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Universidade Federal do Pará

Coordenação do Programa de Pós-Graduação em Geofísica

MÉTODOS DE ELEMENTOS FINITOS PARA AS EQUAÇÕES DE MAXWELL

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Cronograma

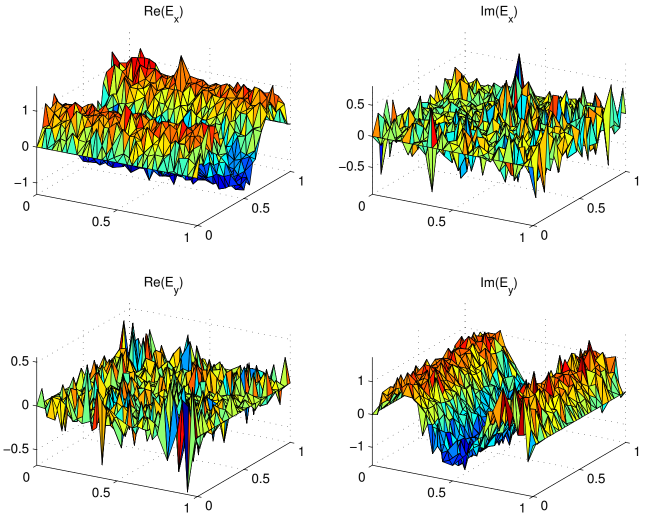
22/08 : Equações de Maxwell
Métodos de elementos finitos para a equação do potencial
Equações de Maxwell no regime harmônico
Métodos de elementos finitos nodais

23/08 : Métodos de elementos finitos de aresta
Panorama da pesquisa na área

Sumário - Quinta-feira 23/08

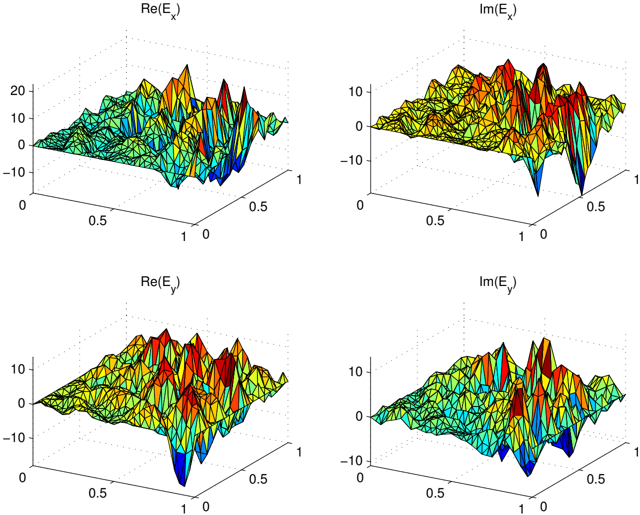
Elementos Finitos Nodais/exemplo

$M = 3, \omega = 100, \sigma = 0$ (malha fina):



Elementos Finitos Nodais/exemplo

$M = 3, \omega = 1, \sigma = 0$ (malha fina):



Em vez de impor a continuidade de $\boldsymbol{N}_{2j-1}$ e $\boldsymbol{N}_{2j}$ nos vértices,

$$\boldsymbol{N}^v_{2j-1}(\boldsymbol{x}) = \begin{pmatrix} N_j(\boldsymbol{x}) \\ 0 \end{pmatrix}, \boldsymbol{N}^v_{2j}(\boldsymbol{x}) = \begin{pmatrix} 0 \\ N_j(\boldsymbol{x}) \end{pmatrix}, N_j(\boldsymbol{x}_k) = \left\{ \begin{array}{ll} 1, & j = k \\ 0, & j \neq k \end{array} \right.$$

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Impomos a continuidade de $\boldsymbol{N}_j \times \boldsymbol{n}$ nas arestas da malha:

$$\int_{\gamma_k} \boldsymbol{N}_j(\boldsymbol{x}) \times \boldsymbol{n} \, d\Gamma = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}$$

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$$\int_{\gamma_k} \boldsymbol{N}_j(\boldsymbol{x}) \cdot \boldsymbol{t} \, d\Gamma = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}$$

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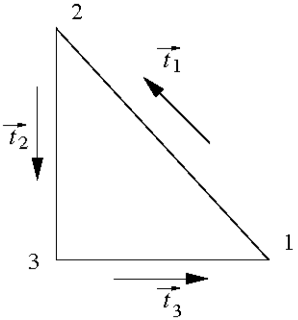
Impomos a continuidade de $\boldsymbol{N}_j \times \boldsymbol{n}$ nas arestas da malha:

$$\int_{\gamma_k} \boldsymbol{N}_j(\boldsymbol{x}) \cdot \boldsymbol{t} \, d\Gamma = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}$$

