Universidade Federal do Paraná
Programa de Pós-Graduação em Geologia

GEOL7048: Tópicos Especiais em Geologia Exploratória II

Métodos semiquantitativos

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UFPR
UNIVERSIDADE FEDERAL DO PARANÁ

Aula 18

- Redução ao polo

Redução ao polo

Segundo Blakely (p. 331), $f_{RTP} = \mathcal{T}^{-1}\left(\Psi\mathcal{T}(f)\right),$

$$\Psi(\boldsymbol{\kappa}) \quad = \quad \frac{|\boldsymbol{\kappa}|^2}{a_1\kappa_x^2 + a_2\kappa_y^2 + a_3\kappa_x\kappa_y + i|\boldsymbol{\kappa}|(b_1\kappa_x + b_2\kappa_y)}, \quad |\boldsymbol{\kappa}| \neq 0,$$

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$$\begin{aligned}
a_1 &= m_z f_z - m_x f_x \\
a_2 &= m_z f_z - m_y f_y \\
a_3 &= -m_y f_x - m_x f_y \\
b_1 &= m_x f_z + m_z f_x \\
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\end{aligned}$$

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$\boldsymbol{f} = (f_1, f_2, f_3)$ mag. total

$\boldsymbol{m} = (m_1, m_2, m_3)$ mag. induzida

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$\boldsymbol{f} = (f_1, f_2, f_3)$ mag. total

$\boldsymbol{m} = (m_1, m_2, m_3)$ mag. induzida

Em termos da declinação e inclinação,

$$\begin{cases} f_x = \cos(D_T)\cos(I_T) \\ f_y = \operatorname{sen}(D_T)\cos(I_T) \\ f_z = \operatorname{sen}(I_T) \end{cases} \qquad \begin{cases} m_x = \cos(D_M)\cos(I_M) \\ m_y = \operatorname{sen}(D_M)\cos(I_M) \\ m_z = \operatorname{sen}(I_M) \end{cases}$$

Redução ao polo

Se não há magnetização remanescente, ou se sua direção é a mesma do campo induzido, $\boldsymbol{f} = \boldsymbol{m}$.

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$$\begin{aligned} a_1 &= m_z^2 - m_x^2 \\ a_2 &= m_z^2 - m_y^2 \\ a_3 &= -2m_y m_x \\ b_1 &= 2m_x m_z \\ b_2 &= 2m_y m_z \end{aligned}$$

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Em termos da declinação $D = D_M$ e inclinação $I = I_M$,

$$\begin{cases} m_x = \cos(D)\cos(I_M) \\ m_y = \operatorname{sen}(D)\cos(I) \\ m_z = \operatorname{sen}(I) \end{cases} \qquad \begin{cases} a_1 = m_z^2 - m_x^2 \\ a_2 = m_z^2 - m_y^2 \\ a_3 = -2m_y m_x \\ b_1 = 2m_x m_z \\ b_2 = 2m_y m_z \end{cases}$$

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Redução ao polo

Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

$$\Psi(\boldsymbol{\kappa}) \quad = \quad \frac{|\boldsymbol{\kappa}|^2}{a_1\kappa_x^2 + a_2\kappa_y^2 + a_3\kappa_x\kappa_y + i|\boldsymbol{\kappa}|(b_1\kappa_x + b_2\kappa_y)}$$

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Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

$$\Psi(\boldsymbol{\kappa}) = \frac{1}{a_1 \left(\frac{\kappa_x}{|\boldsymbol{\kappa}|}\right)^2 + a_2 \left(\frac{\kappa_y}{|\boldsymbol{\kappa}|}\right)^2 + a_3 \frac{\kappa_x \kappa_y}{|\boldsymbol{\kappa}|^2} + i \frac{b_1 \kappa_x + b_2 \kappa_y}{|\boldsymbol{\kappa}|}}$$

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Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

