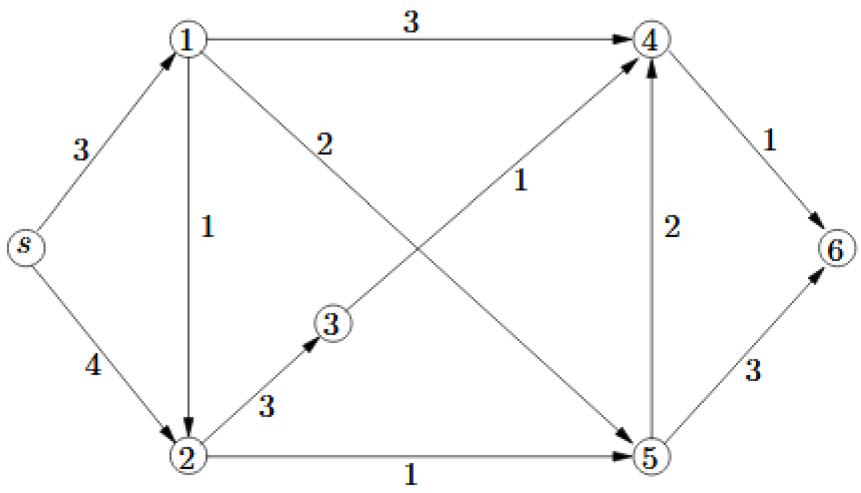
Exercícios

Exercise 5

Compute the shortest-paths tree from s using Dijkstra's algorithm. Sketch each iteration.



https://www.zib.de/groetschel/Project-Quito/cursos/curso_2005_1/final_exercises/1_basics.pdf

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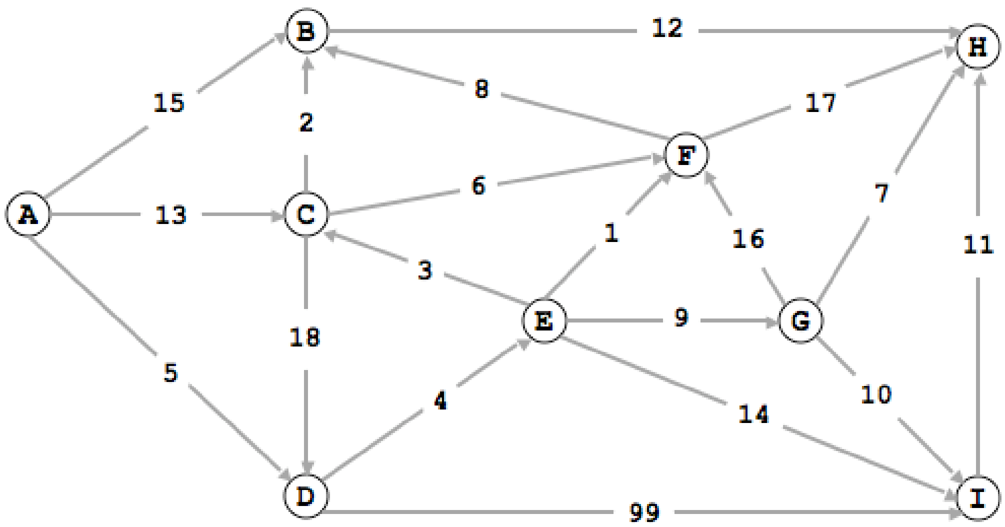
1. Consider the following weighted directed graph.



Run Dijkstra's single source shortest path algorithm on the above digraph using C as the source. Give the order in which the vertices are removed from the priority queue. Note that A is not reachable from C, so it is never added (and thus never removed) from the priority queue.

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0    1    2    3    4    5    6    7
------------------------------------
C
```

https://www.cs.princeton.edu/courses/archive/spr10/cos226/exercises/sp.html

Label the following points in the plane 0 through 5, respectively:

`(1, 3) (2, 1) (6, 5) (3, 4) (3, 7) (5, 3).`

Taking edge lengths to be weights, consider the network defined by the edges

`1-0 3-5 5-2 3-4 5-1 0-3 0-4 4-2 2-3.`

Show all shortest paths in the network defined in Exercise 21.1.

