Universidade Federal do Paraná
Programa de Pós-Graduação em Geologia

GEOL7048: Tópicos Especiais em Geologia Exploratória II

Métodos semiquantitativos

Saulo P. Oliveira

Departamento de Matemática, Universidade Federal do Paraná

UFPR
UNIVERSIDADE FEDERAL DO PARANÁ

Aula 9

- Derivada 2D

- GHT

Derivadas parciais

$$
\begin{aligned}
\frac{\partial f}{\partial x}(x_i, y_i, z_i) &= \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x, y_i, z_i) - f(x_i, y_i, z_i)}{\Delta x} \\
\frac{\partial f}{\partial y}(x_i, y_i, z_i) &= \lim_{\Delta y \to 0} \frac{f(x_i, y_i + \Delta y, z_i) - f(x_i, y_i, z_i)}{\Delta y} \\
\frac{\partial f}{\partial z}(x_i, y_i, z_i) &= \lim_{\Delta z \to 0} \frac{f(x_i, y_i, z_i + \Delta z) - f(x_i, y_i, z_i)}{\Delta z}
\end{aligned}
$$

Derivadas parciais

$$\frac{\partial f}{\partial x}(x_i, y_i, z_i) = \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x, y_i, z_i) - f(x_i, y_i, z_i)}{\Delta x}$$

$$\frac{\partial f}{\partial y}(x_i, y_i, z_i) = \lim_{\Delta y \to 0} \frac{f(x_i, y_i + \Delta y, z_i) - f(x_i, y_i, z_i)}{\Delta y}$$

$$\frac{\partial f}{\partial z}(x_i, y_i, z_i) = \lim_{\Delta z \to 0} \frac{f(x_i, y_i, z_i + \Delta z) - f(x_i, y_i, z_i)}{\Delta z}$$

$$\nabla f(x_i, y_i, z_i) = \begin{bmatrix} \frac{\partial f}{\partial x}(x_i, y_i, z_i) \\ \frac{\partial f}{\partial y}(x_i, y_i, z_i) \\ \frac{\partial f}{\partial z}(x_i, y_i, z_i) \end{bmatrix} \qquad \nabla_x f(x_i, y_i, z_i) = \begin{bmatrix} \frac{\partial f}{\partial x}(x_i, y_i, z_i) \\ \frac{\partial f}{\partial y}(x_i, y_i, z_i) \end{bmatrix}$$

Derivadas parciais

$$\frac{\partial f}{\partial x}(x_i, y_i, z_i) = \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x, y_i, z_i) - f(x_i, y_i, z_i)}{\Delta x}$$

$$\frac{\partial f}{\partial y}(x_i, y_i, z_i) = \lim_{\Delta y \to 0} \frac{f(x_i, y_i + \Delta y, z_i) - f(x_i, y_i, z_i)}{\Delta y}$$

$$\frac{\partial f}{\partial z}(x_i, y_i, z_i) = \lim_{\Delta z \to 0} \frac{f(x_i, y_i, z_i + \Delta z) - f(x_i, y_i, z_i)}{\Delta z}$$

$$\nabla f(x_i, y_i, z_i) = \begin{bmatrix} \frac{\partial f}{\partial x}(x_i, y_i, z_i) \\ \frac{\partial f}{\partial y}(x_i, y_i, z_i) \\ \frac{\partial f}{\partial z}(x_i, y_i, z_i) \end{bmatrix}$$

Gradiente

$$\nabla_x f(x_i, y_i, z_i) = \begin{bmatrix} \frac{\partial f}{\partial x}(x_i, y_i, z_i) \\ \frac{\partial f}{\partial y}(x_i, y_i, z_i) \end{bmatrix}$$

Gradiente horizontal

Diferenças finitas 2D

Vamos estudar o gradiente horizontal em $z = H$ (altura de voo). Por conveniência, vamos omitir a dependência em z:

Diferenças finitas 2D

Vamos estudar o gradiente horizontal em $z = H$ (altura de voo). Por conveniência, vamos omitir a dependência em z:

$$
\begin{aligned}
\frac{\partial f}{\partial x}(x_i, y_i, H) &= \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x, y_i, H) - f(x_i, y_i, H)}{\Delta x} \\
\frac{\partial f}{\partial y}(x_i, y_i, H) &= \lim_{\Delta y \to 0} \frac{f(x_i, y_i + \Delta y, H) - f(x_i, y_i, H)}{\Delta y}
\end{aligned}
$$