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Redução ao polo

Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

$$\Psi(\boldsymbol{\kappa}) = \frac{1}{a_1 \cos^2(\theta) + a_2 \text{sen}^2(\theta) + a_3 \cos(\theta)\text{sen}(\theta) + i\left(b_1 \cos(\theta) + b_2 \text{sen}(\theta)\right)}$$

$$\cos(\theta) = \frac{\kappa_x}{|\boldsymbol{\kappa}|}$$

$$\text{sen}(\theta) = \frac{\kappa_y}{|\boldsymbol{\kappa}|}$$

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Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

$$\Psi(\boldsymbol{\kappa}) = \frac{1}{a_1 \cos^2(\theta) + a_2 \mathrm{sen}^2(\theta) + a_3 \cos(\theta)\mathrm{sen}(\theta) + i\,(b_1 \cos(\theta) + b_2 \mathrm{sen}(\theta))}$$

$$\begin{aligned}
a_1 &= \mathrm{sen}^2(I) - (\cos(D)\cos(I))^2 \\
a_2 &= \mathrm{sen}^2(I) - (\mathrm{sen}(D)\cos(I))^2 \\
a_3 &= -2\mathrm{sen}(D)\cos(D)\cos^2(I) \\
b_1 &= 2\cos(D)\cos(I)\mathrm{sen}(I) \\
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b_1 &= 2\cos(D)\cos(I)\mathrm{sen}(I) \\
b_2 &= 2\mathrm{sen}(D)\cos(I)\mathrm{sen}(I)
\end{aligned}$$

Vejamos as parcelas individualmente:

$$\begin{aligned}
a_1\cos^2(\theta) + a_2\mathrm{sen}^2(\theta) &= (\mathrm{sen}^2(I) - (\cos(D)\cos(I))^2)\cos^2(\theta) \\
&+ (\mathrm{sen}^2(I) - (\mathrm{sen}(D)\cos(I))^2)\mathrm{sen}^2(\theta)
\end{aligned}$$

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\end{aligned}$$

Vejamos as parcelas individualmente:

$$\begin{aligned}
a_1\cos^2(\theta) + a_2\mathrm{sen}^2(\theta) &= \mathrm{sen}^2(I) - (\cos(D)\cos(I))^2\cos^2(\theta) \\
&- (\mathrm{sen}(D)\cos(I))^2\mathrm{sen}^2(\theta)
\end{aligned}$$

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Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

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a_1 \cos^2(\theta) + a_2 \mathrm{sen}^2(\theta) &= \mathrm{sen}^2(I) - \cos^2(D)\cos^2(I)\cos^2(\theta) \\
&- \mathrm{sen}^2(D)\cos^2(I)\mathrm{sen}^2(\theta)
\end{aligned}$$

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\end{aligned}$$

Vejamos as parcelas individualmente:

$$\begin{aligned}
a_1\cos^2(\theta) + a_2\mathrm{sen}^2(\theta) &= \mathrm{sen}^2(I) \\
&- \cos^2(I)[\cos^2(D)\cos^2(\theta) + \mathrm{sen}^2(D)\mathrm{sen}^2(\theta)]
\end{aligned}$$

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$$\Psi(\boldsymbol{\kappa}) = \frac{1}{a_1\cos^2(\theta) + a_2\mathrm{sen}^2(\theta) + a_3\cos(\theta)\mathrm{sen}(\theta) + i\left(b_1\cos(\theta) + b_2\mathrm{sen}(\theta)\right)}$$

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a_3 &= -2\mathrm{sen}(D)\cos(D)\cos^2(I) \\
b_1 &= 2\cos(D)\cos(I)\mathrm{sen}(I) \\
b_2 &= 2\mathrm{sen}(D)\cos(I)\mathrm{sen}(I)
\end{aligned}$$

Vejamos as parcelas individualmente:

$$\begin{aligned}
a_1\cos^2(\theta) + a_2\mathrm{sen}^2(\theta) + a_3\cos(\theta)\mathrm{sen}(\theta) = \mathrm{sen}^2(I) \\
-\cos^2(I)[\cos^2(D)\cos^2(\theta) + \mathrm{sen}^2(D)\mathrm{sen}^2(\theta) \\
+2\mathrm{sen}(D)\cos(D)\cos(\theta)\mathrm{sen}(\theta)]
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a_2 &= \mathrm{sen}^2(I) - (\mathrm{sen}(D)\cos(I))^2 \\
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\end{aligned}$$