Estas bases são conhecidas como elementos finitos de arestas

Funções de base local em $\hat{K}$:

$$\int_{\hat{\gamma}_k} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}\, d\hat{\Gamma} = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}, \qquad j = 1, 2, 3$$



As funções de base são de grau 1 e constantes nas arestas:

$$\hat{\boldsymbol{N}}_j(\hat{x}, \hat{y}) = \begin{pmatrix} a + c\hat{y} \\ b - c\hat{x} \end{pmatrix}$$

Vamos encontrar $\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}})$:

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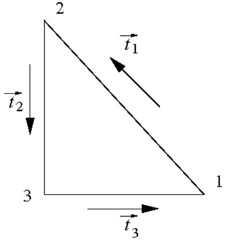

$$\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}_3 = a + c\,\hat{y}$$

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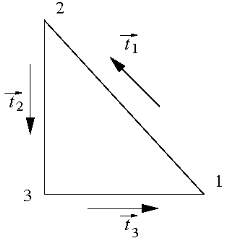

$$\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}\,|_{\gamma_3} = a + c\,\hat{0} = a$$

As funções de base são de grau 1 e constantes nas arestas:

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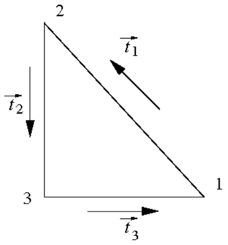

$$\int_{\hat{\gamma}_3} \hat{\mathbf{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}\, d\hat{\Gamma} = a = 0$$

As funções de base são de grau 1 e constantes nas arestas:

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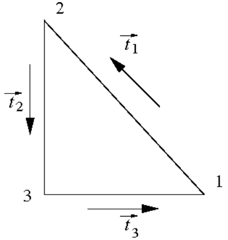
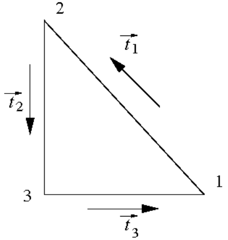


$$a = 0$$

As funções de base são de grau 1 e constantes nas arestas:

$$\hat{\boldsymbol{N}}_j(\hat{x},\hat{y}) = \begin{pmatrix} a + c\hat{y} \\ b - c\hat{x} \end{pmatrix}$$

Vamos encontrar $\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}})$:



$$\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}_2 = -(b - c\,\hat{x})$$

As funções de base são de grau 1 e constantes nas arestas:

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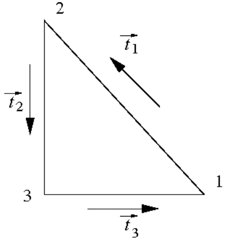
Vamos encontrar $\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}})$:

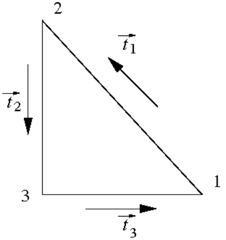


$$\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}|_{\gamma_2} = -(b - c\,\hat{0}) = -b$$

As funções de base são de grau 1 e constantes nas arestas:

$$\hat{\mathbf{N}}_j(\hat{x},\hat{y}) = \begin{pmatrix} a + c\hat{y} \\ b - c\hat{x} \end{pmatrix}$$

Vamos encontrar $\hat{\mathbf{N}}_1(\hat{\boldsymbol{x}})$:



$$\int_{\hat{\gamma}_2} \hat{\mathbf{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}\, d\hat{\Gamma} = -b = 0$$

As funções de base são de grau 1 e constantes nas arestas:

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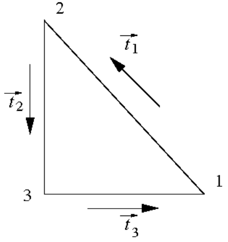
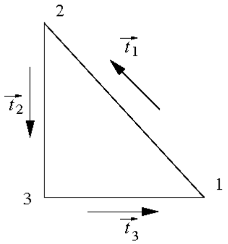


$$b = 0$$

As funções de base são de grau 1 e constantes nas arestas:

$$\hat{\boldsymbol{N}}_j(\hat{x},\hat{y}) = \begin{pmatrix} a + c\hat{y} \\ b - c\hat{x} \end{pmatrix}$$

Vamos encontrar $\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}})$:

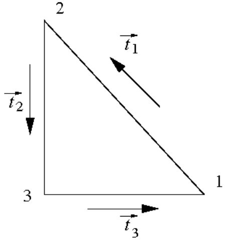


$$\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}_1 = \frac{-(a + c\,\hat{y}) + (b - c\,\hat{x})}{\sqrt{2}}$$

As funções de base são de grau 1 e constantes nas arestas:

$$\hat{\boldsymbol{N}}_j(\hat{x},\hat{y}) = \begin{pmatrix} a + c\hat{y} \\ b - c\hat{x} \end{pmatrix}$$

Vamos encontrar $\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}})$:

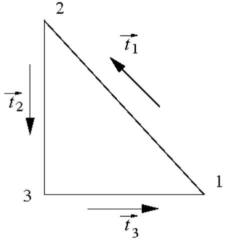


$$\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}_1 = \frac{-c(\hat{x}+\hat{y})}{\sqrt{2}}$$

As funções de base são de grau 1 e constantes nas arestas:

$$\hat{\boldsymbol{N}}_j(\hat{x},\hat{y}) = \begin{pmatrix} a + c\hat{y} \\ b - c\hat{x} \end{pmatrix}$$

Vamos encontrar $\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}})$:

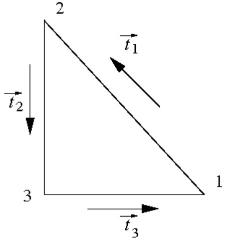


$$\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t}\,|_{\gamma_1} = \frac{-c(1)}{\sqrt{2}} = \frac{-c}{\sqrt{2}}$$

As funções de base são de grau 1 e constantes nas arestas:

$$\hat{\boldsymbol{N}}_j(\hat{x}, \hat{y}) = \begin{pmatrix} a + c\hat{y} \\ b - c\hat{x} \end{pmatrix}$$

Vamos encontrar $\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}})$:



$$\int_{\hat{\gamma}_1} \hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) \cdot \boldsymbol{t} \, d\hat{\Gamma} = -c = 1$$

As funções de base são de grau 1 e constantes nas arestas:

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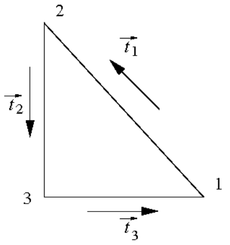

$$\begin{cases} a = 0 \\ b = 0 \\ c = -1 \end{cases} \quad \Longrightarrow \quad \hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) = \begin{pmatrix} -\hat{y} \\ \hat{x} \end{pmatrix}$$

As funções de base são de grau 1 e constantes nas arestas:

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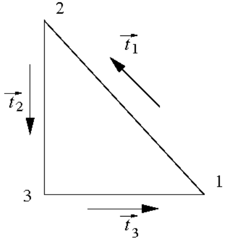
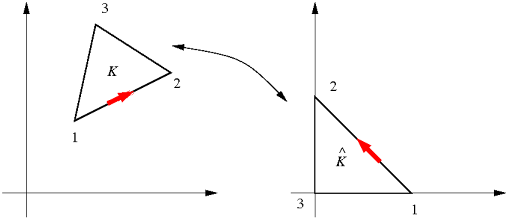
Vamos encontrar $\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}})$:



Prosseguindo, encontramos (Schneebeli, 2003):

$$\hat{\boldsymbol{N}}_1(\hat{\boldsymbol{x}}) = \begin{pmatrix} -\hat{y} \\ \hat{x} \end{pmatrix}, \quad \hat{\boldsymbol{N}}_2(\hat{\boldsymbol{x}}) = \begin{pmatrix} -\hat{y} \\ \hat{x} - 1 \end{pmatrix}, \quad \hat{\boldsymbol{N}}_3(\hat{\boldsymbol{x}}) = \begin{pmatrix} 1 - \hat{y} \\ \hat{x} \end{pmatrix}.$$

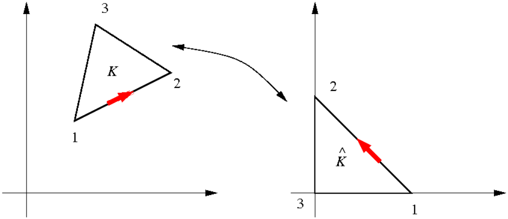
Relação entre os vetores tangentes nos dois elementos ?



Vetor tangente via derivada da parametrização:

$$\hat{\gamma}_1(s) = \begin{pmatrix} 1-s \\ s \end{pmatrix}$$ (sentido anti-horário)

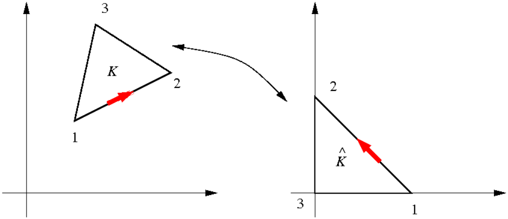
Relação entre os vetores tangentes nos dois elementos ?



Vetor tangente via derivada da parametrização:

$$\hat{\gamma}_1'(s) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

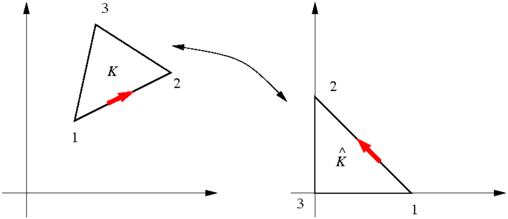
Relação entre os vetores tangentes nos dois elementos ?



