

Universidade Federal do Paraná  
Programa de Pós-Graduação em Geologia

# GEOL7048: Tópicos Especiais em Geologia Exploratória II

## Métodos semiquantitativos

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# Aula 29

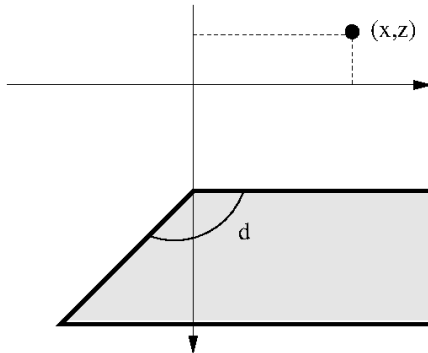
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- Anomalia gerada por um contato geológico
- Método tilt depth

# Anomalia de um contato



Modelo de anomalia gerada por um contato geológico:

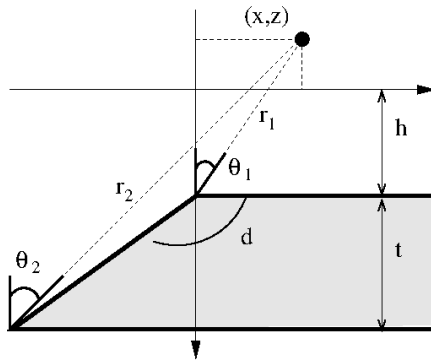


Nabighian (1972) [The analytic signal of of two-dimensional magnetic bodies with polygonal cross-section: Its properties and use for automated anomaly interpretation](#) . Geophysics, 37, 507-517

# Anomalia de um contato



Modelo de anomalia gerada por um contato geológico:



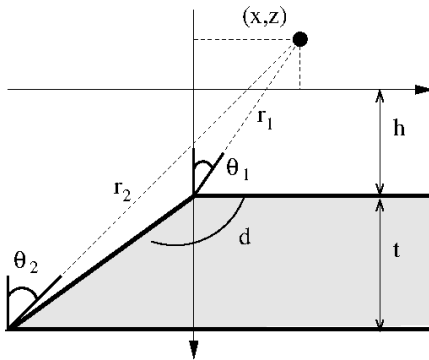
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# Anomalia de um contato



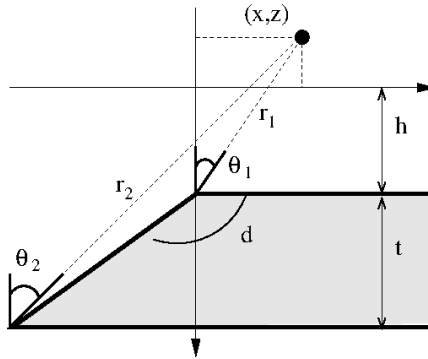
Modelo de anomalia gerada por um contato geológico:

$$M(x, z) = A \left[ (\theta_1 - \theta_2) \cos(\phi) + \ln \frac{r_1}{r_2} \sin(\phi) \right]$$

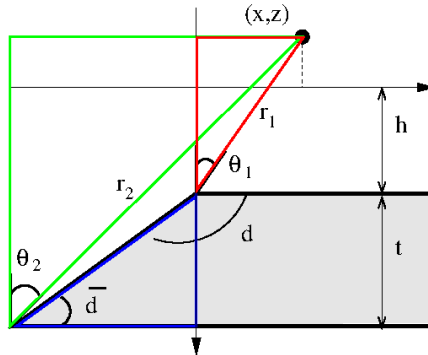


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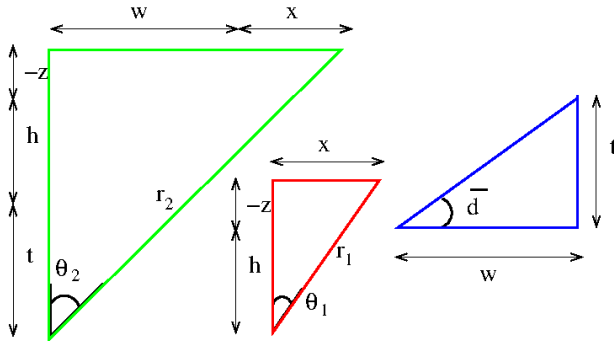
# Anomalia de um contato