Vejamos as parcelas individualmente:

$$\begin{aligned}
a_1 \cos^2(\theta) + a_2 \mathrm{sen}^2(\theta) + a_3 \cos(\theta)\mathrm{sen}(\theta) = {} & \mathrm{sen}^2(I) \\
& - \cos^2(I)[\cos(D)\cos(\theta) + \mathrm{sen}(D)\mathrm{sen}(\theta)]^2
\end{aligned}$$

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$$\begin{aligned}
a_1 &= \mathrm{sen}^2(I) - (\cos(D)\cos(I))^2 \\
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b_2 &= 2\mathrm{sen}(D)\cos(I)\mathrm{sen}(I)
\end{aligned}$$

Vejamos as parcelas individualmente:

$$a_1 \cos^2(\theta) + a_2 \mathrm{sen}^2(\theta) + a_3 \cos(\theta)\mathrm{sen}(\theta) = \mathrm{sen}^2(I) - \cos^2(I)\cos^2(D-\theta)$$

Redução ao polo

Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

$$\Psi(\boldsymbol{\kappa}) = \frac{1}{\text{sen}^2(I) - \cos^2(I)\cos^2(D-\theta) + i\left(b_1\cos(\theta) + b_2\text{sen}(\theta)\right)}$$

$$\begin{aligned}
a_1 &= \text{sen}^2(I) - (\cos(D)\cos(I))^2 \\
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b_1 &= 2\cos(D)\cos(I)\text{sen}(I) \\
b_2 &= 2\text{sen}(D)\cos(I)\text{sen}(I)
\end{aligned}$$

Vejamos as parcelas individualmente:

$$\begin{aligned}
b_1\cos(\theta) + b_2\text{sen}(\theta) = {} & 2\cos(D)\cos(I)\text{sen}(I)\cos(\theta) \\
& +2\text{sen}(D)\cos(I)\text{sen}(I)\text{sen}(\theta)
\end{aligned}$$

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\end{aligned}$$

Vejamos as parcelas individualmente:

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b_1\cos(\theta) + b_2\mathrm{sen}(\theta) = {} & 2\cos(D)\cos(I)\mathrm{sen}(I)\cos(\theta) \\
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$$
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a_1 &= \mathrm{sen}^2(I) - (\cos(D)\cos(I))^2 \\
a_2 &= \mathrm{sen}^2(I) - (\mathrm{sen}(D)\cos(I))^2 \\
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b_1 &= 2\cos(D)\cos(I)\mathrm{sen}(I) \\
b_2 &= 2\mathrm{sen}(D)\cos(I)\mathrm{sen}(I)
\end{aligned}
$$

Vejamos as parcelas individualmente:

$$
b_1\cos(\theta) + b_2\mathrm{sen}(\theta) = 2\cos(I)\mathrm{sen}(I)(\cos(D)\cos(\theta) + \mathrm{sen}(D)\mathrm{sen}(\theta))
$$

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\end{aligned}$$

Vejamos as parcelas individualmente:

$$b_1\cos(\theta) + b_2\operatorname{sen}(\theta) = 2\cos(I)\operatorname{sen}(I)\cos(D-\theta)$$

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b_1 &= 2\cos(D)\cos(I)\mathrm{sen}(I) \\
b_2 &= 2\mathrm{sen}(D)\cos(I)\mathrm{sen}(I)
\end{aligned}$$

Portanto,

$$\Psi(\boldsymbol{\kappa}) = \frac{1}{\mathrm{sen}^2(I) - \cos^2(I)\cos^2(D-\theta) + i2\cos(I)\mathrm{sen}(I)\cos(D-\theta)}$$