Vetor tangente via derivada da parametrização:

$$\frac{1}{\|\hat{\gamma}_1'(s)\|}\hat{\gamma}_1'(s) = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \hat{\boldsymbol{t}}_1$$

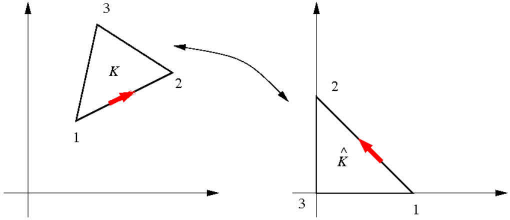
Relação entre os vetores tangentes nos dois elementos ?



Vetor tangente via derivada da parametrização:

$$\hat{\boldsymbol{t}}_j = \frac{1}{\|\hat{\gamma}_j'(s)\|}\hat{\gamma}_j'(s)$$

Relação entre os vetores tangentes nos dois elementos ?



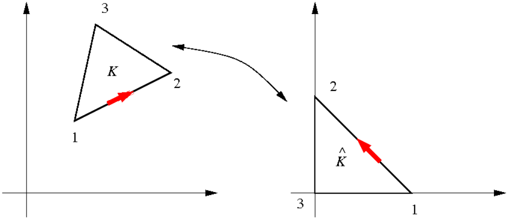
Vetor tangente via derivada da parametrização:

$$\hat{\boldsymbol{t}}_j = \frac{1}{\|\hat{\gamma}_j'(s)\|}\hat{\gamma}_j'(s)$$

Nas coordenadas do elemento K,

$$\boldsymbol{x}(\hat{\boldsymbol{x}}) = \boldsymbol{x}_1\hat{N}_1(\hat{\boldsymbol{x}}) + \boldsymbol{x}_2\hat{N}_2(\hat{\boldsymbol{x}}) + \boldsymbol{x}_3\hat{N}_3(\hat{\boldsymbol{x}}) = \boldsymbol{J}_K\hat{\boldsymbol{x}} + \boldsymbol{x}_0$$

Relação entre os vetores tangentes nos dois elementos ?



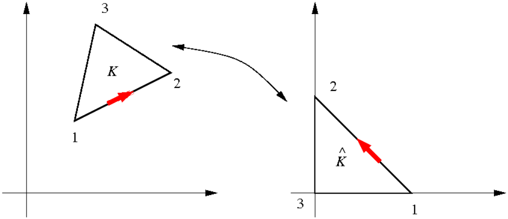
Vetor tangente via derivada da parametrização:

$$\hat{\boldsymbol{t}}_j = \frac{1}{\|\hat{\gamma}_j'(s)\|}\hat{\gamma}_j'(s)$$

Nas coordenadas do elemento K,

$$\boldsymbol{\gamma}_j(s) = \boldsymbol{x}(\hat{\boldsymbol{\gamma}}_j(s))$$

Relação entre os vetores tangentes nos dois elementos ?



Vetor tangente via derivada da parametrização:

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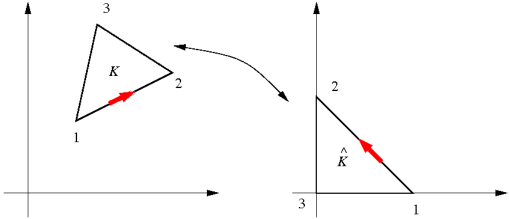
Nas coordenadas do elemento K,

$$\boldsymbol{\gamma}_j(s) = \boldsymbol{J}_K \hat{\boldsymbol{\gamma}}_j(s) + \boldsymbol{x}_0$$

Relação entre os vetores tangentes nos dois elementos ?



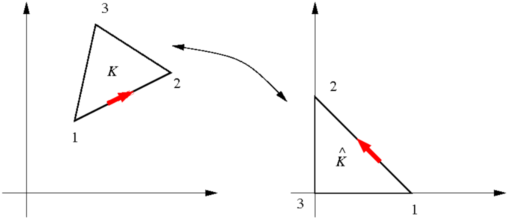
Vetor tangente via derivada da parametrização:

$$\hat{\boldsymbol{t}}_j = \frac{1}{\|\hat{\gamma}'_j(s)\|}\hat{\gamma}'_j(s)$$

Nas coordenadas do elemento K,

$$\gamma'_j(s) = \boldsymbol{J}_K \hat{\gamma}'_j(s)$$

Relação entre os vetores tangentes nos dois elementos ?



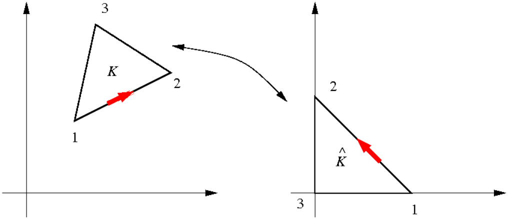
Vetor tangente via derivada da parametrização:

$$\hat{\boldsymbol{t}}_j = \frac{1}{\|\hat{\gamma}_j'(s)\|}\hat{\gamma}_j'(s)$$

Nas coordenadas do elemento K,

$$\boldsymbol{t}_j = \frac{1}{\|\gamma_j'(s)\|}\boldsymbol{J}_K\hat{\gamma}_j'(s)$$

Relação entre os vetores tangentes nos dois elementos ?