http://www.informit.com/articles/article.aspx?p=169575

The following table from a published road map purports to give the length of the shortest routes connecting the cities. It contains an error. Correct the table. Also, add a table that shows how to execute the shortest routes

	Providence	**Westerly**	**New London**	**Norwich**
Providence	–	53	54	48
Westerly	53	–	18	101
New London	54	18	–	12
Norwich	48	101	12	–

http://www.informit.com/articles/article.aspx?p=169575

Exercise 1

Consider the graph below. Compute a minimum spanning tree of this graph.

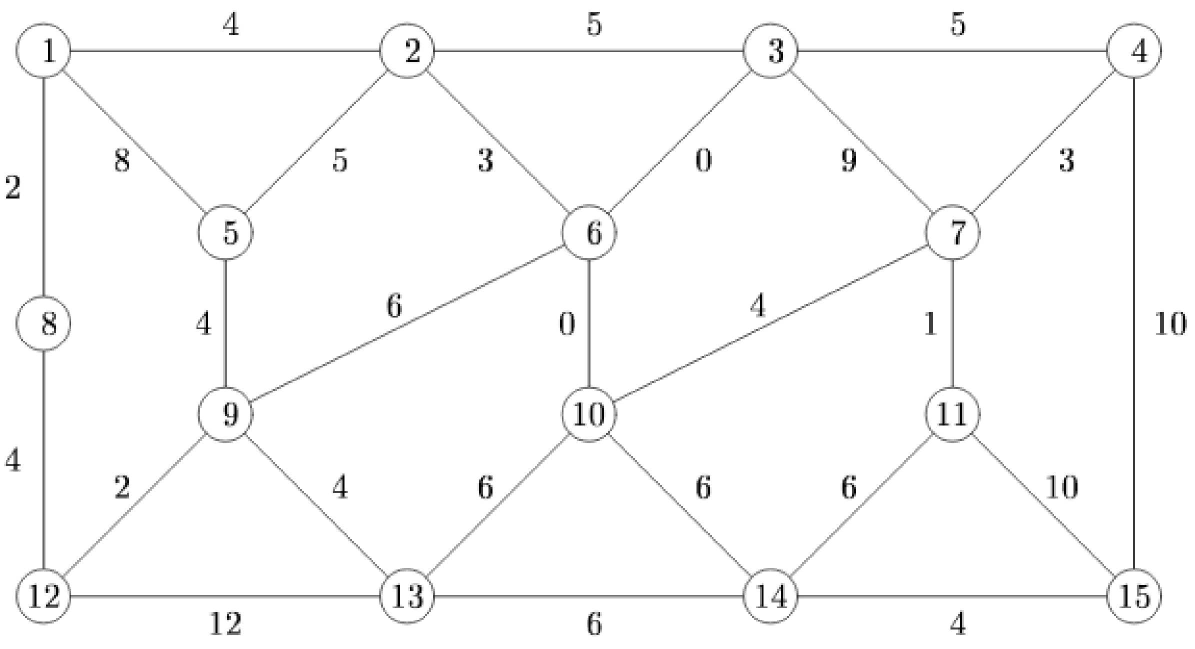


Figure 1: Graaf1

Exercise 2

Consider the graph for exercise 1. Compute the shortest path from point 1 to all other nodes.

https://www.win.tue.nl/~wscor/OW/2V300/Exer.pdf

Exercise 9
Consider the network below. The numbers along the arcs give the value of the current flow, and the capacity of the arc, respectively. Compute the maximum flow through this network, and prove maximality by using a minimum capacity cut. Describe each step of your procedure.

