General structure of the linear programming model

Maximize (or minimize) $Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots\ldots + c_nx_n$

Subject to:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots\ldots + a_{1n}x_n\ (\leq, =, \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots\ldots + a_{2n}x_n\ (\leq, =, \geq)\ b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots\ldots + a_{3n}x_n\ (\leq, =, \geq)\ b_3$$
$$\ldots.$$
$$\ldots.$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots\ldots + a_{mn}x_n\ (\leq, =, \geq)\ b_m$$
$$x_i \geq 0.$$

where

Z ∴ objective function
x_i ∴ decision variables
b_j ∴ available resource for j^{th} constraint
c_i ∴ objective function coefficients
a_{ij} ∴ coefficient for i^{th} decision variable on j^{th} constraint.

Type 1: MIXING PROBLEM

https://www.analyticsvidhya.com/blog/2017/02/lintroductory-guide-on-linear-programming-explained-in-simple-english/

Example: Consider a chocolate manufacturing company which produces only two types of chocolate – A and B. Both the chocolates require Milk and Choco only. To manufacture each unit of A and B, following quantities are required:

- Each unit of A requires 1 unit of Milk and 3 units of Choco
- Each unit of B requires 1 unit of Milk and 2 units of Choco

The company kitchen has a total of 5 units of Milk and 12 units of Choco. On each sale, the company makes a profit of

- $ 6 per unit A sold
- $ 5 per unit B sold.

Now, the company wishes to maximize its profit. How many units of A and B should it produce respectively?

Common terminologies used in Linear Programming

Let us define some terminologies used in Linear Programming using the above example.

- **Decision Variables:** The decision variables are the variables which will decide my output. They represent my ultimate solution. To solve any problem, we first need to identify the decision variables. For the above example, the total number of units for A and B denoted by X & Y respectively are my decision variables.
- **Objective Function:** It is defined as the objective of making decisions. In the above example, the company wishes to increase the total profit represented by Z. So, profit is my objective function.
- **Constraints:** The constraints are the restrictions or limitations on the decision variables. They usually limit the value of the decision variables. In the above example, the limit on the availability of resources Milk and Choco are my constraints.
- **Non-negativity restriction:** For all linear programs, the decision variables should always take non-negative values. Which means the values for decision variables should be greater than or equal to 0.

Type 2: ALLOCATION PROBLEM

http://mu.ac.in/portal/wp-content/uploads/2017/10/dormsem1linearprogramming.pdf

Example: (PRODUCTION ALLOCATION PROBLEM) A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below:

Machine	Time per unit (Minutes)			Machine Capacity (minutes/day)
	Product 1	Product 2	Product 3	
M_1	2	3	2	440
M_2	4	-	3	470
M_3	2	5	-	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is Rs. 4, Rs.3 and Rs.6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (L.P.) model that will maximize the daily profit.

MERITS OF LP

1. Helps management to make efficient use of resources.

2. Provides quality in decision making.

3. Excellent tools for adjusting to meet changing demands.

4. Fast determination of the solution if a computer is used.

5. Provides a natural sensitivity analysis.

6. Finds solution to problems with a very large or infinite number of possible solutions.

DEMERITS OF LP

1. **Existence of non-linear equation**: The primary requirement of Linear Programming is the objective function and constraint function should be linear. Practically linear relationships do not exist in all cases.

2. **Interaction between variables**: LP fails in a situation where non-linearity in the equation emerge due to joint interaction between some of the activities like total effectiveness.

3. **Fractional Value**: In LPP fractional values are permitted for the decision variable.

4. **Knowledge of Coefficients of the equation**: It may not be possible to state all coefficients in the objective function and constraints with certainty.