Diferenças finitas 2D

Vamos estudar o gradiente horizontal em $z = H$ (altura de voo). Por conveniência, vamos omitir a dependência em z:

$$\frac{\partial f}{\partial x}(x_i, y_i) = \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x, y_i) - f(x_i, y_i)}{\Delta x}$$

$$\frac{\partial f}{\partial y}(x_i, y_i) = \lim_{\Delta y \to 0} \frac{f(x_i, y_i + \Delta y) - f(x_i, y_i)}{\Delta y}$$

Diferenças finitas 2D

Vamos estudar o gradiente horizontal em $z = H$ (altura de voo). Por conveniência, vamos omitir a dependência em z:

$$\frac{\partial f}{\partial x}(x_i, y_i) = \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x, y_i) - f(x_i, y_i)}{\Delta x}$$

$$\frac{\partial f}{\partial y}(x_i, y_i) = \lim_{\Delta y \to 0} \frac{f(x_i, y_i + \Delta y) - f(x_i, y_i)}{\Delta y}$$

Aproximação por diferenças progressivas:

$$\frac{\partial f}{\partial x}(x_i, y_i) \approx \frac{f(x_i + \Delta x, y_i) - f(x_i, y_i)}{\Delta x}$$

$$\frac{\partial f}{\partial y}(x_i, y_i) \approx \frac{f(x_i, y_i + \Delta y) - f(x_i, y_i)}{\Delta y}$$

Diferenças finitas 2D

Vamos estudar o gradiente horizontal em $z = H$ (altura de voo). Por conveniência, vamos omitir a dependência em z:

$$\frac{\partial f}{\partial x}(x_i, y_i) = \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x, y_i) - f(x_i, y_i)}{\Delta x}$$

$$\frac{\partial f}{\partial y}(x_i, y_i) = \lim_{\Delta y \to 0} \frac{f(x_i, y_i + \Delta y) - f(x_i, y_i)}{\Delta y}$$

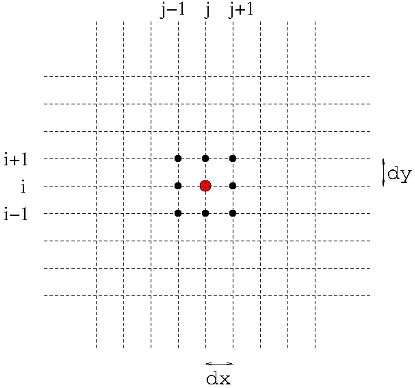
Aproximação por diferenças progressivas:

$$\frac{\partial f}{\partial x}(x_i, y_i) \approx \frac{f(x_i + \Delta x, y_i) - f(x_i, y_i)}{\Delta x}$$

$$\frac{\partial f}{\partial y}(x_i, y_i) \approx \frac{f(x_i, y_i + \Delta y) - f(x_i, y_i)}{\Delta y}$$

Diferenças finitas 2D

Dado um grid bidimensional com pontos $(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$

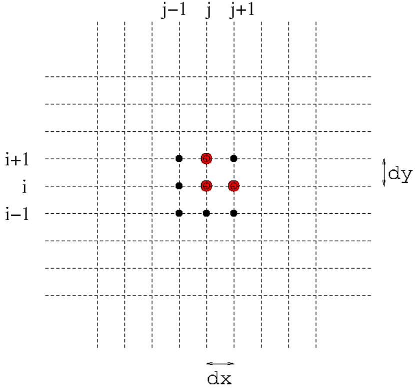


$$\frac{\partial f}{\partial x}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j}) \approx \frac{f(\mathtt{x}_{i,j} + \mathtt{dx}, \mathtt{y}_{i,j}) - f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})}{\mathtt{dx}}$$

$$\frac{\partial f}{\partial y}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j}) \approx \frac{f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j} + \mathtt{dy}) - f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})}{\mathtt{dy}}$$

Diferenças finitas 2D

Dado um grid bidimensional com pontos $(x_{i,j}, y_{i,j})$

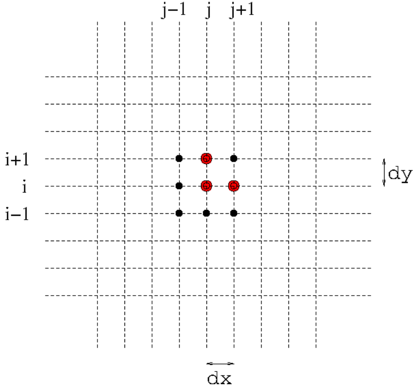


$$\frac{\partial f}{\partial x}(x_{i,j}, y_{i,j}) \approx \frac{f(x_{i,j+1}, y_{i,j+1}) - f(x_{i,j}, y_{i,j})}{dx}$$

$$\frac{\partial f}{\partial y}(x_{i,j}, y_{i,j}) \approx \frac{f(x_{i+1,j}, y_{i+1,j}) - f(x_{i,j}, y_{i,j})}{dy}$$