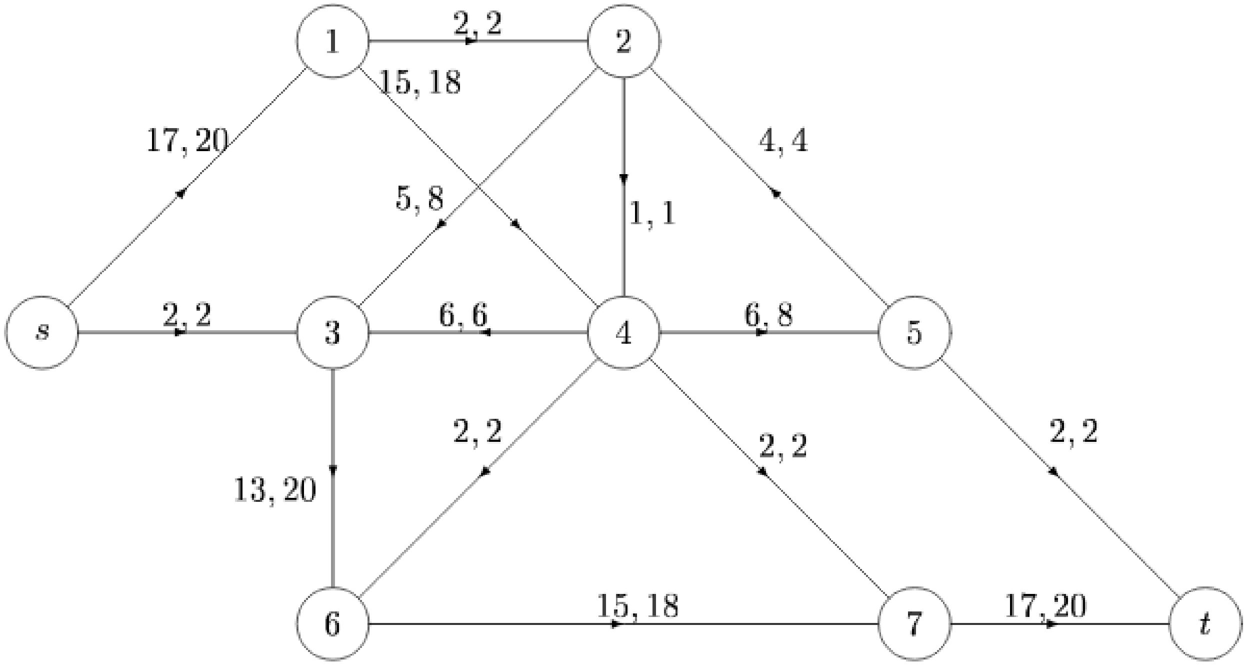


Figure 2: Network

https://www.win.tue.nl/~wscor/OW/2V300/Exer.pdf

Exercise 1 [Dijkstra]Apply the Dijkstra Algorithm to the graph depicted in Figure 1 to compute a shortest-path tree rooted in a and the distance between any vertex and vertex a. The first three steps of the algorithm must be detailed (**at most three or four lines per steps**). Moreover, indicate the order in which vertices are considered during the execution of the algorithm.

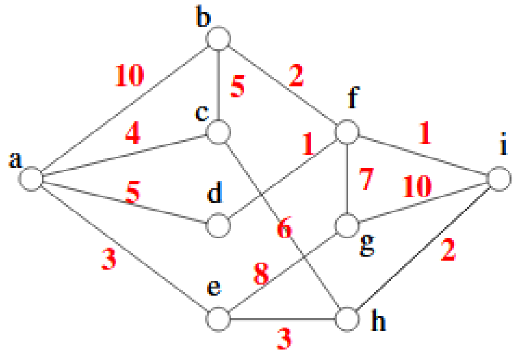
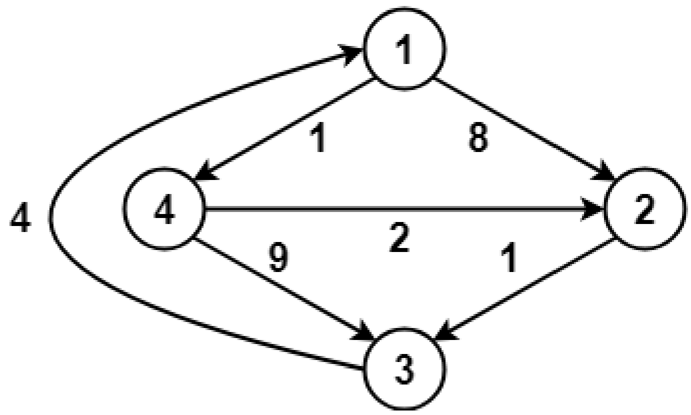


FIGURE 1 – A graph with 9 vertices. A number indicates the length of the arc it is close to.