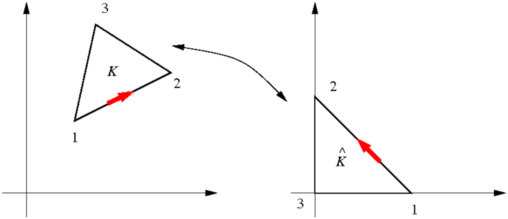
Vetor tangente via derivada da parametrização:

$$\hat{\boldsymbol{t}}_j = \frac{1}{\|\hat{\gamma}_j'(s)\|}\hat{\gamma}_j'(s)$$

Nas coordenadas do elemento K,

$$\boldsymbol{t}_j = \frac{\|\hat{\gamma}_j'(s)\|}{\|\gamma_j'(s)\|}\frac{1}{\|\hat{\gamma}_j'(s)\|}\boldsymbol{J}_K\hat{\gamma}_j'(s)$$

Relação entre os vetores tangentes nos dois elementos ?



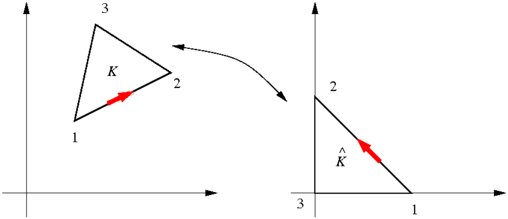
Vetor tangente via derivada da parametrização:

$$\hat{\boldsymbol{t}}_j = \frac{1}{\|\hat{\gamma}_j'(s)\|}\hat{\gamma}_j'(s)$$

Nas coordenadas do elemento K,

$$\boldsymbol{t}_j = \frac{\|\hat{\gamma}_j'(s)\|}{\|\gamma_j'(s)\|}\boldsymbol{J}_K \frac{1}{\|\hat{\gamma}_j'(s)\|}\hat{\gamma}_j'(s)$$

Relação entre os vetores tangentes nos dois elementos ?



Vetor tangente via derivada da parametrização:

$$\hat{\boldsymbol{t}}_j = \frac{1}{\|\hat{\gamma}_j'(s)\|}\hat{\gamma}_j'(s)$$

Nas coordenadas do elemento K,

$$\boldsymbol{t}_j = \frac{\|\hat{\gamma}_j'(s)\|}{\|\gamma_j'(s)\|}\boldsymbol{J}_K\hat{\boldsymbol{t}}_j$$

Objetivo:

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_{\hat{\gamma}_k} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}})\cdot\hat{\boldsymbol{t}}_k \, d\hat{\Gamma} = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}, \qquad j = 1, 2, 3$$

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Temos que

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x}) \cdot \boldsymbol{t}_k \, d\Gamma = \int_0^1 \boldsymbol{N}_j^e(\boldsymbol{\gamma}_k(s)) \cdot \boldsymbol{t}_k \|\boldsymbol{\gamma}_k'(s)\| \, ds$$

Objetivo:

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_{\hat{\gamma}_k} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}})\cdot\hat{\boldsymbol{t}}_k \, d\hat{\Gamma} = \left\{ \begin{array}{ll} 1, & j = k \\ 0, & j \neq k \end{array} \right. , \qquad j = 1,2,3$$

Temos que

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_0^1 \boldsymbol{N}_j^e(\boldsymbol{x}(\hat{\boldsymbol{\gamma}}_k(s)))\cdot\boldsymbol{t}_k \|\boldsymbol{\gamma}_k'(s)\| \, ds$$

Objetivo:

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_{\hat{\gamma}_k} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}})\cdot\hat{\boldsymbol{t}}_k \, d\hat{\Gamma} = \left\{ \begin{array}{ll} 1, & j = k \\ 0, & j \neq k \end{array} \right. , \qquad j = 1, 2, 3$$

Temos que

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_0^1 \boldsymbol{N}_j^e(\boldsymbol{x}(\hat{\boldsymbol{\gamma}}_k(s)))\cdot\left(\frac{\|\hat{\boldsymbol{\gamma}}_k'(s)\|}{\|\boldsymbol{\gamma}_k'(s)\|}\boldsymbol{J}_K\hat{\boldsymbol{t}}_k\right)\|\boldsymbol{\gamma}_k'(s)\| \, ds$$

Objetivo:

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_{\hat{\gamma}_k} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}})\cdot\hat{\boldsymbol{t}}_k \, d\hat{\Gamma} = \left\{ \begin{array}{ll} 1, & j = k \\ 0, & j \neq k \end{array} \right. , \qquad j = 1, 2, 3$$