Diferenças finitas 2D

Dado um grid bidimensional com pontos $(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$



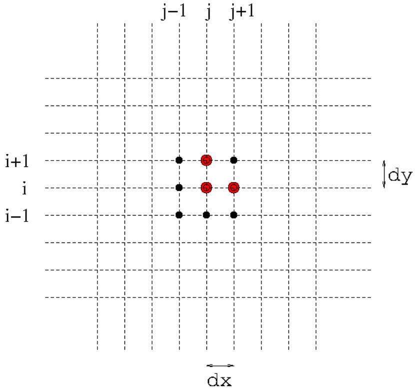
Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$,

$$\frac{\partial f}{\partial x}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j}) \approx \frac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$$

$$\frac{\partial f}{\partial y}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j}) \approx \frac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$$

Diferenças finitas 2D

Dado um grid bidimensional com pontos $(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$



Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$,

$$\mathtt{dxf}_{i,j} = \frac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}} \qquad \mathtt{dyf}_{i,j} = \frac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

$$\mathtt{x} = \begin{bmatrix} \mathtt{x}_{1,1} & \mathtt{x}_{1,2} & \dots & \mathtt{x}_{1,n} \\ \mathtt{x}_{2,1} & \mathtt{x}_{2,2} & \dots & \mathtt{x}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{x}_{m,1} & \mathtt{x}_{m,2} & \dots & \mathtt{x}_{m,n} \end{bmatrix} \quad \mathtt{y} = \begin{bmatrix} \mathtt{y}_{1,1} & \mathtt{y}_{1,2} & \dots & \mathtt{y}_{1,n} \\ \mathtt{y}_{2,1} & \mathtt{y}_{2,2} & \dots & \mathtt{y}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{y}_{m,1} & \mathtt{y}_{m,2} & \dots & \mathtt{y}_{m,n} \end{bmatrix}$$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \dots & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \dots & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \dots & \mathtt{f}_{m,n} \end{bmatrix}$$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \dots & \mathtt{f}_{1,n-1} & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \dots & \mathtt{f}_{2,n-1} & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \dots & \mathtt{f}_{m,n-1} & \mathtt{f}_{m,n} \end{bmatrix}$$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \cdots & \mathtt{f}_{1,n-1} & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \cdots & \mathtt{f}_{2,n-1} & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \cdots & \mathtt{f}_{m,n-1} & \mathtt{f}_{m,n} \end{bmatrix}$$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \ldots & \mathtt{f}_{1,n-1} & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \ldots & \mathtt{f}_{2,n-1} & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \ldots & \mathtt{f}_{m,n-1} & \mathtt{f}_{m,n} \end{bmatrix}$$

$$\mathtt{dxf} = \begin{bmatrix} \mathtt{dxf}_{1,1} & \mathtt{dxf}_{1,2} & \ldots & \mathtt{dxf}_{1,n-1} \\ \mathtt{dxf}_{2,1} & \mathtt{dxf}_{2,2} & \ldots & \mathtt{dxf}_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{dxf}_{m,1} & \mathtt{dxf}_{m,2} & \ldots & \mathtt{dxf}_{m,n-1} \end{bmatrix}$$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \cdots & \mathtt{f}_{1,n-1} & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \cdots & \mathtt{f}_{2,n-1} & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \cdots & \mathtt{f}_{m,n-1} & \mathtt{f}_{m,n} \end{bmatrix}$$

$$\mathtt{dxf} = \frac{1}{\mathtt{dx}} \begin{bmatrix} \mathtt{f}_{1,2} - \mathtt{f}_{1,1} & \mathtt{f}_{1,3} - \mathtt{f}_{1,2} & \cdots & \mathtt{f}_{1,n} - \mathtt{f}_{1,n-1} \\ \mathtt{f}_{2,2} - \mathtt{f}_{2,1} & \mathtt{f}_{2,3} - \mathtt{f}_{2,2} & \cdots & \mathtt{f}_{2,n} - \mathtt{f}_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{f}_{m,2} - \mathtt{f}_{m,1} & \mathtt{f}_{m,3} - \mathtt{f}_{m,2} & \cdots & \mathtt{f}_{m,n} - \mathtt{f}_{m,n-1} \end{bmatrix}$$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \cdots & \mathtt{f}_{1,n-1} & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \cdots & \mathtt{f}_{2,n-1} & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \cdots & \mathtt{f}_{m,n-1} & \mathtt{f}_{m,n} \end{bmatrix}$$