http://www-sop.inria.fr/members/Nicolas.Nisse/lectures/exercises-WeightedGraph.pdf

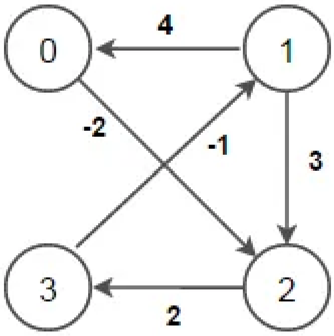
Problem-

Consider the following directed weighted graph-



Using Floyd-Warshall Algorithm, find the shortest path distance between every pair of vertices.

https://www.gatevidyalay.com/floyd-warshall-algorithm-shortest-path-algorithm/



https://www.techiedelight.com/pairs-shortest-paths-floyd-warshall-algorithm/

7. Solve the all-pairs shortest path problem for the digraph with the following weight matrix

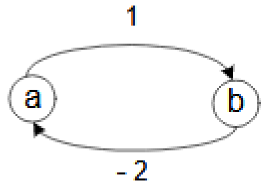
$$\begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

FLOYD

https://sureshvcetit.files.wordpress.com/2018/04/solu8.pdf

Give an example of a graph or a digraph with negative weights for which Floyd's algorithm does not yield the correct result.

9. As a simple counterexample, one can suggest the following digraph:



Floyd's algorithm will yield:

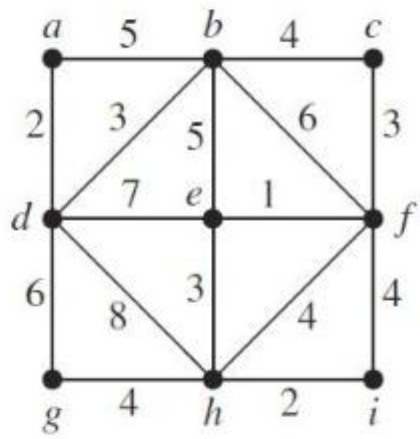
$$D^{(0)} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \qquad D^{(1)} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} \qquad D^{(2)} = \begin{bmatrix} -1 & 0 \\ -3 & -2 \end{bmatrix}$$

None of the four elements of the last matrix gives the correct value of the shortest path, which is, in fact, $-\infty$ because repeating the cycle enough times makes the length of a path arbitrarily small.

Note: Floyd's algorithm can be used for detecting negative-length cycles, but the algorithm should be stopped as soon as it generates a matrix with a negative element on its main diagonal.

https://sureshvcetit.files.wordpress.com/2018/04/solu8.pdf

Use Prim's algorithm to find a minimum spanning tree for the given weighted graph.



Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph in Exercise 3.

http://courses.ics.hawaii.edu/ReviewICS241/morea/trees/MinimumSpanningTrees-QA.pdf

1. Given a complete graph with vertex set $\{A, B, C, D, E, F\}$ and the following weights on the edges.

	A	B	C	D	E	F
A	0	6	9	11	5	9
B	6	0	3	6	5	2
C	9	3	0	0	4	4
D	11	6	0	0	5	6
E	5	5	4	5	0	8
F	9	2	4	6	8	0

a. Determine a minimum spanning tree with Kruskal's algorithm.

b. Determine a minimum spanning tree with Prim's algorithm starting in F.