Redução ao polo

Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

$$\Psi(\boldsymbol{\kappa}) = \frac{1}{\text{sen}^2(I) - \cos^2(I)\cos^2(D-\theta) + i\left(b_1\cos(\theta) + b_2\text{sen}(\theta)\right)}$$

$$\begin{aligned}
a_1 &= \text{sen}^2(I) - (\cos(D)\cos(I))^2 \\
a_2 &= \text{sen}^2(I) - (\text{sen}(D)\cos(I))^2 \\
a_3 &= -2\text{sen}(D)\cos(D)\cos^2(I) \\
b_1 &= 2\cos(D)\cos(I)\text{sen}(I) \\
b_2 &= 2\text{sen}(D)\cos(I)\text{sen}(I)
\end{aligned}$$

Portanto,

$$\Psi(\boldsymbol{\kappa}) = \frac{1}{\text{sen}^2(I) + i^2\cos^2(I)\cos^2(D-\theta) + 2i\text{sen}(I)\cos(I)\cos(D-\theta)}$$

Redução ao polo

Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

$$\Psi(\boldsymbol{\kappa}) \quad = \quad \frac{1}{\text{sen}^2(I) - \cos^2(I)\cos^2(D-\theta) + i\left(b_1\cos(\theta) + b_2\text{sen}(\theta)\right)}$$

$$\begin{aligned}
a_1 &= \text{sen}^2(I) - (\cos(D)\cos(I))^2 \\
a_2 &= \text{sen}^2(I) - (\text{sen}(D)\cos(I))^2 \\
a_3 &= -2\text{sen}(D)\cos(D)\cos^2(I) \\
b_1 &= 2\cos(D)\cos(I)\text{sen}(I) \\
b_2 &= 2\text{sen}(D)\cos(I)\text{sen}(I)
\end{aligned}$$

Portanto,

$$\Psi(\boldsymbol{\kappa}) = \frac{1}{\text{sen}^2(I) + i^2\cos^2(I)\cos^2(D-\theta) + 2\text{sen}(I)\, i\cos(I)\cos(D-\theta)}$$

Redução ao polo

Divindo o numerador e o denominador de $\phi(\boldsymbol{\kappa})$ por $|\boldsymbol{\kappa}|^2$,

$$
\begin{aligned}
\Psi(\boldsymbol{\kappa}) &= \frac{1}{\mathrm{sen}^2(I) - \cos^2(I)\cos^2(D-\theta) + i\left(b_1\cos(\theta) + b_2\mathrm{sen}(\theta)\right)} \\
a_1 &= \mathrm{sen}^2(I) - (\cos(D)\cos(I))^2 \\
a_2 &= \mathrm{sen}^2(I) - (\mathrm{sen}(D)\cos(I))^2 \\
a_3 &= -2\mathrm{sen}(D)\cos(D)\cos^2(I) \\
b_1 &= 2\cos(D)\cos(I)\mathrm{sen}(I) \\
b_2 &= 2\mathrm{sen}(D)\cos(I)\mathrm{sen}(I)
\end{aligned}
$$

Portanto,

$$
\Psi(\boldsymbol{\kappa}) = \frac{1}{\left(\mathrm{sen}(I) + i\cos(I)\cos(D-\theta)\right)^2}
$$

Estimando a magnetização total

Na presença de magnetização remanescente, podemos estimar o vetor $\boldsymbol{f}$ buscando o par (D_T, I_T) para o qual o mínimo da anomalia reduzida é o menor possível (Fedi et al, 1994):

1. Selecione o vetor $\boldsymbol{m}$
2. Defina um grid para (D_T, I_T)
3. Para cada par $(D_{T,i}, I_{T,i})$ no grid,

   $$\boldsymbol{f} = \Big(\cos(D_{T,i})\cos(I_{T,i}), \mathrm{sen}(D_{T,i})\cos(I_{T,i}), \mathrm{sen}(I_{T,i})\Big)$$

   Calcule o campo f_{RTP} usando os vetores $\boldsymbol{f}$ e $\boldsymbol{m}$

   Calcule $v_i = \min(f_{RTP})$
4. Selecione k tal que $(D_{T,k}, I_{T,k}) = \max_i v_i$

Fedi, Florio & Rapolla **(1994)** A method to estimate the total magnetization direction from a distortion analysis of magnetic anomalies, Geoph Prospect, **42**,261-274.