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Temos que

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_0^1 \left(\boldsymbol{J}_K\hat{\boldsymbol{t}}_k\right)^T \boldsymbol{N}_j^e(\boldsymbol{x}(\hat{\gamma}_k(s)))\|\hat{\boldsymbol{\gamma}}_k'(s)\| \, ds$$

Objetivo:

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_{\hat{\gamma}_k} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}})\cdot\hat{\boldsymbol{t}}_k \, d\hat{\Gamma} = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}, \qquad j = 1, 2, 3$$

Temos que

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_0^1 \hat{\boldsymbol{t}}_k^T \boldsymbol{J}_K^T \boldsymbol{N}_j^e(\boldsymbol{x}(\hat{\boldsymbol{\gamma}}_k(s))) \|\hat{\boldsymbol{\gamma}}_k'(s)\| \, ds$$

Objetivo:

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x}) \cdot \boldsymbol{t}_k \, d\Gamma = \int_{\hat{\gamma}_k} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}}) \cdot \hat{\boldsymbol{t}}_k \, d\hat{\Gamma} = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}, \qquad j = 1, 2, 3$$

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Objetivo:

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Para que

$$\int_{\gamma_k} \boldsymbol{N}_j^e(\boldsymbol{x})\cdot\boldsymbol{t}_k \, d\Gamma = \int_0^1 \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{\gamma}}_k(s))\cdot\hat{\boldsymbol{t}}_k \|\hat{\boldsymbol{\gamma}}_k'(s)\| \, ds = \int_{\hat{\gamma}_k} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}})\cdot\hat{\boldsymbol{t}}_k \, d\hat{\Gamma},$$

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Transformação do tipo Piola

$$\boldsymbol{N}_j^e(\boldsymbol{x}(\hat{\boldsymbol{x}})) = \boldsymbol{J}_K^{-T} \hat{\boldsymbol{N}}_j(\hat{\boldsymbol{x}})$$

Elementos Finitos de Aresta

Observações

- Precisamos de uma matriz de conectividade das arestas (convém usar a opção –e no aplicativo triangle)
- Devemos atribuir o sentido do vetor tangente a cada aresta

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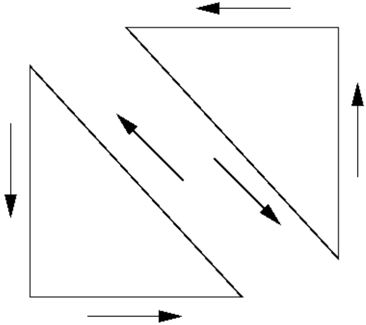

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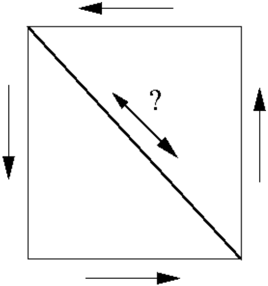

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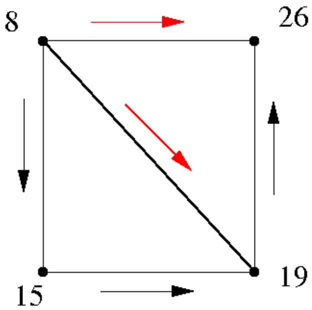
Saída: t aponta para o vértice de maior índice (Jin, 2002)

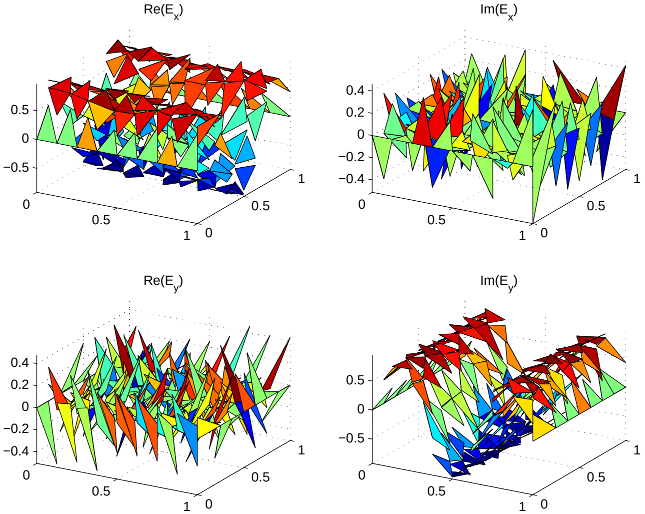
- Precisamos de uma matriz de conectividade das arestas (convém usar a opção –e no aplicativo triangle)
- Devemos atribuir o sentido do vetor tangente a cada aresta