c. Determine the shortest paths from F to all other vertices by use of Dijkstra's algorithm.

https://ittrys.files.wordpress.com/2012/01/u2.pdf

6. The table below gives the weights (lengths) $w(x, y)$ of the arcs in a directed graph (x corresponds to the row, and y to the column).

	A	B	C	D	E	F
A	0	8	8	8	8	2
B	8	0	−2	7	6	2
C	8	4	0	3	2	3
D	8	4	0	0	−1	4
E	8	2	2	3	0	8
F	8	3	2	−1	4	0

a. Use the method of Bellman-Ford to determine a shortest path from A to C, or a negative circuit if such a path does not exist.

b. Use the method of Floyd to determine a shortest path from A to C, or a negative circuit if such a path does not exist.

c. Same questions, but now with $w(C, D) = 0$.

https://ittrys.files.wordpress.com/2012/01/u2.pdf

1. Consider a set of 5 towns. The cost of constraction of a road between towns i and j is a_{ij}. Find the minimum cost road network connecting the towns with each other.

$$\begin{bmatrix} 0 & 3 & 5 & 11 & 9 \\ 3 & 0 & 3 & 9 & 8 \\ 5 & 3 & 0 & +\infty & 10 \\ 11 & 9 & +\infty & 0 & 7 \\ 9 & 8 & 10 & 7 & 0 \end{bmatrix}$$

https://www.lix.polytechnique.fr/~liberti/teaching/isic/isc612-07/ex-gph-sol.pdf

A contractor assigns to one of his building sites a changing number of qualified workers between March and August:

Month	March	April	May	June	July	August
Stuff	4	6	7	4	6	2

Workers can be discharged from the building site only in the beginning of the month. Suppose that in February and in September there is exactly three workers at the building site.

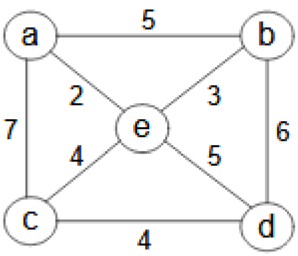
The aim of the contractor is to plan an allocation of workers which minimizes the sum of the following costs :

- Transfer cost. Hiring a worker to the building site costs 50 euros and discharging a worker costs 80 euros.
- Transfer rules. The contractor can hire at most 3 workers at a time and can discharge at most one third of his stuff at a time.
- Over and underpopulation costs. A superfluous worker costs 100 euros, whereas missing of one worker costs 200 euros. When some stuff is missing, workers take additional working hours, but they do not accept to work additionaly more than 1/4 of their normal time.

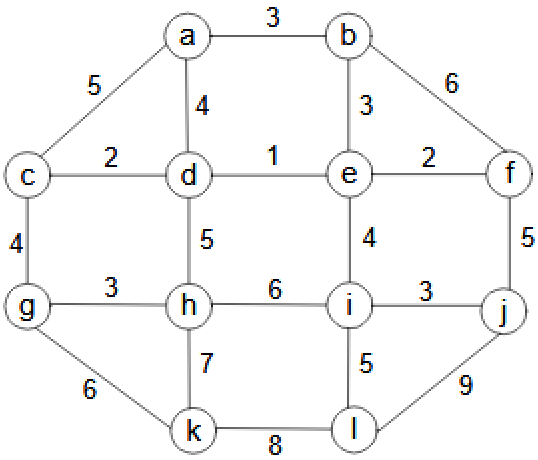
Formulate this problem as a shortest path problem and solve it using the Dijsktra's algorithm.

https://www.lix.polytechnique.fr/~liberti/teaching/isic/isc612-07/ex-gph-sol.pdf

7. a. Apply Prim's algorithm to the following graph. Include in the priority queue all the vertices not already in the tree.



b. Apply Prim's algorithm to the following graph. Include in the priority queue only the fringe vertices (the vertices not in the current tree which are adjacent to at least one tree vertex).



https://sureshvcetit.files.wordpress.com/2018/04/solu9.pdf