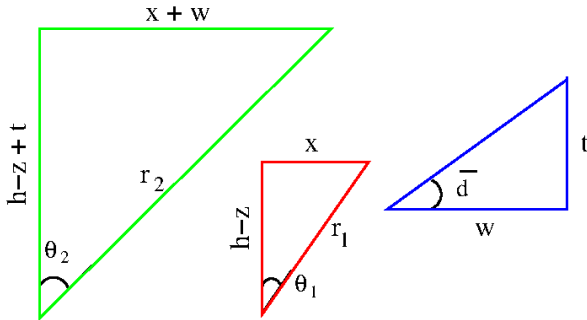
# Anomalia de um contato



# Anomalia de um contato

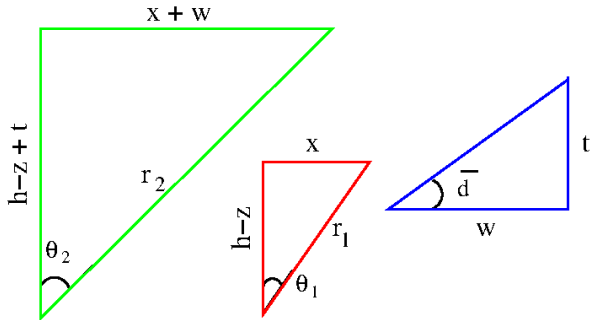


# Anomalia de um contato



$$M(x, z) = A \left[ (\theta_1 - \theta_2) \cos(\phi) + \ln \frac{r_1}{r_2} \sin(\phi) \right]$$

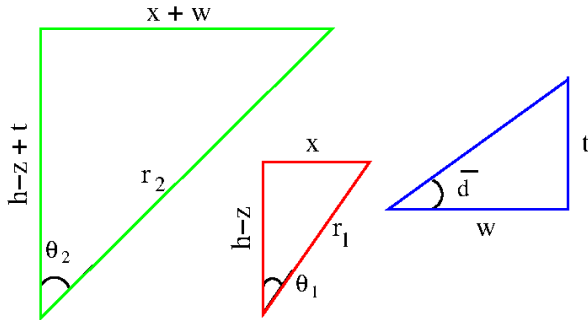
# Anomalia de um contato



$$\tan(\theta_1) = \frac{x}{h-z}, \quad \tan(\theta_2) = \frac{x+w}{h-z+t}, \quad \tan(\bar{d}) = \frac{t}{w}$$

$$r_1^2 = (h-z)^2 + x^2, \quad r_2^2 = (h-z+t)^2 + (x+w)^2$$

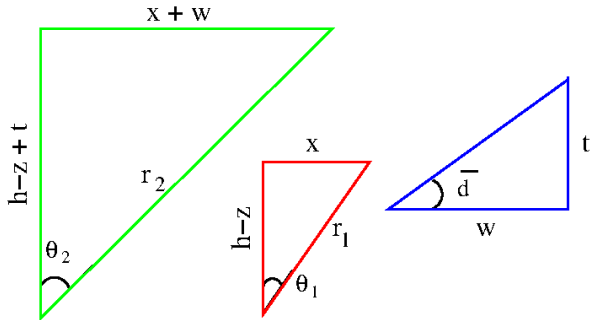
# Anomalia de um contato



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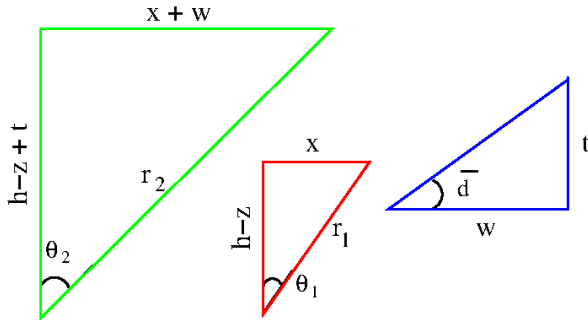
# Anomalia de um contato



$$\tan(\theta_1) = \frac{x}{h - z}, \quad \tan(\theta_2) = \frac{x + w}{h - z + t}, \quad w = t |\cot(\bar{d})|$$

$$r_1^2 = (h - z)^2 + x^2, \quad r_2^2 = (h - z + t)^2 + (x + w)^2$$

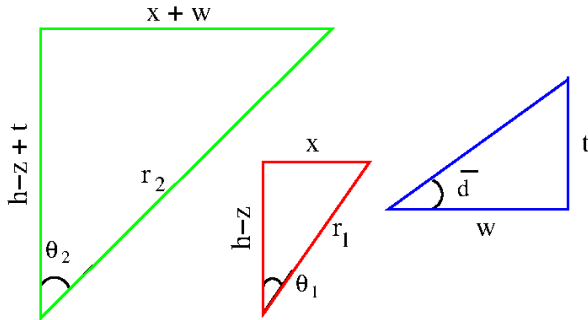
# Anomalia de um contato



$$\tan(\theta_1) = \frac{x}{h-z}, \quad \tan(\theta_2) = \frac{x+t|\cot(d)|}{h-z+t},$$

