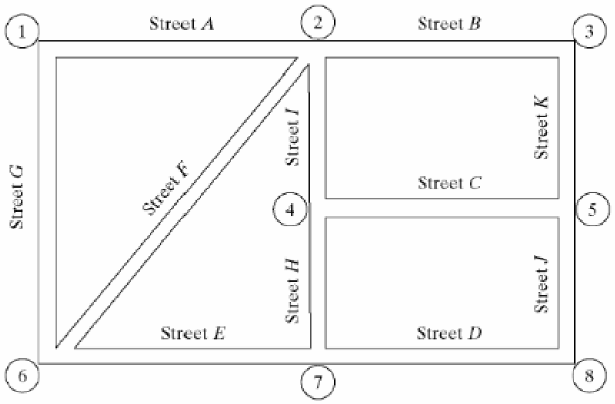
Set Covering Problem

Fonte: http://www.ifp.illinois.edu/~angelia/ge330fall09_ilpmodel_l22.pdf

To promote on-campus safety, the U of A Security Department is in the process of installing cameras at selected locations. The department wants to install the minimum number of cameras while providing a surveillance coverage for each of the campus main streets. It is reasonable to place the cameras at street intersections so that each camera serves at least two streets. The eight candidate locations are given in the figure, indexed by 1 through 8.



We want to minimize the sum of these variables. We have a constraint per street (a requirement that each street has to be covered).

minimize $Z = \sum xj$			
st	$x1+x2\geq 1$,	$x2+x3\geq 1$	streets A and B
	$x4+x5\geq 1$,	$x7+x8\geq 1$	streets C and D
	$x6+x7\geq 1$,	$x2+x6\geq 1$	streets E and F
	$x1+x6\geq 1$,	$x4+x7\geq 1$	streets G and H
	$x2+x4\geq 1$,	$x5+x8\geq 1$	streets I and J
		$x3+x5\geq 1$	street K
	$xj\in\{0,1\}$ for all j		

Com esse modelo busca-se localizar instalações com o menor custo possível, de maneira que todos os pontos de demanda possam ser atendidos por pelo menos uma instalação, dentro de uma distância mínima global.

Set Partitioning and Set Packing

As an example of **set partioning**, consider the problem of delivering orders from a warehouse to n different stores by m trucks. Each store receives its order in exactly one delivery. A truck can deliver at most k(k<n) orders (stores). Because a store may fall on more than one route, a truck may pass a store without delivery of that store's order. It is required that all orders (stores) must be delivered. Here, activity x represents a feasible delivery sequence of orders satisfying the truck capacity. The collection of feasible activities forms a matrix A. The constraint set, Ax = 1, ensures that every order is delivered exactly by one truck. In a busy day, it may be acceptable that some lower priority orders can be postponed to a later day. To

represent this situation, the set of constraints becomes Ax < 1. This problem is known as a set **packing problem**.

Set packing example

Given

- M = {1, 2, 3, 4, 5, 6}
- M1 = {1, 2, 5}; M2 = {1, 3}; M3 = {2, 4}; M3 = {3, 6}; M4 = {2, 3, 6}

Examples

{M1, M2} ≡ {{1, 2, 5}, {3, 6}}: a pack

{M2, M3} ≡ {{1, 3}, {2, 4}}: a pack

{M1, M4} ≡ {{1, 2, 5}, {2, 3, 6}}: not a pack

{M4} ≡ {2, 3, 6}: a pack

Set partitioning example

We are trying to build 2 teams (*k*=2) of people at the office to work in a project but there are some people who don't get along with their coworkers very well so we asked them to assign a score from 1 to 3 to all of their coworkers being 3 "I'd really like to work with him" and 1 being "I'd rather get fired". We'd like to build the teams such that the 'likeness' in every team is the maximum possible. Say we have 4 people (*n*=4) *M*={*Tod*, *Lydia*, *Walter*, *Jesse*} and the scores are as following:

	Tod	Lydia	Walter	Jesse
Tod	−	3	2	1
Lydia	3	−	3	1
Walter	2	3	−	3
Jesse	1	1	3	−

Matching Problem: A Special Type of Set Packing Problem

1 Matching using Linear Programming

We look at the linear programming method for the maximum matching and perfect matching problems. Given a graph $G = (V, E)$, an integer linear program (ILP) for the maximum matching problem can be written by defining a variable x_e for each edge $e \in E$ and a constraint for each vertex $u \in V$ as follows:

$$\begin{aligned} \text{Maximize} \quad & \sum_{e \in E} x_e \text{ subject to} \\ & \forall u \in V \sum_{e \sim u} x_e \leq 1 \\ & \forall e \in E x_e \in \{0, 1\} \end{aligned}$$

Fonte: https://www.imsc.res.in/~meena/matching/lecture5.pdf

12.1 Formulation of General Perfect Matching

Given a weighted general graph $G = (V, E)$, for any edge e, w_e represents the weight of the edge and x_e is the indicator variable of e. If an edge e is in the matching, we set $x_e = 1$, otherwise $x_e = 0$. Base on these definitions, the weighted perfect matching problem can be formulated as follows:

$$\max \sum_{e \in E} w_e \cdot x_e$$

$$\sum_{e \in \delta(v)} x_e = 1 \qquad \forall v \in V \tag{12.1.1}$$

$$x_e = \{0, 1\} \qquad \forall e \in E \tag{12.1.2}$$

Fonte: https://pdfs.semanticscholar.org/1246/a25f9ac7a6d5c5982cb8f4b99d75b60d933a.pdf