$$\mathtt{dxf} = \frac{1}{\mathtt{dx}} \left(\begin{bmatrix} \mathtt{f}_{1,2} & \mathtt{f}_{1,3} & \cdots & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,2} & \mathtt{f}_{2,3} & \cdots & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{f}_{m,2} & \mathtt{f}_{m,3} & \cdots & \mathtt{f}_{m,n} \end{bmatrix} - \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \cdots & \mathtt{f}_{1,n-1} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \cdots & \mathtt{f}_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \cdots & \mathtt{f}_{m,n-1} \end{bmatrix} \right)$$

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \dots & \mathtt{f}_{1,n-1} & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \dots & \mathtt{f}_{2,n-1} & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \dots & \mathtt{f}_{m,n-1} & \mathtt{f}_{m,n} \end{bmatrix}$$

$$\mathtt{dxf} = \frac{1}{\mathtt{dx}} \left(\begin{bmatrix} \mathtt{f}_{1,2} & \mathtt{f}_{1,3} & \dots & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,2} & \mathtt{f}_{2,3} & \dots & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{f}_{m,2} & \mathtt{f}_{m,3} & \dots & \mathtt{f}_{m,n} \end{bmatrix} - \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \dots & \mathtt{f}_{1,n-1} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \dots & \mathtt{f}_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \dots & \mathtt{f}_{m,n-1} \end{bmatrix} \right)$$

```
dxf = ( f(:,2:end) - f(:,1:end-1) )/dx
```

Diferenças finitas 2D

Se $\mathtt{f}_{i,j} = f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})$, $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j}}{\mathtt{dx}}$ e $\mathtt{dyf}_{i,j} = \dfrac{\mathtt{f}_{i+1,j} - \mathtt{f}_{i,j}}{\mathtt{dy}}$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \dots & \mathtt{f}_{1,n-1} & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \dots & \mathtt{f}_{2,n-1} & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \dots & \mathtt{f}_{m,n-1} & \mathtt{f}_{m,n} \end{bmatrix}$$

$$\mathtt{dxf} = \frac{1}{\mathtt{dx}} \left(\begin{bmatrix} \mathtt{f}_{1,2} & \mathtt{f}_{1,3} & \dots & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,2} & \mathtt{f}_{2,3} & \dots & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{f}_{m,2} & \mathtt{f}_{m,3} & \dots & \mathtt{f}_{m,n} \end{bmatrix} - \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \dots & \mathtt{f}_{1,n-1} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \dots & \mathtt{f}_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \dots & \mathtt{f}_{m,n-1} \end{bmatrix} \right)$$

```
dxf = ( f(:,2:end) - f(:,1:end-1) )/dx
dyf = ( f(2:end,:)  - f(1:end-1,:)  )/dy
```

Diferenças finitas centradas 2D

Consideremos agora $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j-1}}{2\mathtt{dx}}$

Diferenças finitas centradas 2D

Consideremos agora $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j-1}}{2\mathtt{dx}}$

$$\mathtt{f} = \begin{bmatrix} \mathtt{f}_{1,1} & \mathtt{f}_{1,2} & \cdots & \mathtt{f}_{1,n-1} & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,1} & \mathtt{f}_{2,2} & \cdots & \mathtt{f}_{2,n-1} & \mathtt{f}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathtt{f}_{m,1} & \mathtt{f}_{m,2} & \cdots & \mathtt{f}_{m,n-1} & \mathtt{f}_{m,n} \end{bmatrix}$$

Diferenças finitas centradas 2D

Consideremos agora $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j-1}}{2\mathtt{dx}}$

$$\mathtt{dxf} = \begin{bmatrix} \mathtt{dxf}_{1,2} & \dots & \mathtt{dxf}_{1,n-1} \\ \mathtt{dxf}_{2,2} & \dots & \mathtt{dxf}_{2,n-1} \\ \vdots & \ddots & \vdots \\ \mathtt{dxf}_{m,2} & \dots & \mathtt{dxf}_{m,n-1} \end{bmatrix}$$