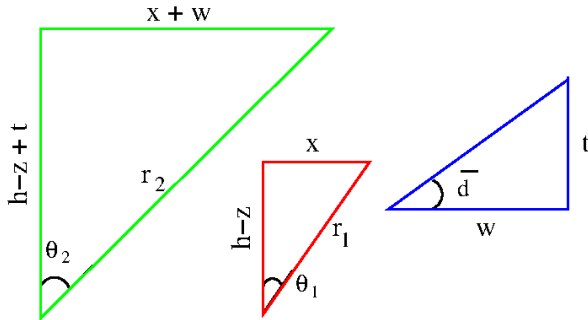
$$r_1^2 = (h-z)^2 + x^2, \quad r_2^2 = (h-z+t)^2 + (x+t|\cot(d)|)^2$$

# Anomalia de um contato



$$M(x, z) = A \left[ \left( \tan^{-1} \frac{x}{h-z} - \tan^{-1} \frac{x+t|\cot(d)|}{t+h-z} \right) \cos(\phi) + \ln \frac{\sqrt{(h-z)^2 + x^2}}{\sqrt{(h-z+t)^2 + (x+t|\cot(d)|)^2}} \sin(\phi) \right]$$

# Anomalia de um contato



$$M(x, z) = A \left[ \left( \tan^{-1} \frac{x}{h-z} - \tan^{-1} \frac{x+t|\cot(d)|}{t+h-z} \right) \cos(\phi) + \right. \\ \left. + \frac{1}{2} \left[ \ln((h-z)^2 + x^2) - \ln((h-z+t)^2 + (x+t|\cot(d)|)^2) \right] \sin(\phi) \right]$$

$$M(x, z) = A \left[ \left( \tan^{-1} \frac{x}{h-z} - \tan^{-1} \frac{x+t|\cot(d)|}{t+h-z} \right) \cos(\phi) + \frac{1}{2} \left[ \ln((h-z)^2 + x^2) - \ln((h-z+t)^2 + (x+t|\cot(d)|)^2) \right] \sin(\phi) \right]$$

Derivada com respeito a  $x$

$$\begin{aligned} \frac{\partial M}{\partial x}(x, z) = & A \left[ \frac{(h-z) \cos(\phi)}{(h-z)^2 + x^2} - \frac{\cos(\phi)}{(h-z+t) \left[ \left( \frac{x+t|\cot(d)|}{h-z+t} \right)^2 + 1 \right]} \right] \\ & + \frac{x \sin(\phi)}{(h-z)^2 + x^2} - \frac{(x+t|\cot(d)|) \sin(\phi)}{(x+t|\cot(d)|)^2 + (h-z+t)^2} \end{aligned}$$

$$M(x, z) = A \left[ \left( \tan^{-1} \frac{x}{h-z} - \tan^{-1} \frac{x+t|\cot(d)|}{t+h-z} \right) \cos(\phi) + \frac{1}{2} \left[ \ln((h-z)^2 + x^2) - \ln((h-z+t)^2 + (x+t|\cot(d)|)^2) \right] \sin(\phi) \right]$$

Derivada com respeito a  $x$ , com a espessura  $t \rightarrow \infty$

$$\frac{\partial M}{\partial x}(x, z) = A \left[ \frac{(h-z) \cos(\phi)}{(h-z)^2 + x^2} + \frac{x \sin(\phi)}{(h-z)^2 + x^2} \right]$$

$$M(x, z) = A \left[ \left( \tan^{-1} \frac{x}{h - z} - \tan^{-1} \frac{x + t |\cot(d)|}{t + h - z} \right) \cos(\phi) + \right. \\ \left. + \frac{1}{2} \left[ \ln((h - z)^2 + x^2) - \ln((h - z + t)^2 + (x + t |\cot(d)|)^2) \right] \sin(\phi) \right]$$