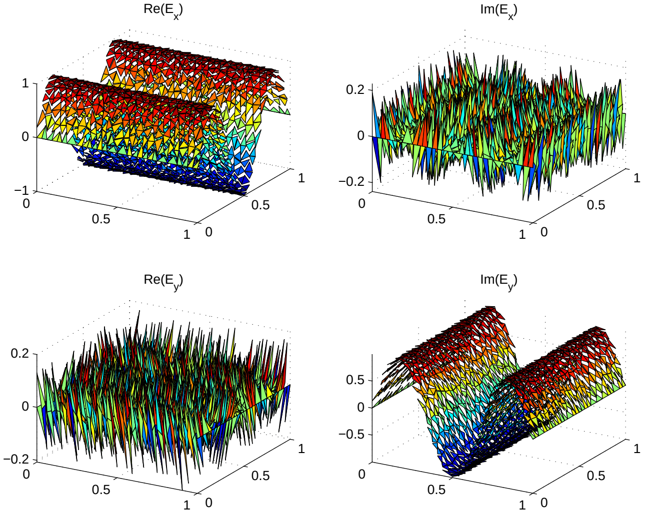
Elementos Finitos de Aresta

Exemplos ($\sigma = 0$)

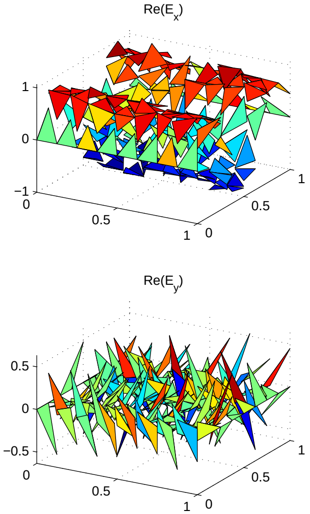
$M = 3, \omega = 100$ (malha grossa):



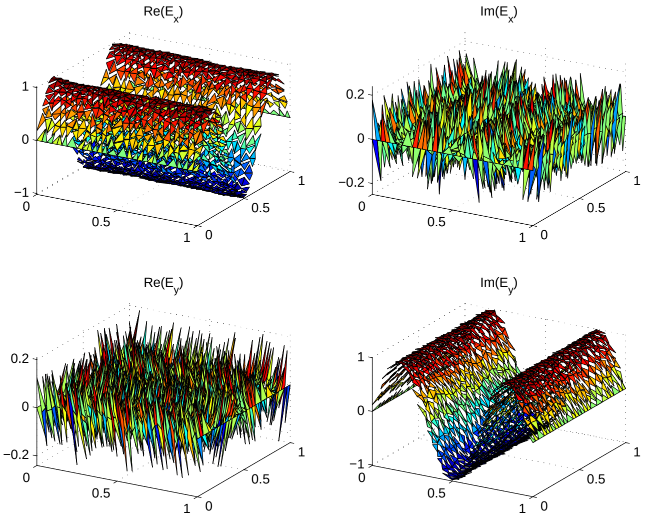
$M = 3, \omega = 100$ (malha fina):



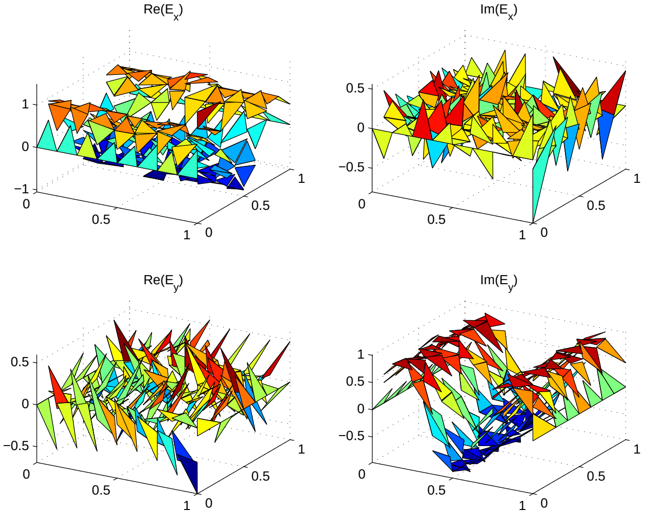
$M = 3, \omega = 10$ (malha grossa):



$M = 3, \omega = 10$ (malha fina):



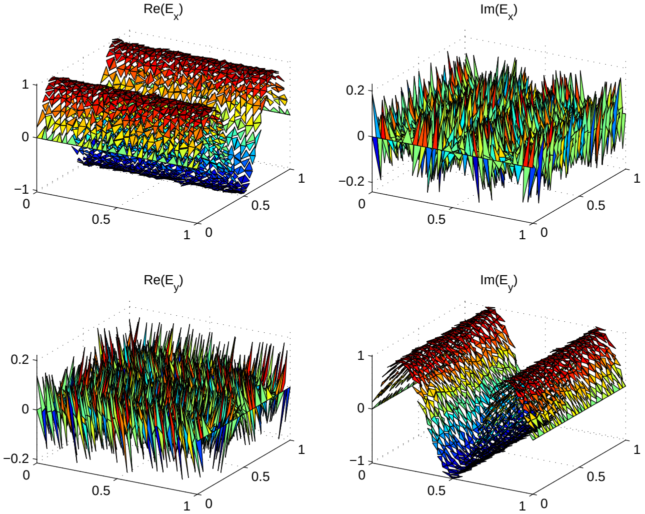
$M = 3, \omega = 1$ (malha grossa):