Diferenças finitas centradas 2D

Consideremos agora $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j-1}}{2\mathtt{dx}}$

$$\mathtt{dxf} = \frac{1}{2\mathtt{dx}} \begin{bmatrix} \mathtt{f}_{1,3} - \mathtt{f}_{1,1} & \dots & \mathtt{f}_{1,n} - \mathtt{f}_{1,n-2} \\ \mathtt{f}_{2,3} - \mathtt{f}_{2,1} & \dots & \mathtt{f}_{2,n} - \mathtt{f}_{2,n-2} \\ \vdots & \ddots & \vdots \\ \mathtt{f}_{m,3} - \mathtt{f}_{m,1} & \dots & \mathtt{f}_{m,n} - \mathtt{f}_{m,n-2} \end{bmatrix}$$

Diferenças finitas centradas 2D

Consideremos agora $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j-1}}{2\mathtt{dx}}$

$$\mathtt{dxf} = \frac{1}{2\mathtt{dx}} \left(\begin{bmatrix} \mathtt{f}_{1,3} & \dots & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,3} & \dots & \mathtt{f}_{2,n} \\ \vdots & \ddots & \vdots \\ \mathtt{f}_{m,3} & \dots & \mathtt{f}_{m,n} \end{bmatrix} - \begin{bmatrix} \mathtt{f}_{1,1} & \dots & \mathtt{f}_{1,n-2} \\ \mathtt{f}_{2,1} & \dots & \mathtt{f}_{2,n-2} \\ \vdots & \ddots & \vdots \\ \mathtt{f}_{m,1} & \dots & \mathtt{f}_{m,n-2} \end{bmatrix} \right)$$

Diferenças finitas centradas 2D

Consideremos agora $\mathtt{dxf}_{i,j} = \dfrac{\mathtt{f}_{i,j+1} - \mathtt{f}_{i,j-1}}{2\mathtt{dx}}$

$$\mathtt{dxf} = \frac{1}{2\mathtt{dx}} \left(\begin{bmatrix} \mathtt{f}_{1,3} & \dots & \mathtt{f}_{1,n} \\ \mathtt{f}_{2,3} & \dots & \mathtt{f}_{2,n} \\ \vdots & \ddots & \vdots \\ \mathtt{f}_{m,3} & \dots & \mathtt{f}_{m,n} \end{bmatrix} - \begin{bmatrix} \mathtt{f}_{1,1} & \dots & \mathtt{f}_{1,n-2} \\ \mathtt{f}_{2,1} & \dots & \mathtt{f}_{2,n-2} \\ \vdots & \ddots & \vdots \\ \mathtt{f}_{m,1} & \dots & \mathtt{f}_{m,n-2} \end{bmatrix} \right)$$

```
dxf = ( f(:,3:end) - f(:,1:end-2) )/(2*dx)
dyf = ( f(3:end,:)  - f(1:end-2,:)  )/(2*dy)
```

GHT

O “Gradiente” Horizontal Total (GHT) é o módulo do vetor gradiente horizontal:

$$
\begin{aligned}
GHT(\mathtt{x}_{i,j}, \mathtt{y}_{i,j}) &= |\nabla_x f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})| \\
&= \sqrt{\left(\frac{\partial f}{\partial x}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})\right)^2 + \left(\frac{\partial f}{\partial y}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})\right)^2}
\end{aligned}
$$

GHT

O "Gradiente" Horizontal Total (GHT) é o módulo do vetor gradiente horizontal:

$$
\begin{aligned}
GHT(\mathtt{x}_{i,j}, \mathtt{y}_{i,j}) &= |\nabla_x f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})| \\
&= \sqrt{\left(\frac{\partial f}{\partial x}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})\right)^2 + \left(\frac{\partial f}{\partial y}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})\right)^2}
\end{aligned}
$$

A aproximação no grid:

```
dxf = ( f(:,3:end) - f(:,1:end-2) )/(2*dx)
dyf = ( f(3:end,:)  - f(1:end-2,:)  )/(2*dy)
GHT = sqrt(dxf.^2 + dyF.^2)
```

GHT

O "Gradiente" Horizontal Total (GHT) é o módulo do vetor gradiente horizontal:

$$
\begin{aligned}
GHT(\mathtt{x}_{i,j}, \mathtt{y}_{i,j}) &= |\nabla_x f(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})| \\
&= \sqrt{\left(\frac{\partial f}{\partial x}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})\right)^2 + \left(\frac{\partial f}{\partial y}(\mathtt{x}_{i,j}, \mathtt{y}_{i,j})\right)^2}
\end{aligned}
$$

A aproximação no grid:

```
dxf = ( f(2:end-1,3:end) - f(2:end-1,1:end-2) )/(2*dx)
dyf = ( f(3:end,2:end-1) - f(1:end-2,2:end-1) )/(2*dy)
GHT = sqrt(dxf.^2 + dyf.^2)
```