Derivada com respeito a  $x$ , com a espessura  $t \rightarrow \infty$

$$\frac{\partial M}{\partial x}(x, z) = A \left[ \frac{(h - z) \cos(\phi) + x \sin(\phi)}{(h - z)^2 + x^2} \right]$$

$$M(x, z) = A \left[ \left( \tan^{-1} \frac{x}{h - z} - \tan^{-1} \frac{x + t |\cot(d)|}{t + h - z} \right) \cos(\phi) + \right. \\ \left. + \frac{1}{2} \left[ \ln((h - z)^2 + x^2) - \ln((h - z + t)^2 + (x + t |\cot(d)|)^2) \right] \sin(\phi) \right]$$

Derivada com respeito a  $x$ , com a espessura  $t \rightarrow \infty$

$$\frac{\partial M}{\partial x}(x, z) = A \left[ \frac{(h - z) \cos(\phi) + x \sin(\phi)}{(h - z)^2 + x^2} \right]$$

Derivada com respeito a  $z$  com  $t \rightarrow \infty$

$$\frac{\partial M}{\partial z}(x, z) = A \left[ \frac{x \cos(\phi) - (h - z) \sin(\phi)}{(h - z)^2 + x^2} \right]$$

# Anomalia de um contato



Derivada com respeito a  $x$ , com  $t \rightarrow \infty$ :

$$\frac{\partial M}{\partial x}(x, z) = A \left[ \frac{(h - z) \cos(\phi) + x \sin(\phi)}{(h - z)^2 + x^2} \right]$$

Derivada com respeito a  $z$  com  $t \rightarrow \infty$ :

$$\frac{\partial M}{\partial z}(x, z) = A \left[ \frac{x \cos(\phi) - (h - z) \sin(\phi)}{(h - z)^2 + x^2} \right]$$

**Parâmetros:**  $A = 2kF[1 - \cos^2(i) \sin^2(\alpha)] \sin(d)$  e  $\phi = 2I - d - 90^\circ$ ,

- $k$ : contraste de susceptibilidade
- $F$ : intensidade do campo magnético terrestre
- $i$ : inclinação do campo magnético terrestre
- $\alpha$ : ângulo entre o norte magnético e o eixo  $x$
- $I$ : ângulo definido por  $\tan(I) = \tan(i) / \cos(\alpha)$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF[1 - \cos^2(i) \sin^2(\alpha)] \sin(d), \quad \phi = 2I - d - 90^\circ, \quad \tan(I) = \frac{\tan(i)}{\cos(\alpha)}$$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF[1 - \cos^2(90^\circ) \sin^2(0^\circ)] \sin(d), \quad \phi = 2I - d - 90^\circ, \quad \tan(I) = \frac{\tan(90^\circ)}{\cos(0^\circ)}$$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF[1 - 0] \sin(d), \quad \phi = 2I - d - 90^\circ, \quad \tan(I) = \frac{\tan(90^\circ)}{1}$$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF \sin(d), \quad \phi = 2I - d - 90^\circ, \quad I = 90^\circ$$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF \sin(d), \quad \phi = 90^\circ - d$$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF \sin(d), \quad \phi = 90^\circ - d$$

Assumindo ainda que o contato é vertical  $d = 90^\circ$ , obtemos

$$A = 2kF \sin(90^\circ), \quad \phi = 90^\circ - 90^\circ$$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF \sin(d), \quad \phi = 90^\circ - d$$

Assumindo ainda que o contato é vertical  $d = 90^\circ$ , obtemos

$$A = 2kF, \quad \phi = 0^\circ$$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF \sin(d), \quad \phi = 90^\circ - d$$

Assumindo ainda que o contato é vertical  $d = 90^\circ$ , obtemos

$$A = 2kF, \quad \phi = 0^\circ$$

Nestas condições,

$$\frac{\partial M}{\partial x}(x, z) = 2kF \left[ \frac{(h - z) \cos(0^\circ) + x \sin(0^\circ)}{(h - z)^2 + x^2} \right]$$

$$\frac{\partial M}{\partial z}(x, z) = 2kF \left[ \frac{x \cos(0^\circ) - (h - z) \sin(0^\circ)}{(h - z)^2 + x^2} \right]$$

