

Exercício 1: PL com única solução

maximizar $Z = x_1 + 3x_2$

sa $-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1 \geq 0, x_2 \geq 0$

maximize $Z - x_1 - 3x_2 = 0$

sa $-x_1 + x_2 + F_1 = 1$

$x_1 + x_2 + F_2 = 2$

$x_1, x_2, F_1, F_2 \geq 0$

Base	x_1	$x_2 \downarrow$	F1	F2	b	
Z	-1	-3				min
F1	-1	1	1		1	1→
F2	1	1		1	2	2

Base	$x_1 \downarrow$	x_2	F1	F2	b	
Z	-4		3		3	min
x_2	-1	1	1		1	---
F2	2		-1	1	1	1/2→

Base	x_1	x_2	F1	F2	b
Z			1	2	5
x_2		1	1/2	1/2	3/2
x_1	1		-1/2	1/2	1/2

Exercício 2: PL com múltiplas soluções

minimizar $Z = -x_1 - x_2$

sa $x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 4$

$-x_1 + x_2 \leq 4$

$x_1 \geq 0, x_2 \geq 0$

maximize $Z = x_1 + x_2$ (maximize $Z - x_1 - x_2 = 0$)

sa $x_1 + x_2 + F_1 = 6$

$x_1 - x_2 + F_2 = 4$

$-x_1 + x_2 + F_3 = 4$

$x_1, x_2, F_1, F_2, F_3 \geq 0$

Base	$x_1 \downarrow$	x_2	F1	F2	F3	b	
Z	-1	-1					min
F1	1	1	1			6	6
F2	1	-1		1		4	4→
F3	-1	1			1	4	---

Base	x_1	x_2	F1	F2	F3	b	
Z		-2↓		1		4	min
F1		2	1	-1		2	1→
x_1	1	-1		1		4	---
F3		0		1	1	8	---

Base	x_1	x_2	F1	F2	F3	b
Z			1	0↓		6
x_2		1	1/2	-1/2		1
x_1	1		1/2	1/2		5
F3			0	1	1	8

Exercício 3: PL com solução ilimitada

$$\text{maximizar } Z = 36x_1 + 30x_2 - 3x_3 - 4x_4$$

$$\text{sa } x_1 + x_2 - x_3 \leq 5$$

$$6x_1 + 5x_2 - x_4 \leq 10$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

$$\text{maximizar } Z = 36x_1 + 30x_2 - 3x_3 - 4x_4$$

$$\text{sa } x_1 + x_2 - x_3 + F_1 = 5$$

$$6x_1 + 5x_2 - x_4 + F_2 = 10$$

$$x_1, x_2, x_3, x_4, F_1, F_2 \geq 0$$

Base	$x_1 \downarrow$	x_2	x_3	x_4	F1	F2	b	
Z	-36	-30	3	4				min
F1	1	1	-1	0	1		5	5
F2	6	5	0	-1		1	10	5/3 →

Base	x_1	x_2	x_3	$x_4 \downarrow$	F1	F2	b	
Z		0	3	-2		6	60	min
F1		1/6	-1	1/6	1	-1/6	10/3	20 →
x_1	1	5/6	0	-1/6		1/6	5/3	---

Base	x_1	x_2	$x_3 \downarrow$	x_4	F1	F2	b
Z		2	-9		12	4	100
x_4		1	-6	1	6	-1	20
x_1	1	1	-1		1	0	5

Exercício 4: PL com solução degenerada

$$\text{minimizar } Z = -5x_1 - 2x_2$$

$$\text{sa } 2x_1 + x_2 \leq 6$$

$$4x_1 - x_2 \leq 12$$

$$x_1 \geq 0, x_2 \geq 0$$

$$\text{maximizar } -Z = 5x_1 + 2x_2$$

$$\text{sa } 2x_1 + x_2 + F_1 = 6$$

$$4x_1 - x_2 + F_2 = 12$$

$$x_1, x_2, F_1, F_2 \geq 0$$

Base	$x_1 \downarrow$	x_2	F1	F2	b	
Z	-5	-2				min
F1	2	1	1		6	3 →
F2	4	-1		1	12	3

Base	x_1	x_2	F1	F2	b
Z		1/2	5/2		15
x_1	1	1/2	1/2		3
F2		-3	-2	1	0

Exercício 5: PL com ciclagem

$$\text{maximizar } Z = 10x_1 - 57x_2 - 9x_3 - 24x_4$$

$$\text{sa } 0,5x_1 - 5,5x_2 - 2,5x_3 + 9x_4 \leq 0$$

$$0,5x_1 - 1,5x_2 - 0,5x_3 + x_4 \leq 0$$

$$x_1 \leq 1$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

$$\text{maximizar } Z - 10x_1 + 57x_2 + 9x_3 + 24x_4 = 0$$

$$\text{sa } 0,5x_1 - 5,5x_2 - 2,5x_3 + 9x_4 + F_1 = 0$$

$$0,5x_1 - 1,5x_2 - 0,5x_3 + x_4 + F_2 = 0$$

$$x_1 + F_3 = 1$$

$$x_1, x_2, x_3, x_4, F_1, F_2, F_3 \geq 0$$

<https://people.orie.cornell.edu/dpw/orie6300/Lectures/lec13.pdf>

Base	$x_1 \downarrow$	x_2	x_3	x_4	F1	F2	F3	b	
Z	-10	57	9	24					min
x_1	1/2	-11/2	-5/2	9	1			0	0 →
F2	1/2	-3/2	-1/2	1		1		0	0
F3	1						1	1	1