Elementos Finitos de Aresta

Exemplos ($\sigma = 0$)

$M = 3, \omega = 1$ (malha fina):



Panorama da Pesquisa na área

Modos espúrios

Histórico

- ”Handwaving” (Lee et al., 1991; Mur, 1994; Hillion, 1997; Jiang et al., 1996; Schroeder and Wolff, 1994; Bossavit, 1990)
- Formulação aumentada (Assous et al., 1993)
- Compacidade discreta (Kikuchi, 1989; Boffi et al., 2006; Demkowicz et al., 2000)
- Métodos mistos (Boffi et al., 1999)

Elementos de arestas

- Elemento de Whitney (Whitney, 1957)
- Nedelec (Nédélec, 1980, 1986)
- Bases mistas nodais e de arestas (Graglia et al., 1997)
- Caso particular: Método de Yee (Monk, 1993)

Alternativas para os modos espúrios

- Formulação aumentada e quadrados mínimos (Assous et al., 1993; Jiang et al., 1996; Vardapetyan and Demkowicz, 1999)
- Interior penalty (Grote et al., 2008)
- Formulações aumentadas ponderadas (Costabel and Dauge, 2002; Bramble and Pasciak, 2004)

Singularidades do domínio

- Elementos de arestas quadrilaterais (Boffi et al., 2006)
- Formulação aumentada com estabilização (Badia and Codina, 2011)

Modelos Gerais

- Sistema de Primeira Ordem (Bramble et al., 2005; Maggio et al., 2004; Rieben et al., 2004; Zhao et al., 2009)
- Potenciais (Hiptmair et al., 2008; Biro and Preis, 1989; Badea et al., 2001)
- CSEM
  - Modos TM e TE (Key and Weiss, 2006)
  - Modelo 2.5D (Li and Key, 2007)
  - Meios anisotrópicos (Li and Key, 2007)
  - Modelo 3D no domínio da frequência / elementos de aresta Fatoração dos sistemas lineares (Silva et al., 2012)

- Resenha dos métodos (Lee et al., 1997)
- Métodos simpléticos (Rieben et al., 2004; Zhao et al., 2009)
- Mass Lumping:
  - contra-exemplo no caso nodal (Lee et al., 1997)
  - combinação com elementos espectrais (Pernet et al., 2005; Cohen and Monk, 1999)

- Discretização de formas diferenciais (Whitney, 1957; Bossavit, 1998; He and Teixeira, 2007; Hiptmair, 2001, 2002)
- Elementos finitos para cálculo exterior (Arnold et al., 2010)
- Compacidade discreta e métodos hp (Boffi et al., 2011)
- Métodos miméticos (Brezzi and Buffa, 2010)

Referências Adicionais

Métodos de alta ordem

- Resenha dos Métodos (Hesthaven, 2003)
- Métodos hp (Vardapetyan and Demkowicz, 1999; Ainsworth and Coyle, 2001)
- Elementos espectrais (arestas)(Lee et al., 2006; Lee and Liu, 2007)
- Elementos espectrais / quadrados mínimos (nodais) (Maggio et al., 2004)
- Elementos espectrais com bases mistas (nodais+arestas) (Lin et al., 2007)
- Partition of Unity (Ledger et al., 2003)

Referências Adicionais

Textos Preliminares

- Introdução aos elementos de Nédélec (Schneebeli, 2003)
- Dissertação sobre elementos de Nédélec (Sebold, 2011)
- livros-texto (Jin, 2002; Monk, 2003)
- Resenha / Demkowicz (Demkowicz, 2004)
- Resenha / Hesthaven (Demkowicz, 2004)
- Mini-curso (Peter Monk) (Monk, 2010)

Referências Adicionais

Artigos Recomendados

- hp / formulação aumentada (Demkowicz and Vardapetyan, 1998)
- Elemento de aresta para quadriláteros em geral (Boffi et al., 2006)
- Elementos espectrais em $H(curl)$ (Lee et al., 2006)
- Elementos espectrais nodais (least squares) (Maggio et al., 2004)
- Formulação aumentada com norma negativa (Bramble et al., 2005)
- Formulação no domínio do tempo (Lee et al., 1997)

Material disponível

Material disponível em

www.ufpr.br/$\sim$saulopo/maxwell

Material disponível

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www.ufpr.br/$\sim$saulopo/maxwell

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