# Anomalia de um contato



No polo,  $\alpha = 0^\circ$  e  $i = 90^\circ$ , de modo que

$$A = 2kF \sin(d), \quad \phi = 90^\circ - d$$

Assumindo ainda que o contato é vertical  $d = 90^\circ$ , obtemos

$$A = 2kF, \quad \phi = 0^\circ$$

Nestas condições,

$$\frac{\partial M}{\partial x}(x, z) = 2kF \frac{(h - z)}{(h - z)^2 + x^2}, \quad \frac{\partial M}{\partial z}(x, z) = 2kF \frac{x}{(h - z)^2 + x^2}$$

# Método tilt depth



Aplicando correção de altitude, podemos considerar  $z = 0$ :

$$M_x(x, 0) = 2kF \frac{(h - 0)}{(h - 0)^2 + x^2}, \quad M_z(x, 0) = 2kF \frac{x}{(h - 0)^2 + x^2}$$

Salem et al (2007) [Tilt-depth method: A simple depth estimation method using first-order magnetic derivatives.](#)

The Leading Edge, 26, 1502-1505

# Método tilt depth



Aplicando correção de altitude, podemos considerar  $z = 0$ :

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Salem et al (2007) observaram que

$$\frac{M_z(x, 0)}{M_x(x, 0)} = \frac{2kF \frac{x}{h^2 + x^2}}{2kF \frac{h}{h^2 + x^2}}$$

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Salem et al (2007) observaram que

$$\tan^{-1} \left( \frac{M_z(x, 0)}{M_x(x, 0)} \right) = \tan^{-1} \left( \frac{x}{h} \right)$$

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# Método tilt depth



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Salem et al (2007) observaram que

$$ISA(x, 0) = \tan^{-1} \left( \frac{x}{h} \right)$$

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The Leading Edge, 26, 1502-1505

# Método tilt depth



Aplicando correção de altitude, podemos considerar  $z = 0$ :

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Salem et al (2007) observaram que

$$ISA(x = 0, 0) = \tan^{-1} \left( \frac{0}{h} \right), \quad ISA(x = h, 0) = \tan^{-1} \left( \frac{h}{h} \right)$$

Salem et al (2007) [Tilt-depth method: A simple depth estimation method using first-order magnetic derivatives](#).

The Leading Edge, 26, 1502-1505

# Método tilt depth



Aplicando correção de altitude, podemos considerar  $z = 0$ :

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Salem et al (2007) observaram que

$$ISA(x = 0, 0) = \tan^{-1}(0), \quad ISA(x = h, 0) = \tan^{-1}(1)$$

Salem et al (2007) [Tilt-depth method: A simple depth estimation method using first-order magnetic derivatives](#).

The Leading Edge, 26, 1502-1505

# Método tilt depth



Aplicando correção de altitude, podemos considerar  $z = 0$ :

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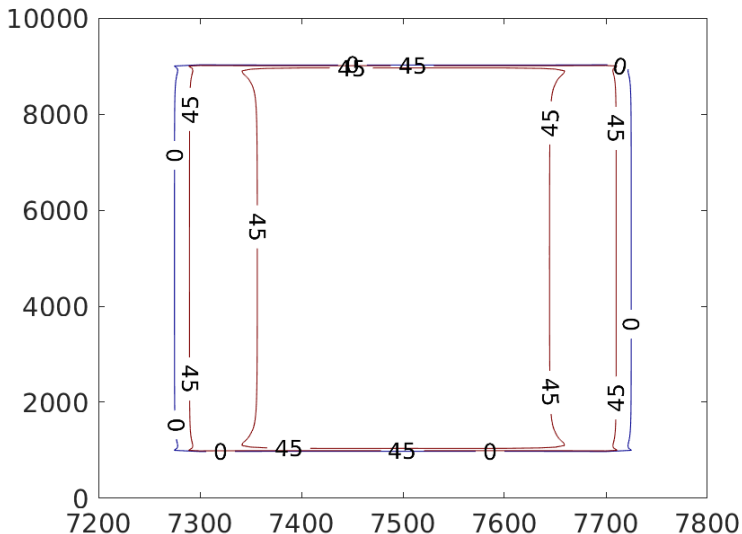
$$ISA(x = 0, 0) = 0^\circ, \quad ISA(x = h, 0) = 45^\circ$$

O método **tilt depth** consiste em estimar  $h$  pela distância entre as isolinhas  $ISA = 0^\circ$  e  $ISA = 45^\circ$ :

Salem et al (2007) [Tilt-depth method: A simple depth estimation method using first-order magnetic derivatives.](#)

The Leading Edge, 26, 1502-1505

# Método tilt depth



# Método tilt depth