Base	x1	x2↓	x3	x4	F1	F2	F3	b	
Z		-53	-41	204	20			0	min
x1	1	-11	-5	18	2			0	---
F2		4	2	-8	-1	1		0	0→
F3		11	5	-18	-2		1	1	1/11

Base	x1	x2	x3↓	x4	F1	F2	F3	b	
Z			-29/2	98	27/4	53/4		0	min
x1	1		1/2	-4	-3/4	11/4		0	0→
x2		1	1/2	-2	-1/4	1/4		0	0
F3			-1/2	4	3/4	-11/4	1	1	---

Base	x1	x2	x3	x4↓	F1	F2	F3	b	
Z	29			-18	-15	93		0	min
x3	2		1	-8	-3/2	11/2		0	---
x2	-1	1		2	1/2	-5/2		0	0→
F3	1			0	0	0	1	1	---

Base	x1	x2	x3	x4	F1↓	F2	F3	b	
Z	20	9			-21/2	141/2		0	min
x3	-2	4	1		1/2	-9/2		0	0→
x4	-1/2	1/2		1	1/4	-5/4		0	0
F3	1	0			0	0	1	1	---

Base	x1	x2	x3	x4	F1	F2↓	F3	b	
Z	-22	93	21			-24		0	min
F1	-4	8	2		1	-9		0	---
x4	1/2	-3/2	-1/2	1		1		0	0→
F3	1	0	0			0	1	1	---

Base	x1	x2	x3	x4	F1	F2	F3	b
Z	-10	57	9	24				0
F1	1/2	-11/2	-5/2	9	1			0
F2	1/2	-3/2	-1/2	1		1		0
F3	1	0	0	0			1	1

PARA EVITAR CILAGEM, UTILIZE A REGRA DE BLAND

Bland's Rule: Choose the entering basic variable x_j such that j is the smallest index with $\bar{c}_j < 0$. Also choose the leaving basic variable i with the smallest index (in case of ties in the ratio test). <https://people.orie.cornell.edu/dpw/orie6300/Lectures/lec13.pdf>

Base	x1↓	x2	x3	x4	F1	F2	F3	b	
Z	-10	57	9	24					min
x1	1/2	-11/2	-5/2	9	1			0	0→
F2	1/2	-3/2	-1/2	1		1		0	0
F3	1						1	1	1

Base	x1	x2↓	x3	x4	F1	F2	F3	b	
Z		-53	-41	204	20			0	min
x1	1	-11	-5	18	2			0	---
F2		4	2	-8	-1	1		0	0→
F3		11	5	-18	-2		1	1	1/11

Base	x1	x2	x3↓	x4	F1	F2	F3	b	
Z			-29/2	98	27/4	53/4		0	min
x1	1		1/2	-4	-3/4	11/4		0	0→
x2		1	1/2	-2	-1/4	1/4		0	0
F3			-1/2	4	3/4	-11/4	1	1	---

Base	x1	x2	x3	x4↓	F1	F2	F3	b	
Z	29			-18	-15	93		0	min
x3	2		1	-8	-3/2	11/2		0	---
x2	-1	1		2	1/2	-5/2		0	0→
F3	1			0	0	0	1	1	---

Base	x1	x2	x3	x4	F1↓	F2	F3	b	
Z	20	9			-21/2	141/2		0	min
x3	-2	4	1		1/2	-9/2		0	0→
x4	-1/2	1/2		1	1/4	-5/4		0	0
F3	1	0			0	0	1	1	---

Base	x1↓	x2	x3	x4	F1	F2	F3	b	
Z	-22	93	21			-24		0	min
F1	-4	8	2		1	-9		0	---
x4	1/2	-3/2	-1/2	1		1		0	0→
F3	1	0	0			0	1	1	---

Base	x1	x2	x3↓	x4	F1	F2	F3	b	
Z		27	-1	44		20		0	min
F1		-4	-2	8	1	-1		0	---
x1	1	-3	-1	2		2		0	---
F3		3	1	-2		-2	1	1	1→

Base	x1	x2	x3	x4	F1	F2	F3	b
Z		30		42		18	1	1
F1		2		4	1	-5	2	2
x1	1	0		0		0	1	1
x3		3	1	-2		-2	1	1