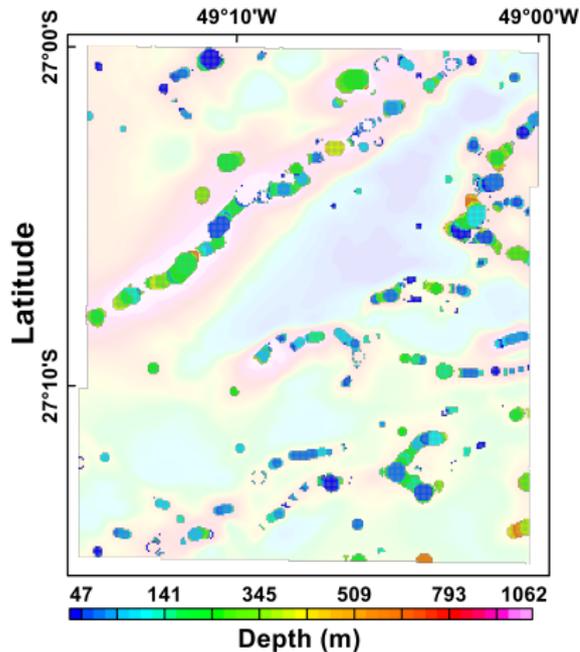


# Exemplos (dados reais)



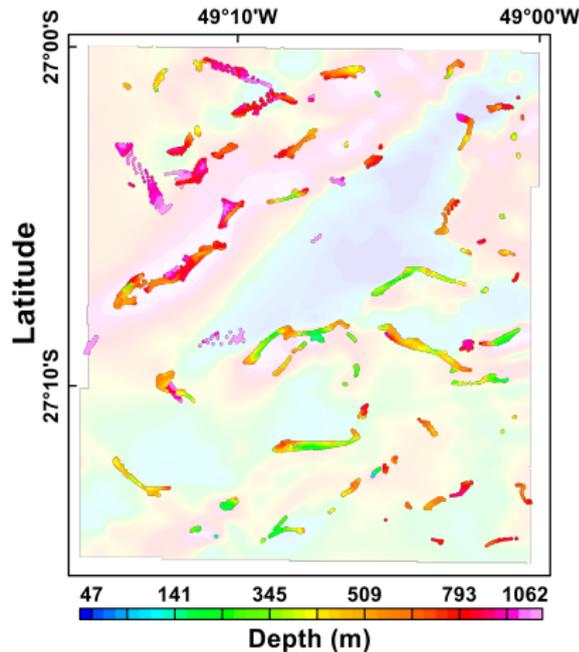
Prof. (ST)

Longitude



Prof. (Euler)

Longitude



# Exemplos (dados reais)



Meia-largura:

